

## CHAPTER 1 □ INTRODUCTION

In many problems of wave propagation and scattering, the medium in which a wave travels can be classified as being either deterministic or random. A great number of investigations have been made on deterministic problems, including various antennas, diffraction and scattering, and guided waves, among others. In contrast to these problems, many natural and biological media are randomly varying in time and space. Thus the waves in such a medium vary randomly in amplitude and phase and must be described in terms of statistical averages and probability densities. In this book, we address ourselves to the question of how a wave interacts with a random medium, how one formulates and expresses mathematically these interactions, and how one obtains solutions for a variety of practical problems.

This treatise is divided into five parts. Volume 1 consists of Parts I and II. Part I (Chapters 2–6) deals with the scattering and propagation of waves in a tenuous distribution of scatterers. Here the single scattering theory and its slight extension are used to explain the fundamentals of wave fluctuations in random media without undue mathematical complexities. This theory also covers many practical problems of wave propagation and scattering in the atmosphere, the ocean, and other random media. Part II (Chapters 7–13) deals with the transport theory, which is also called the theory of radiative transfer. Volume 2 contains Parts III–V. Part III (Chapters 14 and 15) deals with the theory of multiple scattering of waves in randomly distributed scatterers. Part IV (Chapters 16–20) covers weak and strong fluctuation theories in random continuum and turbulence. Part V (Chapters 21 and 22) presents rough surface scattering and remote sensing of random media.

Random media may be grouped into three categories: random scatterers, random continua, and rough surfaces. Random scatterers are a random distribution of many discrete scatterers. Examples of this are rain, fog, smog, hail, aerosols, particles in the ocean, blood cells, polymers, and molecules. Wave propagation and scattering analyses in such random media can be made in two steps. First, we consider the scattering and absorption

characteristics of a single scatterer, and second, we consider the characteristics of a wave when many scatterers are distributed randomly.

The first step is discussed in Chapter 2. It starts with the definitions of scattering amplitude, and absorption and scattering cross sections. Rayleigh scattering, Rayleigh–Debye (Born), WKB approximations, and the Mie theory are summarized. A discussion of polarization effects and the Stokes parameters is included. A brief description of acoustic wave formulations is also given.

Chapter 3 gives examples of scattering and absorption characteristics taken from various practical problems. Particle characteristics in the atmosphere and the ocean are discussed, including those of aerosols, hydrometeors, bubbles, and fish. Also included are optical and acoustic characteristics of red blood cells.

When particle density is low, multiple scattering effects can be neglected and the single scattering theory is adequate. Chapters 4–6 are devoted to the use of the single scattering theory in solving a variety of practical problems. The scattering problems are discussed in Chapter 4 and the problem of pulse propagation and scattering is covered in Chapter 5. Here we develop a fundamental formulation of pulse propagation and scattering in a time-varying random medium. Chapter 6 deals with the fluctuations of a wave in scatterers for a line-of-sight problem.

Chapters 7–13 deal with transport theory and Chapters 14 and 15 present multiple scattering theory. Historically, the problem of wave propagation in random scatterers has been investigated from two distinct points of view. One is “radiative transfer theory” or “transport theory,” and the other is “multiple scattering theory.” Radiative transfer theory deals with the propagation of intensities. It is based on phenomenological and heuristic observations of the transport characteristics of intensities and was initiated by Schuster in 1905 in his study of radiation in foggy atmospheres. The basic differential equation is called the equation of transfer and is equivalent to Boltzmann’s equation in the kinetic theory of gases and in neutron transport theory. It has been successfully employed for the problems of atmospheric and underwater visibility, marine biology, photographic emulsions, and the propagation of radiant energy in the atmospheres of planets, stars, and galaxies. Chapter 7 gives the definitions of fundamental quantities such as specific intensities, fluxes, and energy densities. The differential and integral equations governing these quantities are derived, and general boundary conditions and power conservation relationships are presented.

Exact general solutions of the equations developed in Chapter 7 have not yet been obtained. However, there are some special cases for which simple and useful approximate solutions are available. Chapters 8 and 9

discuss the two such limiting cases of tenuous and dense distributions. For tenuous distributions, the first order multiple scattering theory can be used and for dense distribution, the diffusion approximation is appropriate. These two techniques are applicable to a variety of practical problems. Examples are microwave and optical propagation and scattering through aerosols and hydrometeors, optical scattering from bacteria, and optical diffusion in blood.

Chapters 10 and 11 discuss the case of wave propagation through a medium bounded by parallel planes. This simple geometry represents many physical situations such as planetary atmospheres, layers in the ocean, a slab of blood, or a thin layer of paint, and allows reasonably simple solutions. Chapter 10 discusses two and four flux theory which has been used extensively in the past. This method was developed heuristically by Kubelka and Munk in 1931. In this chapter we include a derivation of this theory from the radiative transfer theory. This should provide a new theoretical basis for two and four flux theory. Chapter 11 presents a theory utilizing many fluxes and the formulations suitable for computer applications. It also includes a general formalism for the solution in many-layered media.

Chapters 12 and 13 present other limiting cases of small and large particles. If the particle size is small compared with a wavelength, the scattering is almost isotropic (uniform in all directions) except for a dipole pattern in the electromagnetic case. If the scattering is isotropic and therefore the scattering amplitude is constant, considerable mathematical simplifications result. These simplifications are covered in Chapter 12 and show clearly the relationship with the diffusion approximation discussed in Chapter 9. Chapter 13 presents an approximation resulting from large particles. If the particle size is large compared with a wavelength, most of the scattering is in the forward direction, and the equation of transfer can be solved exactly by a Fourier transform technique. This is applicable to many problems in which optical beams propagate in water or the atmosphere.

In contrast to the transport theory, analytical theory (or multiple scattering theory) starts with a wave equation, obtains solutions for a single particle, introduces the interaction effects of many particles, and then considers statistical averages. One of the most useful multiple scattering theories was developed by Twersky. A detailed derivation of his theory is given in Chapter 14. The relationships between transport theory and Twersky's multiple scattering theory are also given in this chapter.

Chapter 15 deals with the problem of wave fluctuations and pulse propagation in random scatterers. This is an important problem in communication and remote sensing. However, very few studies on this problem have been reported because of the relative complexities involved. In this

chapter an attempt has been made to present an introduction to the basic formulations for wave fluctuations and pulse propagation in random scatterers. Included are the effects of scatterers on coherence bandwidth, pulse shape, and image resolution.

“Random continuum” means the medium whose index of refraction varies randomly and continuously in time and space. Examples are tropospheric and ionospheric turbulence, planetary atmospheres, solar corona, turbulence in water, turbulent wakes and plumes of aircraft and rocket engines, clear air turbulence, and biological media. If at a given point in the random medium the randomness is smaller than a certain level, the fluctuation of a wave is small. This situation is usually referred to as the “weak fluctuation” approximation. If the randomness is large, it results in the so-called strong fluctuation. For weak fluctuation, a number of simplifications and approximations are possible. Chapters 16–19 are devoted to this case.

Chapter 16 deals with the scattering of continuous waves and pulses from a volume of random medium. It includes the effects of motion of the medium, two-frequency correlation functions, coherence bandwidth, and coherence time, all of which are becoming increasingly important in communication studies.

Chapter 17 is devoted to the problem of line-of-sight propagation through a random medium. It is applicable to microwave and optical propagation in the atmosphere. The formulation is based on the spectral representation of wave fluctuation and refractive index fluctuation. Spatial and spectral filter functions are defined, and the effects of variations of the random medium along the propagation path are discussed.

Chapter 18 deals with spherical and beam wave propagation. Spherical wave representation is necessary for most microwave propagation problems. Beam wave formulation is required for optical and millimeter wave propagation in random media. Temporal fluctuations and spectra of a wave in a time-varying random medium are discussed in Chapter 19. They are applicable to the remote sensing of the atmospheres of planets and solar corona, and are also useful for the remote determination of wind velocity.

Chapter 20 is devoted to the difficult problem of strong fluctuations. Intense research efforts are being conducted on this problem in the United States, the Soviet Union, Japan, the Netherlands, and other countries. This chapter attempts to provide an introduction and a cohesive overview of the strong fluctuation theory. A detailed exposition on the mutual coherence function in a random medium, temporal frequency spectra, and two-frequency correlation functions is given. Included also are the intensity fluctuation, the thin screen theory, the modulation transfer function of the random medium, and adaptive optics.

Chapter 21 presents an introduction to the theory of scattering by rough surfaces. Two major approaches, the perturbation technique and the Kirchhoff method, are discussed along with a summary of recent advances on ocean wave scattering. The concept of scattering cross section per unit area of a rough surface is presented for these two approximations. Coherent and incoherent (diffuse) intensities and their temporal frequency spectra due to the motion of rough surfaces are discussed.

Remote sensing of meteorological parameters such as wind velocity and the strength of turbulence by wave propagation and scattering techniques has become increasingly important in recent years, and several new remote-sensing techniques have recently been proposed. Chapter 22 presents a current account of research on this interesting and important topic. Included are the three common inversion techniques: regularization method, statistical inversion technique, and the Backus–Gilbert inversion technique.

The appendixes give a summary of useful formulas for spectral representations of random functions. They also include a brief account of turbulence characteristics and refractive index fluctuations.