Chapter 1

Signing On with Signed Numbers

In This Chapter

- Using the number line
- Trying absolute value
- Operating on signed numbers: adding, subtracting, multiplying, and dividing

In this chapter you practice the operations on signed numbers and figure out how to make them behave the way you want them to. (Just tell your mother she can't use this chapter on your little brother to make him behave.) The properties are very helpful in making math expressions easier to read and to handle when solving equations in algebra.

Comparing Numbers on the Number Line

You may think that identifying that 16 is bigger than 10 is an easy concept. But what about –16 and –10? Which is bigger?

The easiest way to compare numbers and to tell which is bigger or has a greater value is to find their position on the number line. The number line goes from negatives on the left to positives on the right (see Figure 1-1). Whichever number is farther to the right has the greater value — it's bigger.

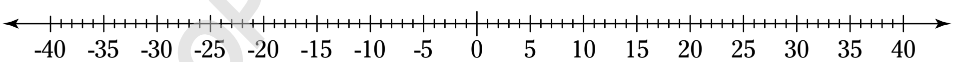


Figure 1-1: A number line.

Q. Using the number line in Figure 1-1, determine which is larger, –16 or –10.

A. **–10**. The number –10 is to the right of –16, so it's the bigger of the two numbers. You write that as $-10 > -16$ (read this as "negative 10 is greater than negative 16"). Or you can write it as $-16 < -10$ (negative 16 is less than negative 10).

Q. Which is larger, –.0023 or –.023?

A. **–.0023**. The number –.0023 is to the right of –.023 so it's larger.

1. Which is larger, –2 or –8?

Solve It

2. Which has the greater value, 0 or –1?

Solve It

3. Which is bigger, –.003 or –.03?

Solve It

4. Which is larger, $-\frac{1}{6}$ or $-\frac{2}{3}$?

Solve It

Absolutely Right — Writing Absolute Value

REMEMBER

The *absolute value* of a number, written $|a|$, is an operation that evaluates whatever is between the vertical bars and then outputs a positive number. Another way of looking at this operation is it can tell you how far a number is from 0 on the number line — with no reference to which side.

The absolute value of a

$|a| = a$, if a is a positive number ($a > 0$) or if $a = 0$

$|a| = -a$, if a is a negative number ($a < 0$). Read this as "The absolute value of a is equal to the *opposite* of a."

EXAMPLE

Q. $|4| =$

A. **4**

Q. $|-3| =$

A. **3**

5. $|8| =$

Solve It

6. $|-6| =$

Solve It

7. $-|-6| =$

Solve It

8. $-|8| =$

Solve It

Adding Signed Numbers

Adding signed numbers involves two different rules.

- You use one when the signs of the two numbers are the same — both positive or both negative.
- You use the other when the two numbers' signs are different.

After you determine whether the signs are the same or different, then you use the absolute values of the numbers.

REMEMBER

To add signed numbers

If the signs are the same, add the absolute values of the two numbers together, and let their common sign be the sign of the answer.

$(+a) + (+b) = +(a + b)$

$(-a) + (-b) = -(a + b)$

If the signs are different, then find the difference between the absolute values of the two numbers (subtract the smaller absolute value from the larger), and let the answer have the sign of the number with the larger absolute value. Assume that $|a| > |b|$.

$(+a) + (-b) = +(a - b)$

$(-a) + (+b) = -(a - b)$

EXAMPLE

Q. $(-6) + (-4) = -(6 + 4) =$

A. **–10**

Q. $(+8) + (-15) = -(15 - 8) =$

A. **–7**

9. $4 + (-3) =$

Solve It

10. $5 + (-11) =$

Solve It

11. $(-18) + (-5) =$

Solve It

12. $47 + (-33) =$

Solve It

13. $(-3) + 5 + (-2) =$

Solve It

14. $(-4) + (-6) + (-10) =$

Solve It

15. $5 + (-18) + (10) =$

Solve It

16. $(-4) + 4 + (-5) + 5 + (-6) =$

Solve It

Making a Difference with Signed Numbers

You really don't use a new set of rules for subtracting signed numbers. You just change the subtraction problem to an addition problem and use the rules for addition of signed numbers. To ensure that your answer to this new addition problem is the answer to the subtraction problem, you not only change the operation from subtraction to addition, but you also change the sign of the second number — the one that's being subtracted.

To subtract two signed numbers

$$a - (+b) = a + (-b)$$

$$a - (-b) = a + (+b)$$

Q. $(-8) - (-5)$

A. **–3**

Q. What's the average annual rainfall for Scottsdale, Arizona?

A. **7.05 inches**. Huh? Where'd that come from, you're wondering? I just wanted to keep you on your toes! Keep this information in mind the next time you plan a trip to Arizona.

17. $5 - (-2) =$

Solve It

18. $-6 - (-8) =$

Solve It

19. $4 - 87 =$

Solve It

20. $0 - (-15) =$

Solve It

21. $2.4 - (-6.8) =$

Solve It

22. $-15 - (-11) =$

Solve It

Multiplying Signed Numbers

When you multiply two expressions with the same sign, the product is positive; when the two expressions have different signs, the product is negative. If you're multiplying more than two factors together, just count the number of negative signs in the problem. If the number of negative signs is an even number, then the answer is positive. If the number of negative signs is odd, then the answer is negative.

REMEMBER

The product of two signed numbers

$(+)(+) = +$

$(-)(-) = +$

$(+)(-) = -$

$(-)(+) = -$

The product of more than two signed numbers

$(+)(+)(+)(-)(-)(-)(-)$ has a *positive* answer for an *even* number of negative factors.

$(+)(+)(+)(-)(-)(-)$ has a *negative* answer for an *odd* number of negative factors.

EXAMPLE

Q. $(-2)(-3) =$

A. **+6**

Q. $(-2)(+3) =$

A. **–6**

23. $(-6)(3) =$

Solve It

24. $(14)(-1) =$

Solve It

25. $(-6)(-3) =$

Solve It

26. $(6)(-3)(4)(-2) =$

Solve It

27. $(-1)(-1)(-1)(-1)(-1)(2) =$

Solve It

28. $(-10)(2)(3)(1)(-1) =$

Solve It

Dividing Signed Numbers

The rules for dividing signed numbers are exactly the same as those for multiplying signed numbers — as far as the sign goes. (See “Multiplying Signed Numbers” earlier in this chapter.) They differ though because you have to divide, of course.

REMEMBER

When dividing signed numbers, just count the number of negative signs that are in the problem — in the numerator, the denominator, and perhaps in front of the problem. If you have an even number of negative signs, the answer is positive. If you have an odd number of negative signs, the answer is negative. This rule works only when factors are multiplied and divided, not added or subtracted.

EXAMPLE

Q. $-36/-9 =$

A. **+4**

Q. $\dfrac{-(-3)(-12)}{4} =$

A. **–9**

29. $-22/-11 =$

Solve It

30. $24/-3 =$

Solve It

31. $\dfrac{-3(-4)}{-2} =$

Solve It

32. $\dfrac{(-5)(2)(3)}{-1} =$

Solve It

33. $\dfrac{(-2)(-3)(-4)}{(-1)(-6)} =$

Solve It

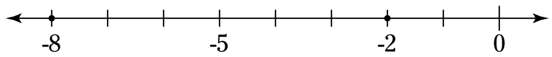
34. $(-1)/(-1) =$

Solve It

Answers to Problems on Signed Numbers

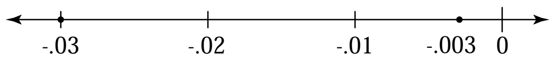
This section provides the answers (in bold) to the practice problems in this chapter.

1 Which is larger, –2 or –8? **–2 is larger**. The following number line shows that the number –2 is to the right of –8. So –2 is bigger than –8 (or $-2 > -8$).

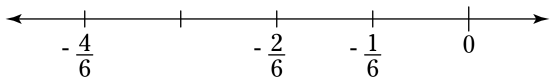


2 Which has the greater value, 0 or –1? **0 is greater**. The number 0 is to the right of –1. So 0 has a greater value than –1 (or $0 > -1$).

3 Which is bigger, –.003 or –.03? **–.003 is bigger**. The following number line shows that the number –.003 is to the right of –.03, which means –.003 is bigger than –.03 (or $-.003 > -.03$).



4 Which is larger, $-\frac{1}{6}$ or $-\frac{2}{3}$? **$-\frac{1}{6}$ is larger**. The number $-\frac{2}{3} = -\frac{4}{6}$, and $-\frac{4}{6}$ is to the left of $-\frac{1}{6}$ on the following number line. So $-\frac{1}{6}$ is larger than $-\frac{2}{3}$ (or $-\frac{1}{6} > -\frac{2}{3}$).



5 $|8| =$ **8** because $8 > 0$.

6 $|-6| =$ **6** because $-6 < 0$ and 6 is the opposite of –6.

7 $-|-6| =$ **–6** because $|-6| = 6$ as in the previous problem.

8 $-|8| =$ **–8** because $|8| = 8$.

9 $4 + (-3) =$ **1** because 4 is the greater absolute value.

$4 + (-3) = +(4 - 3) = 1$

10 $5 + (-11) =$ **–6** because –11 has the greater absolute value of 11.

$5 + (-11) = -(11 - 5) = -6$

11 $(-18) + (-5) =$ **–23** because both of the numbers have negative signs; when the signs are the same, find the sum of their absolute values. $(-18) + (-5) = -(18 + 5) = -23$

12 $47 + (-33) =$ **14** because 47 has the greater absolute value. $47 + (-33) = +(47 - 33) = 14$

13 $(-3) + 5 + (-2) =$ **0**

$(-3) + 5 + (-2) = ((-3) + 5) + (-2) = (2) + (-2) = 0$

14 $(-4) + (-6) + (-10) =$ **–20**

$(-4) + (-6) + (-10) = -(4 + 6) + (-10) = (-10) + (-10) = -(10 + 10) = -20$

15 $5 + (-18) + (10) = \mathbf{-3}$

$5 + (-18) + (10) = -(18 - 5) + 10 = -(13) + 10 = -(13 - 10) = -3$

Or you may prefer to add the two numbers with the same sign, first, like this: $5 + (-18) + (10) = (5 + 10) + (-18) = 15 + (-18) = -(18 - 15) = -3$

16 $(-4) + 4 + (-5) + 5 + (-6) = \mathbf{-6}$

$(-4)+4+(-5)+5+(-6) = \big((-4)+4\big)+\big((-5)+5\big)+(-6) = 0+0+(-6) = -6$

17 $5 - (-2) = \mathbf{7}$

$5 - (-2) = 5 + (+2) = 7$

18 $-6 - (-8) = \mathbf{2}$

$-6 - (-8) = -6 + (+8) = 8 - 6 = 2$

19 $4 - 87 = \mathbf{-83}$

$4 - 87 = -(87 - 4) = -83$

20 $0 - (-15) = \mathbf{15}$

$0 - (-15) = 0 + 15 = 15$

21 $2.4 - (-6.8) = \mathbf{9.2}$

$2.4 - (-6.8) = 2.4 + 6.8 = 9.2$

22 $-15 - (-11) = \mathbf{-4}$

$-15 - (-11) = -15 + 11 = -(15 - 11) = -4$

23 $(-6)(3) = \mathbf{-18}$ because the multiplication problem has one negative, and one is an odd number.

24 $(14)(-1) = \mathbf{-14}$ because the multiplication problem has one negative, and one is an odd number.

25 $(-6)(-3) = \mathbf{18}$ because the multiplication problem has two negatives, and two is even.

26 $(6)(-3)(4)(-2) = \mathbf{144}$ because the multiplication problem has two negatives.

27 $(-1)(-1)(-1)(-1)(-1)(2) = \mathbf{-2}$ because the multiplication problem has five negatives.

28 $(-10)(2)(3)(1)(-1) = \mathbf{60}$ because the multiplication problem has two negatives.

29 $^{-22}/_{-11} = \mathbf{2}$ because the division problem has two negatives, and two is an even number.

30 $^{24}/_{-3} = \mathbf{-8}$ because the division problem has one negative, and one is odd.

31 $\frac{-3(-4)}{-2} = \mathbf{-6}$ because three negatives result in a negative.

32 $\frac{(-5)(2)(3)}{-1} = \mathbf{30}$ because the division problem has two negatives.

33 $\frac{(-2)(-3)(-4)}{(-1)(-6)} = \mathbf{-4}$ because the division problem has five negatives.

34 $\frac{(-1)}{(-1)} = \mathbf{1}$ because the division problem has two negatives.