Contents

Preface **xi**

1 WHY EXTREME VALUE THEORY? 1

1.1 A Simple Extreme Value Problem . . . 1
1.2 Graphical Tools for Data Analysis . . . 3
1.2.1 Quantile-quantile plots . . . 3
1.2.2 Excess plots . . . 14
1.3 Domains of Applications . . . 19
1.3.1 Hydrology . . . 19
1.3.2 Environmental research and meteorology . . . 21
1.3.3 Insurance applications . . . 24
1.3.4 Finance applications . . . 31
1.3.5 Geology and seismic analysis . . . 32
1.3.6 Metallurgy . . . 40
1.3.7 Miscellaneous applications . . . 42
1.4 Conclusion . . . 42

2 THE PROBABILISTIC SIDE OF EXTREME VALUE THEORY 45

2.1 The Possible Limits . . . 46
2.2 An Example . . . 51
2.3 The Fréchet-Pareto Case: $\gamma > 0$ . . . 56
2.3.1 The domain of attraction condition . . . 56
2.3.2 Condition on the underlying distribution . . . 57
2.3.3 The historical approach . . . 58
2.3.4 Examples . . . 58
2.3.5 Fitting data from a Pareto-type distribution . . . 61
2.4 The (Extremal) Weibull Case: $\gamma < 0$ . . . 65
2.4.1 The domain of attraction condition . . . 65
2.4.2 Condition on the underlying distribution . . . 67
2.4.3 The historical approach . . . 67
2.4.4 Examples . . . 67

2.5 The Gumbel Case: $\gamma = 0$ 69
2.5.1 The domain of attraction condition 69
2.5.2 Condition on the underlying distribution 72
2.5.3 The historical approach and examples 72
2.6 Alternative Conditions for $(\mathcal{C}_\gamma)$ 73
2.7 Further on the Historical Approach 75
2.8 Summary 76
2.9 Background Information 76
2.9.1 Inverse of a distribution 77
2.9.2 Functions of regular variation 77
2.9.3 Relation between F and U 79
2.9.4 Proofs for section 2.6 80

3 AWAY FROM THE MAXIMUM 83
3.1 Introduction 83
3.2 Order Statistics Close to the Maximum 84
3.3 Second-order Theory 90
3.3.1 Remainder in terms of U 90
3.3.2 Examples 92
3.3.3 Remainder in terms of F 93
3.4 Mathematical Derivations 94
3.4.1 Proof of (3.6) 95
3.4.2 Proof of (3.8) 96
3.4.3 Solution of (3.15) 97
3.4.4 Solution of (3.18) 98

4 TAIL ESTIMATION UNDER PARETO-TYPE MODELS 99
4.1 A Naive Approach 100
4.2 The Hill Estimator 101
4.2.1 Construction 101
4.2.2 Properties 104
4.3 Other Regression Estimators 107
4.4 A Representation for Log-spacings and Asymptotic Results 109
4.5 Reducing the Bias 113
4.5.1 The quantile view 113
4.5.2 The probability view 117
4.6 Extreme Quantiles and Small Exceedance Probabilities 119
4.6.1 First-order estimation of quantiles and return periods . . . 119
4.6.2 Second-order refinements 121
4.7 Adaptive Selection of the Tail Sample Fraction 123

5 TAIL ESTIMATION FOR ALL DOMAINS OF ATTRACTION 131
5.1 The Method of Block Maxima 132
5.1.1 The basic model 132
5.1.2 Parameter estimation 132

5.1.3 Estimation of extreme quantiles 135
5.1.4 Inference: confidence intervals 137
5.2 Quantile View—Methods Based on $(\mathcal{C}_\gamma)$ 140
5.2.1 Pickands estimator 140
5.2.2 The moment estimator 142
5.2.3 Estimators based on the generalized quantile plot 143
5.3 Tail Probability View—Peaks-Over-Threshold Method 147
5.3.1 The basic model 147
5.3.2 Parameter estimation 149
5.4 Estimators Based on an Exponential Regression Model 155
5.5 Extreme Tail Probability, Large Quantile and Endpoint Estimation Using Threshold Methods 156
5.5.1 The quantile view 156
5.5.2 The probability view 158
5.5.3 Inference: confidence intervals 159
5.6 Asymptotic Results Under $(\mathcal{C}_\gamma)$-$(\mathcal{C}_\gamma^*)$ 160
5.7 Reducing the Bias 165
5.7.1 The quantile view 165
5.7.2 Extreme quantiles and small exceedance probabilities 167
5.8 Adaptive Selection of the Tail Sample Fraction 167
5.9 Appendices 169
5.9.1 Information matrix for the GEV 169
5.9.2 Point processes 169
5.9.3 GRV_2 functions with $\rho < 0$ 171
5.9.4 Asymptotic mean squared errors 172
5.9.5 AMSE optimal k-values 173

6 CASE STUDIES 177
6.1 The Condroz Data 177
6.2 The Secura Belgian Re Data 188
6.2.1 The non-parametric approach 189
6.2.2 Pareto-type modelling 191
6.2.3 Alternative extreme value methods 195
6.2.4 Mixture modelling of claim sizes 198
6.3 Earthquake Data 200

7 REGRESSION ANALYSIS 209
7.1 Introduction 210
7.2 The Method of Block Maxima 211
7.2.1 Model description 211
7.2.2 Maximum likelihood estimation 212
7.2.3 Goodness-of-fit 213
7.2.4 Estimation of extreme conditional quantiles 216

7.3 The Quantile View—Methods Based on Exponential Regression Models . . . 218
7.3.1 Model description . . . 218
7.3.2 Maximum likelihood estimation . . . 219
7.3.3 Goodness-of-fit . . . 222
7.3.4 Estimation of extreme conditional quantiles . . . 223
7.4 The Tail Probability View—Peaks Over Threshold (POT) Method . . . 225
7.4.1 Model description . . . 225
7.4.2 Maximum likelihood estimation . . . 226
7.4.3 Goodness-of-fit . . . 229
7.4.4 Estimation of extreme conditional quantiles . . . 231
7.5 Non-parametric Estimation . . . 233
7.5.1 Maximum penalized likelihood estimation . . . 234
7.5.2 Local polynomial maximum likelihood estimation . . . 238
7.6 Case Study . . . 241

8 MULTIVARIATE EXTREME VALUE THEORY 251
8.1 Introduction . . . 251
8.2 Multivariate Extreme Value Distributions . . . 254
8.2.1 Max-stability and max-infinite divisibility . . . 254
8.2.2 Exponent measure . . . 255
8.2.3 Spectral measure . . . 258
8.2.4 Properties of max-stable distributions . . . 265
8.2.5 Bivariate case . . . 267
8.2.6 Other choices for the margins . . . 271
8.2.7 Summary measures for extremal dependence . . . 273
8.3 The Domain of Attraction . . . 275
8.3.1 General conditions . . . 276
8.3.2 Convergence of the dependence structure . . . 281
8.4 Additional Topics . . . 287
8.5 Summary . . . 290
8.6 Appendix . . . 292
8.6.1 Computing spectral densities . . . 292
8.6.2 Representations of extreme value distributions . . . 293

9 STATISTICS OF MULTIVARIATE EXTREMES 297
9.1 Introduction . . . 297
9.2 Parametric Models . . . 300
9.2.1 Model construction methods . . . 300
9.2.2 Some parametric models . . . 304
9.3 Component-wise Maxima . . . 313
9.3.1 Non-parametric estimation . . . 314
9.3.2 Parametric estimation . . . 318
9.3.3 Data example . . . 321

9.4 Excesses over a Threshold 325
9.4.1 Non-parametric estimation 326
9.4.2 Parametric estimation 333
9.4.3 Data example 338
9.5 Asymptotic Independence 342
9.5.1 Coefficients of extremal dependence 343
9.5.2 Estimating the coefficient of tail dependence 350
9.5.3 Joint tail modelling 354
9.6 Additional Topics 365
9.7 Summary 366

10 EXTREMES OF STATIONARY TIME SERIES 369
10.1 Introduction 369
10.2 The Sample Maximum 371
10.2.1 The extremal limit theorem 371
10.2.2 Data example 375
10.2.3 The extremal index 376
10.3 Point-Process Models 382
10.3.1 Clusters of extreme values 382
10.3.2 Cluster statistics 386
10.3.3 Excesses over threshold 387
10.3.4 Statistical applications 389
10.3.5 Data example 395
10.3.6 Additional topics 399
10.4 Markov-Chain Models 401
10.4.1 The tail chain 401
10.4.2 Extremal index 405
10.4.3 Cluster statistics 406
10.4.4 Statistical applications 407
10.4.5 Fitting the Markov chain 408
10.4.6 Additional topics 411
10.4.7 Data example 413
10.5 Multivariate Stationary Processes 419
10.5.1 The extremal limit theorem 419
10.5.2 The multivariate extremal index 421
10.5.3 Further reading 424
10.6 Additional Topics 425

11 BAYESIAN METHODOLOGY IN EXTREME VALUE STATISTICS 429
11.1 Introduction 429
11.2 The Bayes Approach 430
11.3 Prior Elicitation 431
11.4 Bayesian Computation 433
11.5 Univariate Inference 434

11.5.1 Inference based on block maxima 434
11.5.2 Inference for Fréchet-Pareto-type models 435
11.5.3 Inference for all domains of attractions 445
11.6 An Environmental Application 452

Bibliography **461**

Author Index **479**

Subject Index **485**