

## 1.1 CLAIMS PROCESS

In this book we consider the claims reserving problem for a branch of insurance products known in continental Europe as **non-life insurance**, in the UK as **General Insurance** and in the USA as **Property and Casualty Insurance**. This branch usually contains all kinds of insurance products except life insurance. This separation is mainly for two reasons:

1. Life insurance products are rather different from non-life insurance contracts, for example, terms of contracts, type of claims, risk drivers, etc. This implies that life and non-life products are modeled rather differently.
2. In many countries, for example, Switzerland and Germany, there is a strict legal separation between life insurance and non-life insurance products. This means that a company dealing with non-life insurance products is not allowed to sell life insurance products, and a life insurance company can only sell life-related and disability insurance products. Hence, for example, every Swiss insurance company that sells both life and non-life insurance products, has at least two legal entities.

The non-life insurance branch operates on the following lines of business (LoB):

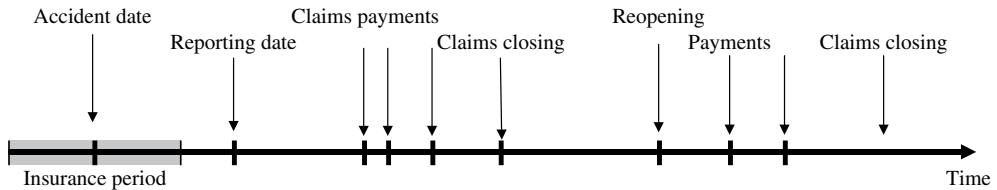
- motor/car insurance (motor third party liability, motor hull);
- property insurance (private and commercial property against fire, water, flooding, business interruption, etc.);
- liability insurance (private and commercial liability including director and officers (D&O) liability insurance);
- accident insurance (personal and collective accident including compulsory accident insurance and workers' compensation);
- health insurance (private personal and collective health);
- marine insurance (including transportation); and
- other insurance products such as aviation, travel insurance, legal protection, credit insurance, epidemic insurance, etc.

A non-life insurance policy is a contract among two parties: the insurer and the insured. It provides the insurer with a fixed/deterministic amount of money (called premium) and the insured with a financial coverage against the random occurrence of well-specified events (or at least a promise that he or she gets a well-defined amount in case such an event happens). The right of the insured to these amounts (in case the event happens) constitutes a **claim** by the insured to the insurer.

The amount which the insurer is obligated to pay in respect of a claim is known as the **claim amount** or the **loss amount**. The payments that make up this claim are known as

- claims payments
- loss payments
- paid claims, or
- paid losses.

The history of a typical non-life insurance claim may take the form shown in Figure 1.1.



**Figure 1.1** Typical time line of a non-life insurance claim

Usually, the insurance company is unable to settle a claim immediately, for three main reasons:

1. There is a reporting delay (time-lag between claims occurrence and claims reporting to the insurer). The reporting of a claim can take several years, especially in liability insurance (e.g. asbestos or environmental pollution claims) – see Example 1.1 and Table 1.1 below.
2. After being reported to the insurer, several years may elapse before the claim is finally settled. For instance, in property insurance we usually have a relatively fast settlement, whereas in liability or bodily injury claims it often takes a long time before the total circumstances of the claim are clear and known (and can be settled).
3. It can also happen that a closed claim needs to be reopened due to (unexpected) new developments, or if a relapse occurs.

### 1.1.1 Accounting Principles and Accident Years

There are three different premium accounting principles:

1. Premium booked
2. Premium written
3. Premium earned.

The choice of principle depends on the kind of business written. Without loss of generality, in this book, we concentrate on the **premium earned** principle.

Usually, an insurance company closes its books at least once a year. Let us assume that we always close our book on 31 December. How should we show a one-year contract that was written on 1 October 2008 with two premium instalments paid on 1 October 2008 and 1 April 2009?

Let us assume that

- premium written 2008 = 100
- premium booked 2008 = 50 (= premium received in 2008)
- pipeline premium 31.12.2008 = 50 (= premium that will be received in 2009, receivables), which gives a premium booked 2009 = 50.

If we assume that the risk exposure is distributed uniformly over time (*pro rata temporis*), then

- premium earned 2008 = 25 (= premium used for exposure in 2008)
- unearned premium reserve (UPR) 31.12.2008 = 75 (= premium that will be used for exposure in 2009), which gives a premium earned 2009 = 75.

If the exposure is not *pro rata temporis*, then, of course, we have a different split of the premium earned into the different accounting years. In order to have a consistent financial statement, it is important that the accident date and the premium accounting principle are compatible (via the exposure pattern). Hence, all claims that have an accident year of 2008 have to be matched to the premium earned 2008, that is, the claims 2008 have to be paid by the premium earned 2008, whereas the claims with an accident year after 2008 have to be paid by the unearned premium reserve (UPR) 31.12.2008.

Hence, on the one hand, we have to build premium reserves for future exposures and on the other, we need to build claims reserves for unsettled claims of past exposures. There are two different types of claims reserves for past exposures:

1. IBNyR reserves (Incurred But Not yet Reported): We need to build claims reserves for claims that have occurred before 31.12.2008, but have not been reported by the end of the year (i.e. the reporting delay laps into the next accounting years).
2. IBNeR reserves (Incurred But Not enough Reported): We need to build claims reserves for claims that have been reported before 31.12.2008, but have not yet been settled – that is, we still expect payments in the future, which need to be financed by the already earned premium.

This means that the claims payments are directly linked to the insurance premium via the exposure pattern. This link determines the building of provisions and reserves for the claims settlement. Other insurance structures are found, for example, in social insurance where one typically pays claims with the so-called pay-as-you-go system. Such systems require a different legal framework – namely, one has to ensure that the current premium income is at least sufficient to cover present claims payments. This is typically done by legal compulsory insurance, but in this text we do not consider such systems.

### 1.1.2 Inflation

The following subsection on inflation follows Taylor (2000).

Claims costs are often subject to inflation. This is seldom related to the typical salary or price inflation, but is very specific to the chosen LoB. For example, in the accident LoB, inflation is driven by medical inflation; whereas, for the motor hull LoB inflation is driven by the complexity of car repairing techniques. The essential point is that claims inflation may continue beyond the occurrence date of the accident up to the point of its final payment or settlement.

**Example 1.1 (reporting delay, IBNyR claims)****Table 1.1** Claims development triangle for number of IBNyR cases (Taylor 2000)

Accident year $i$	Number of reported claims, non-cumulative according to reporting delay reporting year $j$										
	0	1	2	3	4	5	6	7	8	9	10
0	368	191	28	8	6	5	3	1	0	0	1
1	393	151	25	6	4	5	4	1	2	1	0
2	517	185	29	17	11	10	8	1	0	0	1
3	578	254	49	22	17	6	3	0	1	0	0
4	622	206	39	16	3	7	0	1	0	0	0
5	660	243	28	12	12	4	4	1	0	0	0
6	666	234	53	10	8	4	6	1	0	0	0
7	573	266	62	12	5	7	6	5	1	0	1
8	582	281	32	27	12	13	6	2	1	0	
9	545	220	43	18	12	9	5	2	0		
10	509	266	49	22	15	4	8	0			
11	589	210	29	17	12	4	9				
12	564	196	23	12	9	5					
13	607	203	29	9	7						
14	674	169	20	12							
15	619	190	41								
16	660	161									
17	660										

□

If  $(X_{t_i})_{i \in \mathbb{N}}$  denote the positive single claims payments at times  $t_i$  expressed in money value at time  $t_1$  ( $t_k \geq t_l$  for  $k \geq l$ ), then the total claim amount is, in money value at time  $t_1$ , given by

$$C_1 = \sum_{i=1}^{\infty} X_{t_i}$$

If  $\lambda(\cdot)$  denotes the index that measures the claims inflation, the actual (nominal) claims amount is

$$C = \sum_{i=1}^{\infty} \frac{\lambda(t_i)}{\lambda(t_1)} X_{t_i}$$

Whenever  $\lambda(\cdot)$  is an increasing function, we observe that  $C$  is larger than  $C_1$ . Of course, in practice we only observe the unindexed payments  $[\lambda(t_i)/\lambda(t_1)] X_{t_i}$  and, in general, it is difficult to estimate an index function such that we can obtain indexed values  $X_{t_i}$ . Finding an index function  $\lambda(\cdot)$  is equivalent to defining appropriate deflators  $\varphi$ , which is a well-known concept in market consistent actuarial valuation; see, for example, Wüthrich *et al.* (2008).

The basic idea behind indexed values  $C_1$  is that, if two sets of payments relate to identical circumstances except that there is a time translation in the payment, their indexed values will be the same, whereas the unindexed values will differ. For  $c > 0$  we assume that we have a second cashflow satisfying

$$\tilde{X}_{t_i+c} = X_{t_i}$$

Whenever  $\lambda(\cdot)$  is an increasing function we have

$$C_1 = \sum_{i=1}^{\infty} X_{t_i} = \sum_{i=1}^{\infty} \tilde{X}_{t_i+c} = \tilde{C}_1$$

$$\tilde{C} = \sum_{i=1}^{\infty} \frac{\lambda(t_i+c)}{\lambda(t_1)} \tilde{X}_{t_i+c} = \sum_{i=1}^{\infty} \frac{\lambda(t_i+c)}{\lambda(t_1)} X_{t_i} > C$$

That is, the unindexed values differ by the factor  $\lambda(t_i+c)/\lambda(t_i) > 1$ . However, in practice this ratio often turns out to be of a different form, namely

$$\left[ 1 + \tilde{\psi}(t_i, t_i+c) \right] \frac{\lambda(t_i+c)}{\lambda(t_i)}$$

meaning that over the time interval  $[t_i, t_i+c]$  claim costs are inflated by an additional factor  $[1 + \tilde{\psi}(t_i, t_i+c)]$  above the ‘natural’ inflation. This additional inflation is referred to as superimposed inflation and can be caused by, for example, changes in the jurisdiction or an increased claims awareness of the insured. We will not discuss this further in the text.

## 1.2 STRUCTURAL FRAMEWORK TO THE CLAIMS-RESERVING PROBLEM

In this section we follow Arjas (1989). We present a mathematical framework for claims reserving and formulate the claims-reserving problem in the language of stochastic processes. This section is interesting from a theoretical point of view but can be skipped by the more practically oriented reader without loss of understanding for the remainder of the book.

Note that in this section all actions related to a claim are listed in the order of their notification at the insurance company. From a statistical point of view this makes perfect sense; however, from an accounting point of view, one should preferably list the claims according to their occurrence/accident date; this was done, for example, in Norberg (1993, 1999) as we will see below in Subsection 10.1. Of course, there is a one-to-one relation between the two concepts.

We assume that we have  $N$  claims within a fixed time period with reporting dates  $T_1, \dots, T_N$  with  $T_i \leq T_{i+1}$  for all  $i$ . If we fix the  $i$ th claim, then  $T_i = T_{i,0} \leq T_{i,1} \leq \dots \leq T_{i,j} \leq \dots \leq T_{i,N_i}$  denotes the sequence of dates at which some action on claim  $i$  is observed. For example, at time  $T_{i,j}$ , we might have a payment, a new estimation by the claims adjuster, or other new information on claim  $i$ .  $T_{i,N_i}$  denotes the date of the final settlement of the claim. Assume that  $T_{i,N_i+k} = \infty$  for  $k \geq 1$ .

We specify the events that take place at time  $T_{i,j}$  by

$$X_{i,j} = \begin{cases} \text{payment at time } T_{i,j} \text{ for claim } i \\ 0, \text{ if there is no payment at time } T_{i,j} \end{cases}$$

$$I_{i,j} = \begin{cases} \text{new information available at } T_{i,j} \text{ for claim } i \\ \emptyset, \text{ if there is no new information at time } T_{i,j} \end{cases}$$

We set  $X_{i,j} = 0$  and  $I_{i,j} = \emptyset$  whenever  $T_{i,j} = \infty$ . That is, we assume that there are no payments or new information after the final settlement of the claim.

With this structure we can define a number of interesting processes; moreover, our claims-reserving problem can be split into several subproblems. For every claim  $i$  we obtain a marked point processes.

- **Payment process of claim  $i$**   $(T_{i,j}, X_{i,j})_{j \geq 0}$  defines the following cumulative payment process

$$C_i(t) = \sum_{j \in \{k: T_{i,k} \leq t\}} X_{i,j}$$

and  $C_i(t) = 0$  for  $t < T_i$ . The total ultimate claim amount is given by

$$C_i(\infty) = C_i(T_{i,N_i}) = \sum_{j=0}^{\infty} X_{i,j}$$

The total outstanding claims payments for future liabilities of claim  $i$  at time  $t$  are given by

$$R_i(t) = C_i(\infty) - C_i(t) = \sum_{j \in \{k: T_{i,k} > t\}} X_{i,j}$$

Note that  $R_i(t)$  is a random variable at time  $t$  that needs to be predicted. As predictor one often chooses its (conditional) expectation, given the information available at time  $t$ . This (conditional) expectation is called claims reserves for future liabilities at time  $t$ . If the (conditional) expectation is unknown it needs to be estimated with the information available at time  $t$ . Henceforth, this estimator is at the same time used as a predictor for  $R_i(t)$ .

- **Information process of claim  $i$**  is given by  $(T_{i,j}, I_{i,j})_{j \geq 0}$ .
- **Settlement process of claim  $i$**  is given by  $(T_{i,j}, I_{i,j}, X_{i,j})_{j \geq 0}$ .

We denote the aggregate processes of all claims by

$$C(t) = \sum_{i=1}^N C_i(t) \tag{1.1}$$

$$R(t) = \sum_{i=1}^N R_i(t) \tag{1.2}$$

$C(t)$  represents all payments up to time  $t$  for all  $N$  claims, and  $R(t)$  is the amount of the outstanding claims payments at time  $t$  for these  $N$  claims.

Now, we consider claims reserving as a prediction problem. Let

$$\mathcal{F}_t^N = \sigma(\{(T_{i,j}, I_{i,j}, X_{i,j}); 1 \leq i \leq N, j \geq 0, T_{i,j} \leq t\})$$

be a  $\sigma$ -field, which can be interpreted as the information available at time  $t$  from the  $N$  claims settlement processes. Often there is additional exogenous information  $\mathcal{E}_t$  at time  $t$  with

$\mathcal{E}_s \subseteq \mathcal{E}_t$  for  $t \geq s$  such as change of legal practice, high inflation, job market information, etc. Therefore, the total information available to the insurance company at time  $t$  is defined as

$$\mathcal{F}_t = \sigma(\mathcal{F}_t^N \otimes \mathcal{E}_t) \quad (1.3)$$

*Problem/Task* At time  $t$  estimate the conditional distribution

$$P[C(\infty) \in \cdot | \mathcal{F}_t]$$

with the first two moments

$$M_t = E[C(\infty) | \mathcal{F}_t]$$

$$V_t = \text{Var}(C(\infty) | \mathcal{F}_t)$$

### 1.2.1 Fundamental Properties of the Claims Reserving Process

Since  $C(\infty) = C(t) + R(t)$ , and because  $C(t)$  is  $\mathcal{F}_t$ -measurable we have

$$M_t = C(t) + E[R(t) | \mathcal{F}_t]$$

$$V_t = \text{Var}(R(t) | \mathcal{F}_t)$$

**LEMMA 1.2**  $M_t$  is an  $\mathcal{F}_t$ -martingale. That is, we have for  $t > s$

$$E[M_t | \mathcal{F}_s] = M_s \quad \text{a.s.}$$

*Proof* The proof is clear (successive forecasts for increasing information  $\mathcal{F}_t$ ).  $\square$

**LEMMA 1.3** The variance process  $V_t$  is an  $\mathcal{F}_t$ -supermartingale. That is, we have for  $t > s$

$$E[V_t | \mathcal{F}_s] \leq V_s \quad \text{a.s.}$$

*Proof* Using Jensen's inequality for  $t > s$  we get, a.s.,

$$\begin{aligned} E[V_t | \mathcal{F}_s] &= E[\text{Var}(C(\infty) | \mathcal{F}_t) | \mathcal{F}_s] \\ &= E[E[C^2(\infty) | \mathcal{F}_t] | \mathcal{F}_s] - E[E[C(\infty) | \mathcal{F}_t]^2 | \mathcal{F}_s] \\ &\leq E[C^2(\infty) | \mathcal{F}_s] - E[E[C(\infty) | \mathcal{F}_t] | \mathcal{F}_s]^2 \\ &= \text{Var}(C(\infty) | \mathcal{F}_s) = V_s \end{aligned}$$

This completes the proof.  $\square$

Consider  $u > t$  and define the increment from  $t$  to  $u$  by

$$M(t, u) = M_u - M_t$$

Then, a.s., we have

$$\begin{aligned} E[M(t, u)M(u, \infty)|\mathcal{F}_t] &= E[M(t, u)E[M(u, \infty)|\mathcal{F}_u]|\mathcal{F}_t] \\ &= E[M(t, u)(E[C(\infty)|\mathcal{F}_u] - M_u)|\mathcal{F}_t] = 0 \end{aligned} \quad (1.4)$$

This implies that  $M(t, u)$  and  $M(u, \infty)$  are uncorrelated, which is a well-known property referred to as uncorrelated increments.

### ***First Approach to the Claims Reserving Problem***

The use of the martingale integral representation leads to the ‘innovation gains process’, which determines  $M_t$  when updating  $\mathcal{F}_t$ . Although this theory is well understood, this approach has a limited practical value, since one has little idea about the updating process and (statistically) there is not enough data to model this process. Moreover, it is often too complicated for practical applications.

### ***Second Approach to the Claims Reserving Problem***

For  $t < u$  we have that  $\mathcal{F}_t \subset \mathcal{F}_u$ . Since  $M_t$  is an  $\mathcal{F}_t$ -martingale

$$E[M(t, u)|\mathcal{F}_t] = 0, \quad \text{a.s.}$$

We denote the incremental payments from  $t$  to  $u$  by

$$X(t, u) = C(u) - C(t)$$

Hence, almost surely (a.s.),

$$\begin{aligned} M(t, u) &= E[C(\infty)|\mathcal{F}_u] - E[C(\infty)|\mathcal{F}_t] \\ &= C(u) + E[R(u)|\mathcal{F}_u] - (C(t) + E[R(t)|\mathcal{F}_t]) \\ &= X(t, u) + E[R(u)|\mathcal{F}_u] - E[C(u) - C(t) + R(u)|\mathcal{F}_t] \\ &= X(t, u) - E[X(t, u)|\mathcal{F}_t] + E[R(u)|\mathcal{F}_u] - E[R(u)|\mathcal{F}_t] \end{aligned}$$

Henceforth, we have the following two terms:

1. Prediction error for payments within  $(t, t + 1)$

$$X(t, t + 1) - E[X(t, t + 1)|\mathcal{F}_t] \quad (1.5)$$

2. Prediction error for outstanding claims payments  $R(t + 1)$  when updating information

$$E[R(t + 1)|\mathcal{F}_{t+1}] - E[R(t + 1)|\mathcal{F}_t] \quad (1.6)$$

this is the change of conditionally expected outstanding liabilities when updating the information from  $t$  to  $t + 1$ ,  $\mathcal{F}_t \rightarrow \mathcal{F}_{t+1}$ .

In most approaches one focuses on modelling processes (1.5) and (1.6).

## 1.2.2 Known and Unknown Claims

As in Subsection 1.1.1, we distinguish IBNyR (incurred but not yet reported) claims and reported claims. The process  $(N_t)_{t \geq 0}$  defined by

$$N_t = \sum_{i=1}^N 1_{\{T_i \leq t\}} \quad (1.7)$$

counts the number of reported claims. We can split the ultimate claims and the amount of outstanding claims payments at time  $t$  with respect to the fact of whether we have a reported or an IBNyR claim as follows

$$R(t) = \sum_{i=1}^N R_i(t) 1_{\{T_i \leq t\}} + \sum_{i=1}^N R_i(t) 1_{\{T_i > t\}} \quad (1.8)$$

where open liabilities at time  $t$  for reported claims are expressed as

$$\sum_{i=1}^N R_i(t) 1_{\{T_i \leq t\}} \quad (1.9)$$

and open liabilities at time  $t$  for IBNyR claims (including UPR liabilities) are expressed as

$$\sum_{i=1}^N R_i(t) 1_{\{T_i > t\}} \quad (1.10)$$

*Remark* Here we do not understand IBNyR in a strong sense. Observe that at time  $t$  there is, possibly, still a premium exposure (positive UPR liability). Such claims have not yet incurred, and hence are also not yet reported. For the time being we do not distinguish these types of claims for incurred but not yet reported claims – that is, they are all contained in (1.10) (for a more detailed analysis we refer to Subsection 10.1).

Hence, we define the reserves at time  $t$  as

$$R_t^{\text{rep}} = E \left[ \sum_{i=1}^N R_i(t) 1_{\{T_i \leq t\}} \middle| \mathcal{F}_t \right] = E \left[ \sum_{i=1}^{N_t} R_i(t) \middle| \mathcal{F}_t \right] \quad (1.11)$$

$$R_t^{\text{IBNyR}} = E \left[ \sum_{i=1}^N R_i(t) 1_{\{T_i > t\}} \middle| \mathcal{F}_t \right] = E \left[ \sum_{i=N_t+1}^N R_i(t) \middle| \mathcal{F}_t \right] \quad (1.12)$$

where  $N$  is the total (random) number of claims. Since  $N_t$  is  $\mathcal{F}_t$ -measurable

$$R_t^{\text{rep}} = \sum_{i=1}^{N_t} E [ R_i(t) | \mathcal{F}_t ] \quad (1.13)$$

$$R_t^{\text{IBNyR}} = E \left[ \sum_{i=N_t+1}^N R_i(t) \middle| \mathcal{F}_t \right] \quad (1.14)$$

$R_t^{\text{rep}}$  denotes the expected future payments at time  $t$  for reported claims. This term is often called ‘best estimate reserves at time  $t$  for reported claims’.  $R_t^{\text{IBNyR}}$  are the expected future

payments at time  $t$  for IBNyR claims including UPR reserves (or ‘best estimate reserves at time  $t$  for IBNyR and UPR claims’).

**Conclusions**

We can see from (1.13)–(1.14) that the reserves for reported claims and the reserves for IBNyR claims are of a rather different nature:

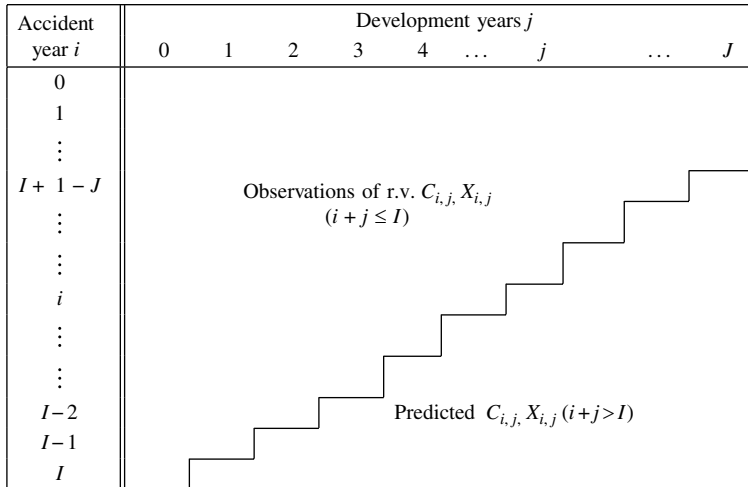
- (i) The reserves for reported claims should/can be determined individually. Often one has a lot of information on reported claims (e.g. case estimates), which suggests estimation on a single claims basis.
- (ii) The reserves for IBNyR claims (including UPR reserves) cannot be decoupled due to the fact that  $N$  is not known at time  $t$  (see (1.12)). Moreover, information on a single claims basis is not available. This shows that IBNyR reserves have to be determined on a collective basis.

Unfortunately, most of the classical claims reserving methods do not distinguish reported claims from IBNyR claims, that is, they estimate the claims reserves for both classes at the same time. In that context, we have to slightly disappoint the reader, since most methods presented in this book do not make this distinction between reported and IBNyR claims. For a first attempt to distinguish these claims categories, we refer to Chapter 10.

**1.3 OUTSTANDING LOSS LIABILITIES, CLASSICAL NOTATION**

In this subsection we introduce the classical claims reserving notation and terminology. In most cases outstanding loss liabilities are studied in so-called claims development triangles which separate insurance claims on two time axes.

Below we use the following notation (see also Figure 1.2):



**Figure 1.2** Claims development triangle

$i$  = accident year, year of occurrence (vertical axis),

$j$  = development year, development period (horizontal axis).

The most recent accident year is denoted by  $I$  while the last development year is denoted by  $J$ . That is,  $i \in \{0, \dots, I\}$  and  $j \in \{0, \dots, J\}$ .

For illustrative purposes, we assume that  $X_{i,j}$  denotes payments. (Alternative meanings for  $X_{i,j}$  are given below.) Then  $X_{i,j}$  denotes all payments in development period  $j$  for claims with accident year  $i$ . That is,  $X_{i,j}$  corresponds to the payments for claims in accident year  $i$  made in accounting year  $i + j$ . Cumulative payments  $C_{i,j}$  for accident year  $i$  after  $j$  development years are then given by

$$C_{i,j} = \sum_{k=0}^j X_{i,k} \quad (1.15)$$

Claims  $X_{i,j}$  and  $C_{i,j}$  are usually studied in claims development triangles: In a claims development triangle, accident years  $i$  are specified on the vertical axis, whereas development periods  $j$  are shown on the horizontal axis (see also Figure 1.2). Typically, at time  $I$ , the claims development tables are split into two parts: the upper triangle/trapezoid containing observations  $X_{i,j}$ ,  $i + j \leq I$ , and the lower triangle with estimates or predicted values of the outstanding payments  $X_{i,j}$ ,  $i + j > I$ . This means that observations are available in the upper triangle/trapezoid

$$\mathcal{D}_I = \{X_{i,j}; i + j \leq I, 0 \leq j \leq J\} \quad (1.16)$$

and the lower triangle  $\mathcal{D}_I^c = \{X_{i,j}; i + j > I, i \leq I, j \leq J\}$  needs to be estimated or predicted.

The accounting years are then given on the diagonals  $i + j = k$ ,  $k \geq 0$ . The incremental claims in accounting year  $k \geq 0$  are denoted by

$$X_k = \sum_{i+j=k} X_{i,j}$$

and are displayed on the  $(k + 1)$ st diagonal of the claims development triangle.

**Incremental claims**  $X_{i,j}$  may represent the incremental payments in cell  $(i, j)$ , the number of reported claims with reporting delay  $j$  and accident year  $i$ , or the change of reported claim amount in cell  $(i, j)$ . **Cumulative claims**  $C_{i,j}$  may represent the cumulative payments, total number of reported claims, or claims incurred (for cumulative reported claims).  $C_{i,j}$  is often called the **ultimate claim amount/load** of accident year  $i$  or the total number of claims in year  $i$ .

#### **Incremental claims**

$X_{i,j}$  incremental payments

$X_{i,j}$  number of reported claims with delay  $j$

$X_{i,j}$  change of reported claim amount

#### **Cumulative claims**

$\iff C_{i,j}$  cumulative payments

$\iff C_{i,j}$  total number of reported claims

$\iff C_{i,j}$  claims incurred

If the  $X_{i,j}$  denote incremental payments then the **outstanding loss liabilities** for accident year  $i$  at time  $j$  are given by

$$R_{i,j} = \sum_{k=j+1}^J X_{i,k} = C_{i,J} - C_{i,j} \quad (1.17)$$

$R_{i,j}$  need to be predicted by so-called **claims reserves**. They essentially constitute the amount that has to be estimated/predicted so that, together with the past claims  $C_{i,j}$ , we obtain the estimator/predictor for the total claims load (ultimate claim)  $C_{i,J}$  for accident year  $i$ .

#### General Assumption 1.4

We assume throughout this book that

$$I = J$$

and that  $X_{i,j} = 0$  for all  $j > J$ .

This general assumption,  $I = J$ , can easily be dropped; however, it substantially simplifies the notation and the formulas. It implies that we have to predict the outstanding loss liabilities for accident years  $i = 1, \dots, I$ .

## 1.4 GENERAL REMARKS

There are several possible ways to consider claims data when constructing loss reserving models, that is, models for estimating/predicting the total ultimate claim amounts. In general, the following data structures are studied:

- cumulative or incremental claims data;
- payments or claims incurred data;
- split small and large claims data;
- indexed or unindexed claims data;
- number of claims and claims averages statistics; etc.

Usually, different methods and differently aggregated data sets lead to very different results. Only an **experienced reserving actuary** is able to tell which is an accurate/good estimate for future liabilities for a specific data set, and which method applies to which data set.

Often there are many phenomena in the data that first need to be understood before applying any claims reserving method (we cannot simply project past observations to the future by applying one stochastic model). Especially in direct insurance, the understanding of the data can even go down to single claims and to the personal knowledge of the managing claims adjusters.

With this in mind, we will describe different methods that can be used to estimate loss reserves, but only practical experience will tell which of the methods should be applied in any particular situation. That is, the focus of this book is on the mathematical description of relevant stochastic models and we will derive various properties of these models. The

question of an *appropriate model choice* for a specific data set is only *partially* treated here. In fact, the model choice is probably one of the most difficult questions in any application in practice. Moreover, the claims reserving literature on the topic of choosing a model is fairly limited – for example, for the chain-ladder method certain aspects are considered in Barnett and Zehnwirth (2000) and Venter (1998), see also Chapter 11. From this point of view we will apply the different methods rather mechanically (without deciding which model to use). We always use the same data set (provided in Table 2.2, which gives an example for cumulative claims  $C_{i,j}$ ). On this data set we will then apply stochastic methods, some of which will be purely data based, while others will incorporate expert opinions and external knowledge.

In classical claims reserving literature, claims reserving is often understood to be providing a best estimate to the outstanding loss liabilities. Providing a best estimate means that one applies an algorithm that gives this number/amount. In recent years, especially under new solvency regimes, one is also interested in the development of adverse claims reserves, and estimating potential losses that may occur in the future in these best estimate reserves. Such questions require stochastic claims reserving models that (1) justify the claims reserving algorithms and (2) quantify the uncertainties in these algorithms.

From this point of view one should always be aware of the fact that stochastic claims reserving models do not provide solutions where deterministic algorithms fail, they rather quantify the uncertainties in deterministic claims reserving algorithms using appropriate stochastic models.

The focus in this chapter is always on estimating total claims reserves and to quantify the total prediction uncertainty in these reserves (prediction errors in total ultimate claims). This is a long-term view that is important in solvency questions; however, there are other views such as short-term views which quantify uncertainties, for example, in profit-and-loss statements. Such short-term views are important if one is to make long term or intertemporal management decisions. We do not consider such short-term views here, but for the interested reader they are partially treated in Merz and Wüthrich (2007a) and Wüthrich *et al.* (2007b).

Moreover, claims reserves are always measured on the nominal scale. From an economic point of view one should also study discounted reserves since these relate to financial markets and market values of insurance liabilities. Unfortunately, market values of non-life insurance run-off portfolios are only barely understood and (probably) their derivation requires a whole set of new mathematical tools, where one needs to understand the influence of financial market drivers on non-life reserves, etc. In the literature there are only few papers that treat the topic of discounted reserves – see, for example, Taylor (2004) and Hoedemakers *et al.* (2003, 2005).

### *Remark on Claims Figures*

When we speak about claims development triangles (paid or incurred data), these usually contain loss adjustment expenses, which can be allocated/attributed to single claims (and therefore are contained in the claims figures). Such expenses are called **allocated loss adjustment expenses** (ALAE). These are typically expenses for external lawyers, external expertise, etc. Internal loss adjustment expenses (income of claims handling department, maintenance of claims handling systems, management fees, etc.) are typically not contained in the claims figures and therefore have to be estimated separately. These costs are called **unallocated loss adjustment expenses** (ULAE). The New York method (paid-to-paid

method) is the most popular for estimating reserves for ULAE expenses. The New York method is rather rough as it only works well in stationary situations, and one could therefore think of more sophisticated systems. Since ULAE are usually small compared to the other claims payments, the New York method is often sufficient in practical applications. As we will not comment further on ULAE reserving methods in this text, we refer the reader to Foundation CAS (1989), Feldblum (2002), Kittel (1981), Johnson (1989) and Buchwalder *et al.* (2006a).