Chapter 1

Introducing Geometry

In This Chapter

- Surveying the geometric landscape: Shapes and proofs
- Finding out "What is the point of geometry, anyway?"
- Getting psyched to kick some serious geometry butt

Studying geometry is sort of a Dr. Jekyll-and-Mr. Hyde thing. You have the ordinary, everyday geometry of shapes (the Dr. Jekyll part) and the strange world of geometry proofs (the Mr. Hyde part).

Every day, you see various shapes all around you (triangles, rectangles, boxes, circles, balls, and so on), and you're probably already familiar with some of their properties: area, perimeter, and volume, for example. In this book, you discover much more about these basic properties and then explore more-advanced geometric ideas about shapes.

Geometry proofs are an entirely different sort of animal. They involve shapes, but instead of doing something straightforward like calculating the area of a shape, you have to come up with an airtight mathematical argument that proves something about a shape. This process requires not only mathematical skills but verbal skills and logical deduction skills as well, and for this reason, proofs trip up many, many students. If you're one of these people and have already started singing the geometry-proof blues, you might even describe proofs — like Mr. Hyde — as monstrous. But I'm confident that, with the help of this book, you'll have no trouble taming them.

This chapter is your gateway into the sensational, spectacular, and super-duper (but sometimes somewhat stupefying) subject of this book: geometry. If you're tempted to ask, "Why should I care about geometry?" this chapter will give you the answer.

Studying the Geometry of Shapes

Have you ever reflected on the fact that you're literally surrounded by shapes? Look around. The rays of the sun are — what else? — rays. The book in your hands has a shape, every table and chair has a shape, every wall has an area, and every container has a shape and a volume; most picture frames are rectangles, CDs and DVDs are circles, soup cans are cylinders, and so on and so on. Can you think of any solid thing that doesn't have a shape? This section gives you a brief introduction to these one-, two-, and three-dimensional shapes that are all-pervading, omnipresent, and ubiquitous — not to mention all around you.

One-dimensional shapes

There aren't many shapes you can make if you're limited to one dimension. You've got your lines, your segments, and your rays. That's about it. But it doesn't follow that having only one dimension makes these things unimportant — not by any stretch. Without these one-dimensional objects, there'd be no two-dimensional shapes; and without 2-D shapes, you can't have 3-D shapes. Think about it: 2-D squares are made up of four 1-D segments, and 3-D cubes are made up of six 2-D squares. And it'd be very difficult to do much mathematics without the simple 1-D number line or without the more sophisticated 2-D coordinate system, which needs 1-D lines for its *x*- and *y*-axes. (I cover lines, segments, and rays in Chapter 2; Chapter 18 discusses the coordinate plane.)

Two-dimensional shapes

As you probably know, two-dimensional shapes are flat things like triangles, circles, squares, rectangles, and pentagons. The two most common characteristics you study about 2-D shapes are their area and perimeter. These geometric concepts come up in countless situations in the real world. You use 2-D geometry, for example, when figuring the acreage of a plot of land, the number of square feet in a home, the size and shape of cloth needed when making curtains or clothing, the length of a running track, the dimensions of a picture frame, and so on. The formulas for calculating the area and perimeter of 2-D shapes are covered in Parts III through V.

I devote many chapters in this book to triangles and *quadrilaterals* (shapes with four sides); I give less space to shapes that have more sides, like pentagons and hexagons. Shapes of any number of straight sides, called *polygons*, have more-advanced features such as diagonals, apothems, and exterior angles, which you explore in Part IV.

Historical highlights in the study of shapes

The study of geometry has impacted architecture, engineering, astronomy, physics, medicine, and warfare, among other fields, in countless ways for well over 5,000 years. I doubt anyone will ever be able to put a date on the discovery of the simple formula for the area of a rectangle (Area = length · width), but it likely predates writing and goes back to some of the earliest farmers. Some of the first known writings from Mesopotamia (in about 3500 B.C.) deal with the area of fields and property. And I'd bet that even pre-Mesopotamian farmers knew that if one farmer planted an area three times as long and twice as wide as another farmer, then the bigger plot would be 3 · 2, or 6 times as large as the smaller one.

The architects of the pyramids at Giza (built around 2500 B.C.) knew how to construct right angles using a 3-4-5 triangle (one of the right triangles I discuss in Chapter 8). Right angles are necessary for the corners of the pyramid's square base, among other things. And of course, you've probably heard of Pythagoras (circa 570–500 B.C.) and the famous right-triangle theorem named after him (see Chapter 8). Archimedes (287–212 B.C.) used geometry to invent the pulley. He developed a system of compound pulleys that could lift an entire warship filled with men (for more of Archimedes's accomplishments, see Chapter 22). The Chinese knew how to calculate the area and volume of many different geometric shapes and how to construct a right triangle by 100 B.C.

In more recent times, Galileo Galilei (1564–1642) discovered the equation for the motion of a projectile (see Chapter 22) and designed and built the best telescope of his day. Johannes Kepler (1571–1630) measured the area of sections of the elliptical orbits of the planets as they orbit the sun. René Descartes (1596–1650) is credited with inventing coordinate geometry, the basis for most mathematical graphing (see Chapter 18). Isaac Newton (1642–1727) used geometrical methods in his *Principia Mathematica,* the famous book in which he set out the principle of universal gravitation.

Closer to home, Ben Franklin (1706–1790) used geometry to study meteorology and ocean currents. George Washington (1732–1799) used trigonometry (the advanced study of triangles) while working as a surveyor before he became a soldier. Last but certainly not least, Albert Einstein discovered one of the most bizarre geometry rules of all: that gravity warps the universe. One consequence of this is that if you were to draw a giant triangle around the sun, the sum of its angles would actually be a little larger than 180°. This contradicts the 180° rule for triangles (see Chapter 7), which works until you get to an astronomical scale. The list of highlights goes on and on.

You may be familiar with some shapes that have curved sides, such as circles, ellipses, and parabolas. The circle is the only curved 2-D shape covered in this book. In Part V, you investigate all sorts of interesting circle properties involving diameters, radii, chords, tangent lines, and so on.

Three-dimensional shapes

I cover three-dimensional shapes in Part VI. You work with prisms (a box is one example), cylinders, pyramids, cones, and spheres. The two major characteristics of these 3-D shapes, which you study in Chapter 17, are their *surface area* and *volume.*

Three-dimensional concepts like volume and surface area come up frequently in the real world; examples include the volume of water in a fish tank or backyard pool. The amount of wrapping paper you need to wrap a gift box depends on its surface area. And if you wanted to calculate the surface area and volume of the Great Pyramid in Egypt — you've been dying to do this, right? — you couldn't do it without 3-D geometry.

Here are a couple of ideas about how the three dimensions are interrelated. Two-dimensional shapes are enclosed by their sides, which are 1-D segments; 3-D shapes are enclosed by their faces, which are 2-D polygons. And here's a nifty real-world example of the relationship between 2-D area and 3-D volume: A gallon of paint (a 3-D volume quantity) can cover a certain number of square feet of area on a wall (a 2-D area quantity). (Well, okay, I have to admit it — I'm playing a bit fast and loose with my dimensions here. The paint on the wall is actually a 3-D shape. There's the length and width of the wall, and the third dimension is the thickness of the layer of paint. If you multiply these three dimensions together, you get the volume of the paint.)

Getting Acquainted with Geometry Proofs

Geometry proofs are an oddity in the mathematical landscape, and just about the only place you find geometry proofs is in a geometry course. If you're in a course right now and you're wondering what's the point of studying something you'll never use again, I get back to that in a minute in the section "When Am I Ever Going to Use This?" For now, I just want to give you a very brief description of what a geometry proof is.

A *geometry proof* — like any mathematical proof — is an argument that begins with known facts, proceeds from there through a series of logical deductions, and ends with the thing you're trying to prove.

Mathematicians have been writing proofs — in geometry and all other areas of math — for over 2,000 years. (See the sidebar about Euclid and the history of geometry proofs.) The main job of a present-day mathematician is proving things by writing formal proofs. This is how the field of mathematics progresses: As more and more ideas are proved, the body of mathematical knowledge grows. Proofs have always played, and still play, a significant role in mathematics. And that's one of the reasons you're studying them. Part II delves into all the details on proofs, but in the sections that follow, I get you started in the right direction.

Easing into proofs with an everyday example

You probably never realized it, but sometimes when you think through a situation in your day-to-day life, you use the same type of deductive logic that's used in geometry proofs. Although the topics are different, the basic nature of the argument is the same.

Here's an example of real-life logic. Say you're at a party at Sandra's place. You have a crush on Sandra, but she's been dating Johnny for a few months. You look around at the partygoers and notice Johnny talking with Judy, and a little later you see them step outside for a few minutes. When they come back inside, Judy's wearing Johnny's ring. You weren't born yesterday, so you put two and two together and realize that Sandra's relationship with Johnny is in trouble and, in fact, may end any minute. You glance over in Sandra's direction and see her leaving the room with tears in her eyes. When she comes back, you figure it might not be a bad idea to go over and talk with her.

(By the way, this story about a party gone bad is based on Lesley Gore's No. 1 hit from the '60s, "It's My Party." The sequel song, also a hit, "Judy's Turn to Cry," relates how Sandra got back at Judy. Check out the lyrics online.)

Now, granted, this party scenario might not seem like it involves a deductive argument. Deductive arguments tend to contain many steps or a chain of logic like, "If A, then B; and if B, then C; if C, then D; and so on." The party fiasco might not seem like this at all because you'd probably see it as a single incident. You see Judy come inside wearing Johnny's ring, you glance at Sandra and see that she's upset, and the whole scenario is clear to you in an instant. It's all obvious — no logical deduction seems necessary.

Turning everyday logic into a proof

Imagine that you had to explain your entire thought process about the party situation to someone with absolutely no knowledge of how people usually behave. For instance, imagine that you had to explain your thinking to a hypothetical Martian who knows nothing about our Earth ways. In this case, you *would* need to walk him through your reasoning step by step.

Here's how your argument might go. Note that each statement comes with the reasoning in parentheses:

1. Sandra and Johnny are going out (this is a given fact).
2. Johnny and Judy go outside for a few minutes (also given).

3. When Judy returns, she has a new ring on her finger (a third given).
4. Therefore, she's wearing Johnny's ring (*much* more probable than, say, that she found a ring on the ground outside).
5. Therefore, she's going out with Johnny (because when a boy gives a girl his ring, it means they're going out).
6. Therefore, Sandra and Johnny will break up soon (because a girl will not continue to go out with a guy who's just given another girl his ring).
7. Therefore, Sandra will soon be available (because that's what happens after someone breaks up).
8. Therefore, I should go over and talk with her (duh).

This eight-step argument shows you that there really is a chain of logical deductions going on beneath the surface, even though in real life your reasoning and conclusions about Sandra would come to you in an instant. And the argument gives you a little taste for the type of step-by-step reasoning you use in geometry proofs. You see your first geometry proof in the next section.

Sampling a simple geometrical proof

Geometry proofs are like the party argument in the preceding section, only with a lot less drama. They follow the same type of series of intermediate conclusions that lead to the final conclusion: Beginning with some given facts, say A and B, you go on to say *therefore,* C; then *therefore,* D; then *therefore,* E; and so on till you get to your final conclusion. Here's a very simple example using the line segments in Figure 1-1.

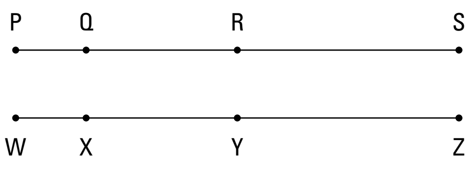

Figure 1-1: $\overline{PS}$ and $\overline{WZ}$, each made up of three pieces.

For this proof, you're told that segment $\overline{PS}$ is *congruent to* (the same length as) segment $\overline{WZ}$, that $\overline{PQ}$ is congruent to $\overline{WX}$, and that $\overline{QR}$ is congruent to $\overline{XY}$. (By the way, instead of saying *is congruent to* all the time, you can just use the symbol ≅ to mean the same thing.) You have to prove that $\overline{RS} \cong \overline{YZ}$.

Hate proofs? Blame Euclid.

Euclid (circa 385–275 B.C.) is usually credited with getting the ball rolling on geometry proofs. (If you're having trouble with proofs, now you know who to blame!) His approach was to begin with a few undefined terms such as *point* and *line* and then to build from there, carefully defining other terms like *segment* and *angle*. He also realized that he'd need to begin with some unproved principles (called *postulates*) that he'd just have to assume were true.

He started with ten postulates, such as "a straight line segment can be drawn by connecting any two points" and "two things that each equal a third thing are equal to one another." After setting down the undefined terms, the definitions, and the postulates, his real work began. Using these three categories of things, he proved his first *theorem* (a proven geometric principle), which was the side-angle-side method of proving triangles congruent (see Chapter 9). And then he proved another theorem and another and so on.

Once a theorem had been proved, it could then be used (along with the undefined terms, definitions, and postulates) to prove other theorems. If you're working on proofs in a standard high school geometry course, you're walking in the footsteps of Euclid, one of the giants in the history of mathematics — lucky you!

Now, you may be thinking, "That's obvious — if $\overline{PS}$ is the same length as $\overline{WZ}$ and both segments contain these equal short pieces and the equal medium pieces, then the longer third pieces have to be equal as well." And of course, you'd be right. But that's not how the proof game is played. You have to spell out every little step in your thinking so your argument doesn't have any gaps. Here's the whole chain of logical deductions:

1. $\overline{PS} \cong \overline{WZ}$ (this is given).
2. $\overline{PQ} \cong \overline{WX}$ and

   $\overline{QR} \cong \overline{XY}$ (these facts are also given).
3. Therefore, $\overline{PR} \cong \overline{WY}$ (because if you add equal things to equal things, you get equal totals).
4. Therefore, $\overline{RS} \cong \overline{YZ}$ (because if you start with equal segments, the whole segments $\overline{PS}$ and $\overline{WZ}$, and take away equal parts of them, $\overline{PR}$ and $\overline{WY}$, the parts that are left must be equal).

In formal proofs, you write your statements (like $\overline{PR} \cong \overline{WY}$ from Step 3) in one column, and your justifications for those statements in another column. Chapter 4 shows you the setup.

When Am I Ever Going to Use This?

You'll likely have plenty of opportunities to use your knowledge about the geometry of shapes. And what about geometry proofs? Not so much. Read on for details.

When you'll use your knowledge of shapes

Shapes are everywhere, so every educated person should have a working knowledge of shapes and their properties. The geometry of shapes comes up often in daily life, particularly with measurements.

In day-to-day life, if you have to buy carpeting or fertilizer or grass seed for your lawn, you should know something about area. You might want to understand the measurements in recipes or on food labels, or you may want to help a child with an art or science project that involves geometry. You certainly need to understand something about geometry to build some shelves or a backyard deck. And after finishing your work, you might be hungry — a grasp of how area works can come in handy when you're ordering pizza: a 20-inch pizza is four, not two, times as big as a 10-incher, and a 14-inch pizza is twice as big as a 10-incher. (Check out Chapter 15 to see why this is.)

Careers that use geometry

Here's a quick alphabetical tour of careers that use geometry. Artists use geometry to measure canvases, make frames, and design sculptures. Builders use it in just about everything they do; ditto for carpenters. For dentists, the shape of teeth, cavities, and fillings is one big geometry problem. Diamond cutters use geometry every time they cut a stone. Dairy farmers use geometry when calculating the volume of milk output in gallons.

Eyeglass manufacturers use geometry in countless ways whenever they use the science of optics. Fighter pilots (or quarterbacks or anyone else who has to aim something at a moving target) have to understand angles, distance, trajectory, and so on. Grass-seed sellers have to know how much seed customers need to use per square yard or per acre. Helicopter pilots use geometry (actually, their computerized instruments do the work for them) for all calculations that affect taking off and landing, turning, wind speed, lift, drag, acceleration, and the like. Instrument makers have to use geometry when they make trumpets, pianos, violins — you name it. And the list goes on and on . . .

When you'll use your knowledge of proofs

Will you ever use your knowledge of geometry proofs? In this section, I give you two answers to this question: a politically correct one and a politically incorrect one. Take your pick.

First, the politically correct answer (which is also *actually* correct). Granted, it's extremely unlikely that you'll ever have occasion to do a single geometry proof outside of a high school math course (college math majors are about the only exception). However, doing geometry proofs teaches you important lessons that you can apply to non-mathematical arguments. Among other things, proofs teach you the following:

- Not to assume things are true just because they seem true at first glance
- To very carefully explain each step in an argument even if you think it should be obvious to everyone
- To search for holes in your arguments
- Not to jump to conclusions

And in general, proofs teach you to be disciplined and rigorous in your thinking and in how you communicate your thoughts.

If you don't buy that PC stuff, I'm sure you'll get this politically incorrect answer: Okay, so you're never going to use geometry proofs, but you do want to get a decent grade in geometry, right? So you might as well pay attention in class (what else is there to do, anyway?), do your homework, and use the hints, tips, and strategies I give you in this book. They'll make your life much easier. Promise.

Why You Won't Have Any Trouble with Geometry

TIP

Geometry, especially proofs, can be difficult. Mathwise, it's foreign territory with some very rocky terrain. But it's far from impossible, and you can do a bunch of things to make your geometry experience as easy as pi (get it?):

- **Powering through proofs.** If you get stuck on a proof, check out the helpful tips and warnings that I give you throughout each chapter. You may also want to look at Chapter 21 to make sure you keep the ten most important ideas for proofs fresh in your mind. Finally, you can go to Chapter 6 to see how to reason your way through a long, complicated proof.

- **Figuring out formulas.** If you can't figure out a problem that uses a geometry formula, you can look in Appendix A at the back of the book to make sure that you have the formula right.
- **Coming to terms with the lingo.** If you need help with any geometry terms, the glossary in Appendix B can come in handy.
- **Sticking it out.** My main piece of advice to you is never to give up on a problem. The greater number of sticky problems that you finally beat, the more experience you gain to help you beat the next one. After you take in all my expert advice — no brag, just fact — you should have all the tools you need to face down whatever your geometry teacher or math-crazy friends can throw at you.