

Index

Numerics

- 1-D (one-dimensional) geometry, 12, 14
- 2-D (two-dimensional) geometry, 12–14, 326–331
- 3 : 4 : 5 family of Pythagorean triples, 115–117
- 3-4-5 Pythagorean triple, 114
- 3-D (three-dimensional) geometry
 - base, 368
 - cone. *See* cone
 - cylinder. *See* cylinder
 - determining a plane, 269
 - flat-top figures, 273–279
 - formulas and properties, 365
 - height, 354–355, 370
 - intersecting line and plane, 270–272
 - line perpendicular to plane, 265–268
 - locus (loci), 332–333
 - overview, 12, 13–14
 - pointy-top figures, 279–285
 - problems, 332–333
 - pyramid. *See* pyramid
 - spheres. *See* sphere
 - surface area. *See* surface area
 - volume. *See* volume
- 5-12-13 Pythagorean triple, 114
- 7-24-25 Pythagorean triple, 114
- 8 : 15 : 17 family of Pythagorean triples, 116
- 8-15-17 Pythagorean triple, 114
- 30°- 60°- 90° (right) triangle, 94, 120–122, 190–191, 196–197
- 45°- 45° -90° (isosceles right) triangle, 118–120, 197–198

• A •

- AA (Angle-Angle), 209, 210, 350
- AAS (Angle-Angle-Side), 137–139, 349
- acute angle, 30, 31, 96, 367
- acute triangle, 96, 367
- adding
 - angles, 40
 - arcs, 235
 - segments, 39
- addition theorem, 39, 62–67, 70
- adjacent angles, 31, 32, 367
- adjacent sides, 367
- algebraic proof, coordinate geometry, 300–302
- all-radii-are-congruent theorem, 229, 347
- alternate exterior angles, 152, 367
- alternate interior angles, 152, 367
- altitude. *See also* height
 - defined, 96, 367
 - drawing, 190, 232
 - of triangle, 96–98, 104, 106, 111
- Altitude-on-Hypotenuse Theorem, 216–218, 362
- analytical proof, coordinate geometry, 14, 298–299
- angle
 - adding and subtracting, 40
 - addition theorem, 63–64, 70
 - base. *See* base angle
 - bisecting, 42–43, 337–338
 - of circle. *See* angle, of circle
 - copying, 335–336
 - defined, 23, 367
 - diagrams, 23, 44–45
 - equilateral triangle, 93–94
 - if-angles-then-sides theorem, 135, 348
 - if-sides-then-angles theorem, 135, 348
 - measurement, 30, 36–39, 42–44, 154, 199–201
 - pairs of, 31–33, 156
 - Parallel-Lines Theorems, 346–347
 - parallelogram, 163, 164
 - in polygon, 199–201, 363
 - rotation, 317
 - subtraction theorem, 67–68
 - transversal. *See* transversal
 - triangle, 96. *See also* triangle
 - trisecting, 42–43, 338
 - vertex of, 23, 374
 - writing and reading, 23, 31
- angle, of circle
 - angle-arc theorems and formulas, 250–257
 - central angles, 233–235
 - chord-chord angles, 252–253, 257–259, 364
 - defined, 250–252, 363
 - inside circle, 252–254, 363, 364
 - measurements, 250–256, 363, 364
 - on circle, 250–252, 363, 364
 - outside circle, 254–256, 363, 364
 - secant-secant, 254, 260–262, 364
 - secant-tangent, 254
 - size of, and distance from circle's center, 256–257
 - tangent-chord, 250–251
 - tangent-secant, 254, 259–260, 364
 - tangent-tangent, 254
- angle, type of
 - acute, 30, 31, 96, 367
 - adjacent, 31, 32, 367
 - alternate exterior, 152, 367

- angle, type of (*continued*)
 alternate interior, 152, 367
 base. *See* base angle
 central, 233–235, 368
 complementary,
 32, 59–62, 369
 congruent, 38, 60, 73–75,
 251, 369
 corresponding, 152,
 204–205, 369
 exterior. *See* exterior angle
 included, 126
 inscribed, 250, 251–252
 interior. *See* interior angle
 obtuse, 30, 31, 96, 371
 reflex, 30, 31
 right, 30, 31, 32, 373
 rotation angle, 317
 straight, 30, 31, 374
 supplementary, 32,
 59–62, 374
 vertex, 93
 vertical, 33, 73–74, 346, 374
 Z-angle, 161–162
- angle bisector, 42, 104,
 105, 367
- Angle-Angle (AA), 209, 210,
 350
- Angle-Angle-Side (AAS),
 137–139, 349
- Angle-Bisector Theorem,
 223–224
- Angle-Side-Angle (ASA),
 128–130, 349
- apothem, 195–197,
 245–246, 367
- arc. *See also* circle
 and central angle theorems,
 233–234
 and chords theorems, 234
 defined, 233, 367
 formulas, 363
 length of, 244–246,
 248–249, 363
 measurement, 244
- Archimedes
 (mathematician), 13,
 351–352
- area
 circle, 176–177, 244,
 247–248, 363
 defined, 96, 367
 formulas, 98–100, 188–202,
 361–362
 hexagon, 110–111, 196–197
 kite, 188, 189, 192–193, 362
 lateral, 276, 280
 octagon, 197–199
 parallelogram, 188–191, 362
 with Pythagorean Theorem,
 109–113
 quadrilateral, 187–195
 rectangle, 188, 189
 regular polygon, 195–199,
 245, 362
 rhombus, 188, 191–192
 sector of circle, 247–249, 363
 segment of circle, 248
 square, 188
 surface. *See* surface area
 trapezoid, 188, 189–190,
 194–195
 triangle, 96–101, 109–113,
 196, 300–302, 361
- ASA (Angle-Side-Angle),
 128–130, 349
- auxiliary lines, in geometry
 proof, 159–161
- **B** •
- base
 3-D diagram, 368
 area formula, 98–100
 cylinder, 274
 isosceles triangle, 93, 368
 prism, 273
 trapezoid, 157, 368
 triangle, 96
- base angle
 defined, 93
 trapezoid and isosceles
 trapezoid, 171–172, 368
 triangle, 93, 348, 368
- bisect(ing)
 angle, 42–43, 337–338
 defined, 368
 Like Divisions Theorem,
 70–73
 segment, 40–41, 338–339
- bisector
 angle, 42, 104, 105, 367
 perpendicular. *See*
 perpendicular bisector
 reflecting line, 310
 segment, 338–339
- bubble logic in proofs,
 55–57, 88
- buckyballs, 357
- **C** •
- careers using geometry, 18
- Cartesian coordinate system.
See coordinate system
- Cartesian plane, 289–291, 369
- CASTC (corresponding angles
 of similar triangles are
 congruent), 213–214
- catenary curve, 356–357
- center of rotation,
 317, 318–320
- center of triangle, 102–106
- central angle, 233–235, 368
- centroid of triangle,
 102–104, 368
- China, geometry use in, 13
- chord. *See also* circle
 arcs theorems, 234
 bisected theorem, 234
 central angle theorems, 234
 chord-chord angles,
 252–253, 257–259, 364
 congruency and equidis-
 tance theorem, 229
 defined, 228–229, 368
 inscribed angle, 250, 251–252
 tangent-chord angle, 250
- Chord-Chord Power Theorem,
 257–259, 262, 364
- Chung-Chin Tsu
 (mathematician), 352
- circle
 all-radii-are-congruent
 theorem, 347
 angle measuring. *See* angle,
 of circle

- arc. *See* arc
area, 176–177, 244,
247–248, 363
chord. *See* chord
circumference, 244, 246,
363, 368
circumscribed, 104, 105, 368
as common shape, 227
common-tangent problem
proof, 238–241
concentric, 369
congruent, 229, 233–234,
347, 369
defined, 228, 368
equations of, 303–305, 366
formulas, 243–262, 363–364
inscribed, 105
pi (π). *See* pi (π)
as polygon, 245–246
proof, 229–230, 235–236
properties, 363–364
radius, 231–232, 347, 372
secant-secant, 254,
260–262, 364
sectors, 363
segments inside, 228–229
tangent to, 237–241,
259–260, 374
tangent-secant, 254,
259–260, 364
theorems, 229, 233–235,
257–262, 347
walk-around problem,
241–242
circumcenter, 104–105, 368
circumcircles, 105
circumference of circle,
244, 363, 368
circumscribed circle, 104,
105, 368
collinear points, 25, 26, 369
Columbus, Christopher, 354
common tangent,
238–241, 369
compass, 334–342, 369
complementary angle,
32, 59–62, 369
complementary angle
theorems, 59–62
concentric circles, 333, 369
cone
circular, right, 279
defined, 279, 369
lateral area, 280
pointy-top objects, 279
surface area, 284–285, 365
volume, 280, 284, 365
congruence. *See also*
isometry
AAS (Angle-Angle-Side),
137–139, 349
all-radii-are-congruent
theorem, 229, 347
angle, 38, 60, 73–75, 251, 369
angle on circle, 251–252
ASA (Angle-Side-Angle),
128–130, 349
circles, 229, 233–234,
347, 369
complementary angle
theorems, 59–62
complementary triangle,
60–61
CPCTC (corresponding
parts of congruent
triangles are congruent),
131–134
defined, 369
HLR (hypotenuse-leg-right
angle) theorem,
139–141, 349
if-sides-then-angles
theorem, 135, 348
proving that angles are
congruent (theorem),
152, 153, 156
radius, 347
segments, 36, 369
and similarity, 203
SSS (Side-Side-Side),
124–126, 130, 349
Substitution Property,
75–78
supplementary angle
theorems, 59–62, 153, 156
Transitive Property, 75–78
triangle. *See* triangle
congruence
of vertex angle, 346
vertical angle, 346
vertical-angles-as-congruent
theorem, 73–75, 346
congruent, defined, 369
congruent angle, 38, 60,
73–75, 152, 251, 369
congruent circles, 229,
233–234, 347, 369
congruent segments, 36, 369
construction
bisecting an angle, 337–338
bisecting a segment,
338–339
copying an angle, 335–336
copying a segment, 334–335
copying a triangle, 336–337
defined, 369
parallel line, 341–342
perpendicular bisector,
338–339
perpendicular line, 339–342
trisecting an angle, 338
coordinate (Cartesian) plane,
289–291, 369
coordinate system
algebraic proof, 300–302
analytical proof, 298–299
coordinate plane, 289–291
distance formula,
294–295, 366
drawing general figure,
298–299
equations of circle,
303–306, 366
equations of line,
302–303, 366
formulas, 291–295, 365–366
midpoint formula, 295, 365
problem, 295–297
properties, 365–366
slope and slope formula,
291–294, 365
coordinates, 289
coplanar. *See also*
intersection; parallel line
defined, 369
line, 27–28
points, 25, 26

- copying. *See also* locus (loci), locating and constructing
 an angle, 335–336
 a segment, 334–335
 a triangle, 336–337
- corresponding angle,
 152, 204–205,
 213–214, 369
- corresponding angles of
 similar triangles are
 congruent (CASTC),
 213–214
- corresponding parts of
 congruent triangles are
 congruent (CPCTC),
 131–134, 349
- corresponding sides,
 204–205, 213–216
- corresponding sides of
 similar triangles are
 proportional (CSSTP),
 213–216
- CPCTC (corresponding parts
 of congruent triangles
 are congruent), 131–134,
 349
- cross diagonal, 170
- CSSTP (corresponding sides
 of similar triangles are
 proportional), 213–216
- cube, volume of, 274, 285–286
- cylinder
 circular, right, 274
 defined, 273, 274, 369
 lateral area, 276
 surface area,
 274, 278–279, 365
 volume, 274, 279, 365
- **D** •
- da Vinci, Leonardo (artist), 91
- deductive reasoning,
 2, 15, 43, 54, 56
- definition
 as concept, 22
 use in proofs, 53–54
- degree, 37, 244, 369
- Descartes, René
 (mathematician),
 13, 65, 289, 303
- determining a line, 347
- determining a plane, 269
- diagonal
 for area of kite, 192–193
 defined, 369
 kite, 170
 midpoint formula and
 bisection, 297
 n -gon, 201–202
 number that can be drawn,
 363
 parallelogram, 163, 164–165
 polygon, 201–202, 363
 Pythagorean Theorem,
 108–109
 quadrilateral, 162
- diagram. *See also specific
 topics*
 angle, 23, 44–45
 construction of. *See*
 construction
 flat-top object, 273–279
 general drawing, 298–299
 line, 23
 parallel line, 27
 plane, 24
 point, 23
 pointy-top object, 279–285
 proof, 43–46, 50, 51
 Pythagorean theorem, 108
 ray, 23
 right angle, 31
 segment, 23, 44–45
 straight line, 44
- diameter of circle, 228, 369
- disjoint, 170
- displacement, 351–352
- distance
 chord congruency and
 equidistance theorem,
 229
 circle equation, 304–306
 formula, 294–295, 297, 366
 to horizon, 260, 355
 problem solving with,
 297, 300–301
 translation distance,
 313, 315
- dividing a segment, 41, 342
- dome, geodesic, 357
- drawing constructions.
See construction
- drawing general figure,
 298–299
- Dunce Cap Theorem, 241–242
- **E** •
- Earth, 354, 355
- edge, 273, 279, 369
- Egyptian pyramids at Giza, 13
- Einstein, Albert (physicist), 13
- endpoint
 equidistance theorem,
 143–144
 midpoint formula, 295, 297,
 300, 365
 point excluded from
 problem solving, 330
- equation of circle,
 303–306, 366
- equation of line. *See* line,
 equation of
- equiangular, 93, 369
- equiangular polygons, 363
- equidistance theorems,
 141–144
- equilateral, defined, 93, 369
- equilateral triangle
 30°–60°–90° (right) triangle,
 94, 120–122, 190–191,
 196–197
 altitude, 98
 area, 100, 196, 361
 defined, 91, 92, 93–94, 370
 icosahedron, 357–358
- Eratosthenes
 (mathematician), 354
- Euclid (Greek
 mathematician), 17
- “Eureka” moment,
 Archimedes, 351–352
- exterior angle
 alternate, 152, 367
 defined, 370
 diagram, 23
 parallel-line theorems,
 152–153, 178–179,
 346–347
 polygon angles, 199–201, 363
 same-side, 153, 373
 sum measures, 363
- external tangent, 238

• F •

face, 273, 279, 370
 families of Pythagorean triple triangle, 115–118
 finished geometry proof, 88
 flat-top object
 cylinder, 274
 defined, 273, 365
 diagram, 273–279
 lateral area, 276
 prism, 273
 shortest distance between two points, 275–276
 surface area, 274, 278–279
 volume, 274, 277
 foot, 265, 370
 formulas. *See also* properties
 3-D geometry, 365
 angle-arc, 250–257
 arcs, 250–257, 363
 area, 98–100, 188–202, 361–362
 circle, 243–262, 363–364
 circumference, 363
 coordinate system, 291–295, 365–366
 distance formula, 294–295, 297, 366
 equilateral triangle area, 100, 196, 361
 Hero's area, 100, 113, 361
 horizontal distance, 294
 kite area, 188, 189, 192–193
 midpoint, 295, 297, 300, 365
 parallelogram area, 188–191, 362
 polygons, 362–363
 quadrilateral area, 188–190
 rectangle area, 188, 189, 362
 rhombus area, 188, 191–192, 362
 right triangle, 361
 slope, 291–294, 365
 trapezoid area, 188, 189–190, 194–195, 362
 triangles, 361–362
 vertical distance formula, 294

45°-45°-90° (isosceles right) triangle, 118–120, 197–198
 four-step problem solving process, 326
 Franklin, Ben (inventor), 13
 Fuller, Buckminster (scientist), 357

• G •

Galileo Galilei (mathematician), 13, 355
 game plan for geometry proofs, 2, 61, 80, 145–146
 Gauss, Carl Friedrich (mathematician), 351
 generality, loss of, 299
 geodesic dome, 357
 geometry, overview. *See also* specific topics
 definitions in, 22
 dimensions, 12–14
 historical view, 13
 proof importance, 11, 14–20
 ten interesting problems, 351–358
 uses for, 18–19
 geometry proof
 analytical, 14, 298–302
 auxiliary lines in, 159–161
 bubble logic in, 55–57, 88
 circle, 229–230, 235–236
 common-tangent problem, 238–241
 components of, 49–51
 congruent lines, 16–17
 congruent triangles. *See* triangle congruence
 defined, 14
 definitions for, 53–54
 diagram, 43–46, 50, 51
 Euclid and, 17
 everyday approach (song lyrics), 15–16
 example, 56–57
 finished proof, 88
 format, 88
 game plan for, 2, 61, 80, 145–146

gaps, bridging, 86–87
 getting stuck, 83–85
 given statement, 16–17, 50, 51, 65, 81
 if-then statement, 51–53, 56, 81–83
 importance of, 11, 14–20
 indirect, 146–148
 introduction, 14–17
 jumping ahead, 85–86
 overview, 11, 14–20
 parallel-line theorems, 152–153, 178–179, 346–347
 parallelogram, 162–186
 plane, intersecting, 266–272
 postulate use in, 54–55, 173–175
 property-proof connection, 173–186
 prove statement, 50, 51, 66, 141
 quadrilaterals, 162–186
 reason column, 50, 51, 53–54
 Reflexive Property, 345–346
 similarity, 207–224
 statement column, 50, 51
 Substitution Property, 75–78
 “therefore,” 16–17
 Transitive Property, 75–78
 transversal theorem, 153–155
 triangle congruence, 123–141, 145–148
 two-column format, 17, 55–57
 uses for, 18–19
 working backward, 85–87, 145–146
 given points, 330
 given statement, geometry proof, 16–17, 50, 51, 65, 81
 glide reflection
 defined, 321, 370
 equals three reflections (theorem), 309, 322–323
 main reflecting line of (theorem), 322–324

Golden Gate Bridge, 356–357
 golden ratio, 353
 golden rectangle, 353
 “Goldilocks rule” for scalene triangles, 92
 Great Pyramid of Khufu, 354–355
 Greek mathematicians, 1, 108, 334, 351–353

• H •

half properties, 170
 height. *See also* altitude;
specific geometric shapes
 3-D geometry, 354–355, 370
 area formula, 98–100
 defined, 370
 distance to horizon, 355
 golden ratio (ϕ), 353
 Great Pyramid of Khufu, 354–355
 plane, 269
 quadrilaterals, 190, 232
 slant, 280, 373
 triangle, 96–98, 104, 106, 111
 Hero’s area formula, 100, 113, 361
 hexagon, area of, 110–111, 196–197
 historical view of geometry, 1, 13, 108, 334, 351–353
 HLR (hypotenuse-leg-right angle) theorem, 139–141, 349
 horizon, distance to, 355
 horizontal, 26
 horizontal (x) axis, 290. *See also* coordinate system
 horizontal distance formula, 294
 horizontal line
 equation of, 303, 366
 slope, 292, 293, 303
 hypotenuse. *See also* Pythagorean Theorem
 30°- 60°- 90° (right) triangle, 94, 120–122, 190–191, 196–197
 45°- 45°- 90° (isosceles right) triangle, 118–120, 197–198

Altitude-on-Hypotenuse Theorem, 216–218, 362
 defined, 108, 370
 distance formula, 294–295, 366
 right triangle, 96
 hypotenuse-leg-right angle (HLR) theorem, 139–141, 349

• I •

icosahedron, 357–358
 if-angles-then-sides theorem, 135, 348
 if-sides-then-angles theorem, 135, 348
 if-then statement, geometry proof, 51–53, 56, 81–83
 image, 307, 370
 incenter of triangle, 104–105, 370
 incircles, 105
 incline and slope, 292–293
 indirect geometry proof, 146–148
 infinite Pythagorean triple triangle, 115
 infinite series of triangles, 103
 inscribed angle, circle, 250, 251–252
 inscribed circle, 352
 inscribed shapes, 370
 interior angle
 alternate, 152, 367
 defined, 370
 diagram, 23
 parallel-line theorems, 152–153, 178–179, 346–347
 polygon angles, 199–201, 363
 same-side, 153, 373
 sum measures, 363
 internal tangent, 238
 intersection
 defined, 370
 determining a plane, 269–271
 of diagonals at midpoint, 297
 lines, 28, 270–272

origin, 290
 perpendicular. *See* perpendicular line
 plane, 29, 266–272
 point, 28
 transversal. *See* transversal vertex.
See vertex (vertices)
 irreducible Pythagorean triple triangle, 114–115
 isometry. *See also* reflection
 defined, 307, 370
 glide reflection, 309, 321–324, 370
 rotation, 317–321
 translation, 312–316
 isosceles trapezoid
 defined, 157, 370
 properties, 172
 quadrilateral relationships, 158–159
 isosceles triangle
 45°- 45°- 90° (isosceles right) triangle, 118–120, 197–198
 altitude, 98
 base, 93, 368
 base angle, 368
 defined, 91, 92, 93, 370
 if-angles-then-sides theorem, 135, 348
 if-sides-then-angles theorem, 135, 348
 leg, 371
 theorems, 134–137, 348

• J •

jumping ahead, geometry proof, 85–86
 justification in *given* statement, 16–17, 50, 51, 65, 81

• K •

Kepler, Johannes (mathematician), 13
 Khufu, Great Pyramid of, 354–355

kite
 area formula, 188, 189,
 192–193, 362
 defined, 157
 proofs, 185–186
 properties, 169–171
 quadrilateral relationships,
 158–159

• L •

lateral area
 cone, 280
 cylinder, 276
 defined, 276
 flat-top objects, 276
 pointy-top objects, 280–281
 prism, 276
 pyramid, 280
 leg. *See also* side
 isosceles triangle, 93, 371
 Pythagorean theorem, 108
 right triangle, 96
 trapezoid, 157, 371
 of triangle, 108
 length
 arc, 244–246, 248–249, 363
 of segment, 36
 Like Divisions Theorem, 70–73
 Like Multiples Theorem, 70–73
 line
 auxiliary lines in proof,
 159–161
 defined, 22, 371
 determining a plane, 269
 diagram, 23
 equation of. *See* line,
 equation of
 horizontal, 26, 27
 intersecting plane and,
 270–272
 Line-Plane Perpendicularity
 Theorem, 265–268
 pairs, 27–29
 and planes, 270–272
 point-slope form, 303, 306,
 366

points, determining, 347
 slope, 291–294
 slope-intercept form,
 302, 306, 366
 translation, 313
 two points determine a line
 postulate, 347
 type of. *See* line, type of
 writing and reading, 22
 line, equation of
 coordinatizing system,
 302–303, 366
 horizontal, 303, 366
 point-slope, 303, 306, 366
 reflecting line, 316, 318–321
 slope, 302–303
 slope-intercept, 302, 306, 366
 tangent line, 305–306
 translation line, 313, 315
 vertical, 303, 366
 line, type of
 coplanar, 27–28
 intersecting, 269
 main reflecting line, 322–324
 non-coplanar, 28–29
 parallel. *See* parallel line
 perpendicular. *See*
 perpendicular line
 ray. *See* ray
 reflecting. *See* reflecting line
 segment. *See* segment
 tangent, 237–238
 translation line, 313, 315
 transverse. *See* transversal
 vertical, 26, 27
 linear pair of angles, 32
 Line-Plane Perpendicularity
 Theorem, 265–268
 locus (loci), locating and
 constructing. *See also*
 point
 2-D geometry, 326–331
 3-D geometry, 332–333
 defined, 326, 371
 four-step problem solving
 process, 326
 problem solving, 326–333

three-dimensional
 problems, 332–333
 two-dimensional problems,
 326–331

logic
 bubble, in proof, 55–57, 88
 everyday approach (song
 lyrics), 15–16
 if-then, 51–53, 56, 81–83

• M •

main diagonal, 170
 mathematikoī, 108
 measurement. *See also* pi (π);
specific topics
 angle, 30, 36–39, 42–44, 154,
 199–201
 angle of circle, 250–256,
 363, 364
 arc, 233, 244
 segment, 35–36, 154
 median, 102, 188, 371
 Mesopotamia, geometry in, 13
 Midline Theorem, 211–212
 midpoint
 center of rotation, 319–320
 centroid of triangle,
 102–104, 368
 defined, 41, 371
 formula, 295, 297, 300, 365
 Like Divisions Theorem,
 70–73
 main reflecting line of glide
 reflection (theorem),
 322–324
 point excluded from
 problem solving, 330
 problem solving with,
 297, 300
 reflecting line, 311–312, 316
 minute, 371
 mirror. *See* reflection
 moon, size of, 229
 motion of projectile, 355–356

• **N** •

- nautilus, golden ratio, 353
- negative slope, 293
- Newton, Isaac
 - (mathematician), 13, 351
- n -gon, diagonals in, 201–202, 363. *See also* polygon
- non-collinear points, 25, 269, 371
- non-coplanar, 28–29, 371
- non-coplanar points, 26
- numerics
 - 1-D (one-dimensional) shapes, 12, 14
 - 2-D (two-dimensional) geometry, 12–14, 326–331
 - 3 : 4 : 5 family of
 - Pythagorean triple, 115–117
 - 3-4-5 Pythagorean triple, 114
 - 3-D (three-dimensional) geometry. *See* 3-D (three-dimensional) geometry
 - 5-12-13 Pythagorean triple, 114
 - 7-24-25 Pythagorean triple, 114
 - 8 : 15 : 17 family of
 - Pythagorean triple, 116
 - 8-15-17 Pythagorean triple, 114
 - 30°- 60°- 90° (right) triangle, 94, 120–122, 190–191, 196–197
 - 45°- 45°- 90° (isosceles right) triangle, 118–120, 197–198

• **O** •

- oblique, 28
- obtuse angle, 30, 31, 96, 371
- obtuse triangle, 96, 98, 371
- octagon, 197–199
- 1-D (one-dimensional) geometry, 12, 14
- ordered pair, 290
- orientation, 308, 309–310, 371

- origin, 290, 304–306
- orthocenter of triangle, 104, 106, 371

• **P** •

- pairs
 - angles, 31–33, 156
 - ordered, on coordinate plane, 290
 - triangle congruence, 123–134
- parabola, 355–356, 357
- parallel line
 - constructing, 341–342
 - construction, 341–342
 - coplanar, 27
 - defined, 27, 371
 - determining a plane, 269–271
 - diagrams, 27
 - line parallel to a given line through point not on given line, 341
 - naming, 27
 - properties, 151–157
 - proving that angles are parallel (theorem), 153
 - slopes, 293, 294
 - transversal. *See* transversal
- parallel plane, determining, 269–271
- parallel-line theorems, 152–153, 178–179, 346–347
- parallelogram
 - angles, 163, 164
 - area, 188–191, 362
 - defined, 157, 372
 - diagonals, 163, 164–165
 - main reflecting line of glide reflection, 322–324
 - proofs, 162–186
 - properties, 162–166, 173–186
 - property-proof connection, 173–186
 - quadrilateral relationships, 158–159
 - reversible properties, 174
 - sides, 163, 164
 - special, 166–169

- Parthenon, 353
- pentagon, 372
- perimeter
 - circumference of circle, 244, 246, 363, 368
 - defined, 372
 - distance formula, 297
 - similar polygon, 205, 208–209
- perpendicular, defined, 372
- perpendicular bisector
 - circumcenter of triangle, 104, 105
 - constructions, 338–339
 - defined, 372
 - equidistance theorems, 141–144
 - reflecting line as, 310–312
 - of a segment, 338–339
 - triangle congruence, 141–144
- perpendicular line
 - bisected chords theorem, 229
 - construction, 339–342
 - line perpendicular to plane, 265–268
- Line-Plane Perpendicularity Theorem, 265–268
- perpendicular to plane, 265–268
- radius-tangent perpendicularity, 237–238
- right angle, 30
- slopes, 293, 294
- through a point not on a given line, 340
- through a point on a given line, 339–340
- pi (π)
 - area of circle, 176–177, 244, 247–248
 - calculating, 352
 - circumference of circle, 244, 246, 363, 368
 - defined, 5
 - Pi Day (March 14), 123
 - pizza slice. *See* sector of circle

- plane
- 2-D (two-dimensional)
 - geometry, 12–14, 326–331
 - 3-D locus problems, 333
 - coordinate (Cartesian), 289–291, 369
 - defined, 24, 265, 372
 - determining, 269
 - diagram, 24
 - intersecting line, 270–272
 - intersecting plane, 29, 266–272
 - Line-Plane Perpendicularity Theorem, 265–268
 - lines, 265–268, 270–272
 - naming, 24
 - parallel, 29, 270
 - types, 29
- point. *See also* locus (loci), locating and constructing
- circle centered at, and circle equation, 304–306
 - defined, 22, 25, 372
 - determining a plane, 269
 - diagram, 23
 - equidistance theorem, 143–144
 - excluded from problem solving, 330
 - intersection, 28
 - line, determining, 347
 - point-slope form, 303, 306, 366
 - shortest distance between two points, 275–276
 - two points determine a line postulate, 160, 347
 - types of. *See* point, type of
- point, type of
- collinear, 25, 26, 369
 - coplanar, 25, 26
 - endpoint. *See* endpoint
 - equidistant, 141, 143
 - midpoint. *See* midpoint
 - non-collinear, 25, 269, 371
 - non-coplanar, 26
 - of tangency, 237, 330
- point-slope form, 303, 306, 366
- pointy-top object
- cone, 279
 - defined, 365
 - diagram, 279–285
 - lateral area, 280–281
 - pyramid, 279
 - surface area, 281–285, 365
 - volume, 280, 281–282, 284, 365
 - water into wine problem, 283
- polygon
- align similar, 205–207
 - angles of, 199–201, 363
 - area, 195–199, 245, 362
 - defined, 12, 372
 - formulas, 362–363
 - pentagon, 372
 - perimeter, 205, 208–209
 - properties, 362–363
 - radius, 372
 - regular, 195–199, 362, 373
 - similar, 204–209
 - vertex, 374
- polygon, type of
- 3-sided (triangle). *See* triangle
 - 4-sided (quadrilateral). *See* quadrilateral
 - 5-sided (pentagon), 372
 - 6-sided (hexagon), 110–111, 196–197
 - 8-sided (octagon), 197–199
 - n -sided, 201–202, 363
 - regular, 195–199, 245, 279, 362, 373
- positive slope, 293
- postulates
- AA (Angle-Angle), 209, 210, 350
 - ASA (Angle-Side-Angle), 128–130, 349
 - defined, 17, 372
 - Euclid's, 17
 - SAS. *See* Side-Angle-Side (SAS)
 - SSS. *See* Side-Side-Side (SSS)
 - two points determine a line, 347
 - use in proofs, 54–55, 173–175
- practical uses of geometry, 18–19
- pre-image, 307, 372
- prime mark, for reflections, 308
- Principia Mathematica* (Newton), 13
- prism
- defined, 273, 372
 - flat-top objects, 273
 - lateral area, 276
 - right, 273
 - surface area, 274, 278, 365
 - volume, 274, 277, 365
- problem solving, 326–333
- projectile motion, 355–356
- proof. *See* geometry proof
- properties. *See also* formulas
- circle, 363–364
 - coordinate system, 365–366
 - defined, 162
 - half, 170
 - isosceles trapezoid, 172
 - kite, 169–171
 - parallel line, 151–157
 - parallelogram, 162–166, 173–186
 - polygon, 362–363
 - property-proof connection, 173–186
 - quadrilateral, 151–157, 162–172
 - rectangle, 167–169, 174
 - Reflexive Property, 132–133, 345–346
 - reversible, 174–175
 - rhombus, 167–169, 174
 - square, 167–169
 - Substitution, 75–78
 - 3-D (three-dimensional) geometry, 365
 - Transitive Property, 75–78
 - transversal, 152–157
 - trapezoid, 171–172
 - triangle, 361–362
- property-proof connection, quadrilaterals, 173–186

proportions
 Angle-Bisector Theorem, 223–224
 CSSTP (corresponding sides of similar triangles are proportional), 213–216
 Side-Splitter Theorem, 219–222
 similar figures, 204–205
prove statement, geometry proof, 50, 51, 66, 141
 pyramid
 defined, 279, 372
 Egyptian, 13, 354–355
 lateral area, 280
 pointy-top objects, 279
 regular, 279
 surface area, 281, 282–283, 365
 volume, 280, 281, 282, 365
 Pythagoras (mathematician), 13, 108
 Pythagorean Theorem
 coordinate system, 291, 294–295
 defined, 108, 361
 distance formula, 294
 historical view, 107–108
 problem solving with, 108–113
 Pythagorean triple triangles, 113–118, 122

• Q •

quadrant, coordinate system, 290
 quadrilateral. *See also specific quadrilaterals*
 area, 187–195, 362
 area formula, 188–190
 child and parent, 174
 coordinate geometry proof, 296–297
 defined, 12, 151, 157, 372
 parallel-line properties, 151–157
 proofs, 162–186

properties of, 151–157, 162–172
 property-proof connection, 173–186
 transversal theorems, 152–157
 type of, 157–159

• R •

radius of circle
 all-radii-are-congruent theorem, 229, 347
 and bisected chords theorem, 229
 defined, 228, 229, 372
 extra, for problem solving, 231–232
 radius of regular polygon, 372
 radius of sphere, 285
 radius-tangent perpendicularity, 237–238
 ratio. *See also* slope
 3 : 4 : 5 family of Pythagorean triples, 115–117
 8 : 15 : 17 family of Pythagorean triples, 116
 for area of rhombus, 191–192
 families of Pythagorean triple triangle, 115–118
 scalene triangle, 93
 special right triangles, 122, 362
 ray
 angle bisector as, 42, 104, 105, 367
 as angle side, 39
 defined, 23, 372
 diagram, 23
 horizontal, 26, 27
 naming, 23
 parallel, 27
 type of, 26–27
 vertical, 26, 27
 reason column, geometry proof, 50, 51, 53–54
 reasoning
 deductive, 2, 15, 43, 54, 56
 indirect proofs, 148

rectangle
 area formula, 188, 189, 362
 coordinate geometry proof, 296–297
 defined, 157, 372
 golden, 353
 lateral, and flat-top figures, 274
 proof that quadrilateral is, 180–182
 properties, 167–169, 174
 Pythagorean Theorem, 109
 quadrilateral relationships, 158–159, 166–169
 reversible properties, 174
 reflecting line
 defined, 310
 equation of, 316, 318–321
 as perpendicular bisector, 310–312
 reflection. *See also* glide reflection
 defined, 308, 372
 lines, 310–312, 318–324
 main reflecting line, 322–324
 number of, 309
 orientation, 309–310
 reflex angle, 30, 31
 Reflexive Property, 132–133, 345–346
 regular polygon
 area, 195–199, 245, 362
 defined, 195, 373
 pointy-top figures, 279
 reversible properties, 174–175
 rhombus
 area, 188, 191–192, 362
 defined, 157, 373
 proof that quadrilateral is, 182–184
 properties, 167–169, 174
 quadrilateral relationships, 158–159, 166–169
 reversible properties, 174
 right angle, 30, 31, 32, 373
 right circular cone, 279

- right circular cylinder, 274
right prism, 273
right triangle
 30° - 60° - 90° (right) triangle,
 94, 120–122, 190–191,
 196–197
 45° - 45° - 90° (isosceles right)
 triangle, 118–120, 197–198
altitude, 98
Altitude-on-Hypotenuse
 Theorem, 216–218, 362
area, 110–111, 361
for area of parallelogram,
 188–191, 362
defined, 96, 373
formulas, 361
hypotenuse, 108
leg (side), 108
pyramid, 281
Pythagorean triple triangle,
 113–118, 122
ratio of sides, 362
special, 122, 362
and trapezoids, 194–195
rise, 292, 295
rotation
 center of, 317, 318–320
 defined, 317, 373
 equals two reflections
 (theorem), 309, 317–318
 equations of two reflecting
 lines, 318–321
rotation angle, 317
round-robin tennis, 202
run, 292, 295
- S ●
- same-side exterior angle,
 153, 373
same-side interior angle,
 153, 373
SAS (Side-Angle-Side)
 triangle congruence,
 126–128, 130, 349
 triangle similarity, 209,
 212–213, 350
scalene triangle, 91, 92–93,
 98, 373
secant, 254, 373
secant-secant angles,
 254, 260–262, 364
Secant-Secant Power
 Theorem, 260–262, 364
secant-tangent angle, of
 circle, 254
second, 373
sector of circle
 area of, 247–249, 363
 defined, 247, 373
segment
 addition theorem,
 39, 63–64, 70
 auxiliary lines in proof,
 159–161
 bisecting, 40, 338–339
 bisector of, 338–339
 of circle, 243–244, 247, 248,
 249, 373
 congruent, 36, 369
 copying, 334–335
 defined, 22, 243, 371
 diagram, 23, 44–45
 dividing into equal
 parts, 342
 equidistance theorem,
 143–144
 horizontal, 26, 27
 length of, 36
 measurement, 35–36, 154
 midpoint, 41
 naming, 22, 36
 parallel, 27
 perpendicular bisector of a
 segment, 338–339
 subtraction theorem,
 39, 67–68
 transversal theorems,
 152–157, 341
 trisecting, 41
 vertical, 26, 27
semiperimeter, 100
set (locus), 325
shapes, 12–14
side. *See also* leg; *specific
geometric shapes*
 adjacent, 367
 corresponding, 204–205
 defined, 23
 equilateral triangle, 93–91
 if-angles-then-sides
 theorem, 135, 348
 if-sides-then-angles
 theorem, 135, 348
 included, 128
 isosceles triangle, 93
 parallelogram, 163, 164
 polygon. *See* polygon
 right triangle, 362
 Side-Splitter Theorem,
 219–222
 triangle, 91–94, 362
 Side-Angle-Side (SAS)
 triangle congruence,
 126–128, 130, 349
 triangle similarity,
 212–213, 350
 Side-Side-Side (SSS)
 triangle congruence,
 124–126, 130, 349
 triangle similarity, 209,
 211–212, 350
 Side-Splitter Theorem,
 219–222
 similar, defined, 373
 similarity. *See also* isometry
 aligning polygon, 205–207
 Altitude-on-Hypotenuse
 Theorem, 216–218
 Angle-Bisector Theorem,
 223–224
 and congruence, 203
 defined, 203, 373
 polygons, 204–209
 Side-Splitter Theorem,
 219–222
 triangle. *See* triangle
 similarity
 skew, 28
 slant height, 280, 373
 slide. *See* glide reflection
slope
 center of rotation, 319–320
 defined, 291, 373
 formula, 292, 365
 incline, 292–293
 line equations, 302–303
 negative, 293

- slope (*continued*)
 - point-slope form, 303, 306, 366
 - positive, 293
 - problem solving with, 296
 - reflecting line as perpendicular bisector, 311–312, 316
 - undefined, 292, 293, 303
 - slope-intercept form, 302, 306, 366
 - slope-intercept form, 302, 306, 366
 - soccer balls, 357–358
 - solid geometry. *See also* 3-D (three-dimensional) geometry
 - flat-top figures, 273–279
 - pointy-top objects, 279–285
 - space, 3-D, 24
 - special figures
 - parallelogram, 166–169
 - right triangle, 122, 362
 - sphere
 - defined, 285, 373
 - surface area, 285–286, 365
 - volume, 285, 365
 - spiral, golden ratio, 353
 - square
 - area, 188, 362
 - area of octagon, 198–199
 - defined, 157, 374
 - proof that quadrilateral is, 184
 - properties, 167–169
 - Pythagorean triple triangle, 114
 - quadrilateral relationships, 158–159, 166–169
 - square roots, Pythagorean Theorem, 112, 113, 118, 122
 - SSS (Side-Side-Side) triangle congruence, 124–126, 130, 349
 - triangle similarity, 209, 211–212, 350
 - statement column, geometry proof, 50, 51
 - straight angle, 30, 31, 374
 - straight line, 44
 - straightedge, 334–342, 369
 - Substitution Property, 75–78
 - subtracting
 - angles, 40
 - arcs, 235
 - segments, 39, 67–68
 - supplementary angle, 32, 59–62, 374
 - supplementary angle theorems, 59–62, 153, 156
 - surface area. *See also* area
 - 3-D geometry, 365
 - cone, 284–285, 365
 - cylinder, 274, 278–279, 365
 - flat-top objects, 274, 278–279, 365
 - pointy-top objects, 281–285, 365
 - prism, 274, 278, 365
 - pyramid, 282–283, 365
 - sphere, 285–286, 365
- T ●
- tangent
 - to circle, 237–241, 259–260, 369, 374
 - common-tangent, 238–241, 369
 - Dunce Cap Theorem, 241–242
 - equation of, 305–306
 - walk-around problem, 241–242
 - tangent line, 237–238
 - tangent-chord angle, 250–251
 - tangent-secant angle, 254, 259–260, 364
 - Tangent-Secant Power Theorem, 259–260, 262, 364
 - tangent-tangent angle, of circle, 254
 - theorem
 - AAS (Angle-Angle-Side), 137–139, 349
 - addition, 39, 62–67, 70
 - all-radii-are-congruent, 229, 347
 - Altitude-on-Hypotenuse, 216–218, 362
 - angle-arc, 250–257
 - Angle-Bisector, 223–224
 - arc theorems, 233–234
 - bisected chords, 229
 - central angle, 233–234
 - chord theorems, 229, 234
 - Chord-Chord Power, 257–259, 262, 364
 - circle, 229, 233–235, 257–262, 347
 - complementary angle, 59–62
 - defined, 17, 374
 - Dunce Cap, 241–242
 - equidistance, 141–144, 229
 - Euclid's, 17
 - glide reflection equals three reflections, 309, 322–323
 - HLR (hypotenuse-leg-right angle), 139–141, 349
 - if-angles-then-sides, 135, 267, 348
 - if-sides-then-angles, 135, 348
 - isosceles triangle, 134–137
 - Like Divisions, 70–73
 - Like Multiples, 70–73
 - Line-Plane Perpendicularity, 265–266
 - main reflecting line of glide reflection, 322–324
 - Midline, 211–212
 - parallel-line, 152–153, 178–179, 346–347
 - perpendicularity and bisected chords theorem, 229
 - proving that angles are congruent, 152, 153, 156
 - Pythagorean. *See* Pythagorean Theorem
 - radius, 229
 - rotation equals two reflections, 309, 317–318
 - Secant-Secant Power, 260–262, 364
 - Side-Splitter, 219–222
 - subtraction, 39, 67–68
 - supplementary angle, 59–62, 153, 156
 - Tangent-Secant Power, 259–260, 262, 364
 - translation equals two reflections, 309, 312–314

- transversal theorems, 152–157, 341
- triangle, 134–136, 139–141
- use in proofs, 54–55, 59–75
- vertical-angles-as-congruent, 73–75, 346
- 30°-60°-90° (right) triangle, 94, 120–122, 190–191, 196–197
- 3-D (three-dimensional) geometry
- base, 368
- cone. *See* cone
- cylinder. *See* cylinder
- determining a plane, 269
- flat-top figures, 273–279
- formulas and properties, 365
- height, 354–355, 370
- intersecting line and plane, 270–272
- line perpendicular to plane, 265–268
- locus (loci), 332–333
- overview, 12, 13–14
- pointy-top figures, 279–285
- problems, 332–333
- pyramid. *See* pyramid
- spheres. *See* sphere
- surface area. *See* surface area
- volume. *See* volume
- 3-D (three-dimensional) space, 24
- torus, 333
- transformation, 307, 374
- Transitive Property, 75–78
- translation
- defined, 309, 312
- distance, 313, 315
- elements of, 314–316
- equals two reflections (theorem), 309, 312–314
- line, 313, 315–316
- proof, 313–316
- transversal
- angles are congruent (theorem), 152, 156
- angles are parallel (theorem), 153
- defined, 152, 374
- multiple, parallel lines with, 155–157
- parallel-line properties, 152–157
- supplementary angle theorem, 59–62, 153, 156
- theorems, 152–157, 341
- trapezoid
- area, 188, 189–190, 194–195, 362
- bases, 157, 368
- defined, 157, 374
- isosceles, 157, 368, 370
- legs, 157, 371
- median, 188, 371
- properties, 171–172
- quadrilateral relationships, 158–159
- triangle
- altitude, 96–98, 104, 106, 111
- angles, 96
- area, 96–101, 109–113, 196, 300–302, 361
- for area of rhombus, 191–192
- for area of trapezoid, 194–195
- base, 96
- center of, 102–106
- centroid, 102–104, 368
- congruence. *See* triangle congruence
- copying, 336–337
- defined, 374
- equiangular, 93
- formulas, 361–362
- geodesic domes, 357
- if-angles-then-sides theorem, 348
- if-sides-then-angles theorem, 348
- inequality principle, 94–95
- infinite series of, 103
- lateral, and pointy-top figures, 280
- median, 102, 371
- properties, 361–362
- Pythagorean triple, 113–118
- semiperimeter, 100
- sides of, 91–94
- similar. *See* triangle similarity
- type of. *See* triangle, type of
- triangle, type of
- acute, 96, 98, 367
- equilateral. *See* equilateral triangle
- isosceles. *See* isosceles triangle
- obtuse, 96, 98, 371
- right. *See* right triangle
- scalene, 91, 92–93, 98, 373
- triangle congruence
- AAS (Angle-Angle-Side), 137–139, 349
- ASA (Angle-Side-Angle), 128–130, 349
- CPCTC (corresponding parts of congruent triangles are congruent), 131–134, 349
- defined, 124, 131–132, 349
- versus equidistance theorem, 141–144
- geometric proofs, 123–141, 145–148
- HLR (Hypotenuse-leg-right angle) theorem, 139–141
- isosceles triangle theorems, 134–137
- for pairs, 123–134
- SAS (Side-Angle-Side), 126–128, 130, 349
- SSS (Side-Side-Side), 124–126, 130, 349
- triangle inequality principle, 94–95
- triangle similarity
- AA (Angle-Angle), 209, 210, 350
- CASTC (corresponding angles of similar triangles are congruent), 213–214
- CSSTP (corresponding sides of similar triangles are proportional), 213–216
- defined, 350
- proof, 209–216

triangle similarity (*continued*)
 Pythagorean triple
 triangle, 116
 SAS (Side-Angle-Side), 209,
 212–213, 350
 SSS (Side-Side-Side), 209,
 211–212, 350
 trigonometry, 13
 trisect(ing)
 angle, 42–43, 338
 defined, 41, 374
 Like Divisions Theorem,
 70–73
 segment, 41
 trisection point, 41
 trisector, angle, 42, 338
 Tsu Chung-Chin
 (mathematician), 352
 two equidistant points
 determine the
 perpendicular bisector
 (equidistance theorem),
 141–143
 two points determine a line
 postulate, 160
 2-D (two-dimensional)
 geometry, 12–14,
 326–331
 two-column format,
 geometry proof,
 17, 55–57

• u •

undefined slope, 292, 293, 303
 union of two objects, 374
 unit segment, 36

• v •

vertex (vertices)
 of angle, 23, 374
 of polygon, 374
 of prism, 273
 of pyramid, 279
 vertex angle, 42, 93, 223, 346
 vertical, as term, 26
 vertical (y) axis, 290. *See also*
 coordinate system
 vertical angle, 73
 vertical distance formula, 294
 vertical line
 equation of, 303, 366
 slope, 292, 293, 303
 vertical-angles-as-congruent
 theorem, 73–75, 346
 volume
 3-D objects, 365
 cone, 280, 284, 365
 crown in Archimedes'
 bathtub, 351–352
 cube, 274, 285–286
 cylinder, 274, 279, 365
 defined, 374
 flat-top objects, 274, 277, 365
 pointy-top objects, 280,
 281–282, 284, 365
 prism, 274, 277, 365
 pyramid, 280, 282, 365
 sphere, 285, 365

• w •

walk (glide) reflection, 309,
 321–322, 370
 walk-around problem, 241–242

Wantzel, Pierre
 (mathematician), 338
 Washington, George (U.S.
 President), 13
 wine-into-water volume
 problem, 283

• x •

X angles, 73
 x-axis, 290. *See also*
 coordinate system
 x-intercept
 circle equation, 305
 and slope, 303
 x-y coordinate system. *See*
also coordinate system
 defined, 289–290, 369
 distance formula, 294

• y •

y-axis, 290. *See also*
 coordinate system
 y-intercept
 circle equation, 305
 and slope, 302–303

• z •

Z-angle, 161–162
 zero slope, 293, 294, 303