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MAGNETIC FIELD IMAGING

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INTRODUCTION

Magnetic field mapping is the production of maps or images of magnetic fields in space. Magnetic field maps are needed for designing and optimizing of magnets used in particle accelerators, spectrometers (mass, nuclear magnetic resonance, and electron paramagnetic resonance), and magnetic resonance imaging systems. Magnetic field maps are also used in geologic exploration where the variations in the magnitude and direction of the earth's magnetic field are indicative of subsurface features and objects. Field mapping relies on various methods of measuring the magnetic field, generally one point at a time. These measurement methods are the main focus of this article.

It is curious to note that most measurement methods have remained virtually unchanged for a very long period, but the equipment has been subject to continual development. In the following, only the more commonly used methods will be discussed. These methods are complementary and a wide variety of the equipment is readily available from industry. For the many other existing measurement methods, a more complete discussion can be found in two classical bibliographical reviews (1,2). An interesting description of early measurement methods can be found in (3). Much of the following material was presented at the CERN Accelerator School on Measurement and Alignment of Accelerator and Detector Magnets (4). Those proceedings contain a recent compendium of articles in this field and form a complement to the classical reviews.

MEASUREMENT METHODS

Before computers became common tools, electromagnets were designed by using analytical calculations or by measuring representative voltage maps in electrolytic tanks and resistive sheets. Magnetic measurements on the final magnets and even on intermediate magnet models were imperative at that time.

Nowadays, it has become possible to calculate the strength and quality of magnetic fields with impressive accuracy. However, the best and most direct way to verify that the expected field quality has been reached is magnetic measurements on the finished magnet. It is also the most efficient way of verifying the quality of series produced electromagnets to monitor tooling wear during production.

Choice of Measurement Method

The choice of measurement method depends on several factors. The field strength, homogeneity, variation in time, and the required accuracy all need to be considered. The number of magnets to be measured can also determine the method and equipment to be deployed. As a guide, Fig. 1 shows the accuracy that can be obtained in an absolute measurement as a function of the field level, using commercially available equipment. An order of magnitude may be gained by improving the methods in the laboratory.

Magnetic Resonance Techniques

The nuclear magnetic resonance technique is considered the primary standard for calibration. It is frequently used for calibration purposes and also for high precision field mapping. The method was first used in 1938 (5,6) for measuring the nuclear magnetic moment in molecular beams. A few years later two independent research teams observed the phenomenon in solids (7–9). Since then, the method has become the most important way of measuring

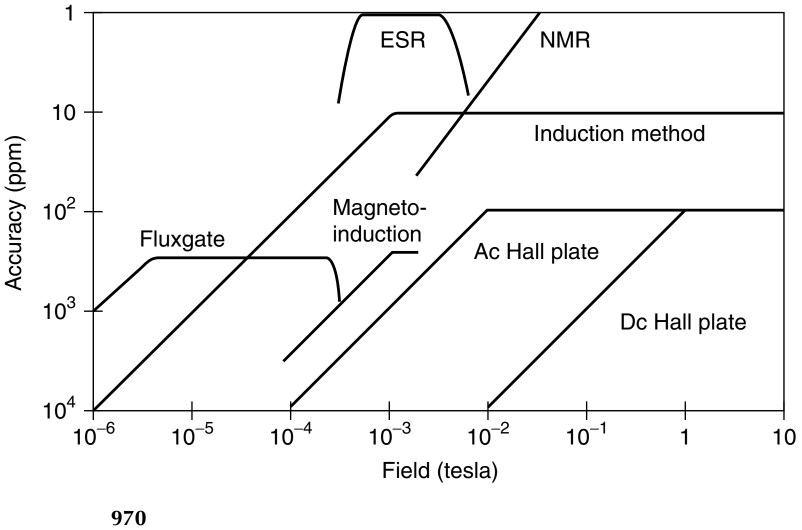

Figure 1. Measurement methods: Accuracies and ranges.

magnetic fields with very high precision. Because it is based on an easy and precise frequency measurement, it is independent of temperature variations. Commercially available instruments measure fields in the range from 0.011 T up to 13 T at an accuracy better than 10 ppm. Commercial units are also available for measuring weaker magnetic fields, such as the earth's magnetic field (30 to 70 μT), but at lower accuracy.

In practice, a sample of water is placed inside an excitation coil, powered from a radio-frequency oscillator. The precession frequency of the nuclei in the sample is measured either as nuclear induction (coupling into a detecting coil) or as resonance absorption (10). The measured frequency is directly proportional to the strength of the magnetic field whose coefficients are 42.57640 MHz/T for protons and 6.53569 MHz/T for deuterons. The magnetic field is modulated with a low-frequency signal to determine the resonance frequency (11).

The advantages of the method are its very high accuracy, its linearity, and the static operation of the system. The main disadvantage is the need for a rather homogeneous field to obtain a sufficiently coherent signal. A small compensation coil that is formed on a flexible printed circuit board and provides a field gradient may be placed around the probe when used in a slightly inhomogeneous field. A correction of the order of 0.2 T/m may be obtained (11). The limited sensitivity and dynamic range also set limits to this method's suitability. It is, however possible to use several probes with multiplexing equipment, if a measurement range of more than half a decade is needed.

Pulsed NMR measurements have been practiced for various purposes (12,13), even at cryogenic temperatures (14). But equipment for this type of measurement is not yet commercially available.

Finally, it should be mentioned that a rather exotic method of NMR measurement using water flowing in a small tube has given remarkably good results in low fields (15–17). It fills the gap in the measurement range up to 11 mT, for which NMR equipment is not yet commercially available. In addition, it provides a method of measurement in strong ionizing radiation such as in particle accelerators. It was tested for measurements in the bending magnets installed in the CERN Large Electron Positron collider (LEP). A resolution of 0.0001 mT was reached in the range from the remanent field of 0.5 mT up to the maximum field of 112.5 mT, and corresponding reproducibility was observed (18). The remarkable sensitivity and resolution of this measurement method makes it suitable for absolute measurements in low fields. In fact, it was even possible to detect the earth's magnetic field outside the magnet, corresponding to an excitation frequency of about 2 kHz. However, the operation of this type of equipment is rather complicated due to the relatively long time delays in the measurement process.

Electron spin resonance (ESR) (19–22) is a related and very precise method for measuring weak fields. It is now commercially available in the range from 0.55–3.2 mT, has a reproducibility of 1 ppm, and is a promising tool in geology applications.

Magnetic resonance imaging (MRI) has been proposed for accelerator magnet measurements (23). It is a very promising technique, which has proven its quality in other applications. However, the related signal processing requires powerful computing facilities, which were not so readily available in the past.

The Fluxmeter Method

This method is based on the induction law. The change of flux in a measurement coil will induce a voltage across the coil terminals. It is the oldest of the currently used methods for magnetic measurements, but it can be very precise (24). It was used by Wilhelm Weber in the middle of the last century (25) when he studied the variations in the strength and direction of the earth's magnetic field. Nowadays, it has become the most important measurement method for particle accelerator magnets. It is also the most precise method for determining the direction of magnetic flux lines; this is of particular importance in accelerator magnets. The coil geometry is often chosen to suit a particular measurement. One striking example is the Fluxball (26) whose complex construction made it possible to perform point measurements in inhomogeneous fields.

Measurements are performed either by using fixed coils in a dynamic magnetic field or by moving the coils in a static field. The coil movement may be rotation through a given angle, continuous rotation, or simply movement from one position to another. Very high resolution may be reached in field mapping by using this method (27).

Very high resolution may also be reached in differential fluxmeter measurements using a pair of search coils connected in opposition, where one coil moves and the other is fixed, thus compensating for fluctuations in the magnet excitation current and providing a much higher sensitivity when examining field quality. The same principle is applied in harmonic coil measurements, but both coils move. A wide variety of coil configurations is used, ranging from the simple flip-coil to the complex harmonic coil systems used in fields of cylindrical symmetry.

Induction Coils

The coil method is particularly suited for measurements with long coils in particle accelerator magnets (28,29), where the precise measurement of the field integral along the particle trajectory is the main concern. Long rectangular coils were usually employed and are still used in magnets that have a wide horizontal aperture and limited gap height. In this case, the geometry of the coil is chosen to link with selected field components (30). The search coil is usually wound on a core made from a mechanically stable material to ensure a constant coil area, and the wire is carefully glued to the core. Special glass or ceramics that have low thermal dilatation are often used as core materials. During coil winding, the wire must be stretched so that its residual elasticity assures well-defined geometry and mechanical stability of the coil.

Continuously rotating coils that have commutating polarity were already employed in 1880 (3). The harmonic coil method has now become very popular for use in circular cylindrical magnets, in particular, superconducting beam transport magnets. The coil support is usually a rotating cylinder. This method has been developed since 1954 (31,32). The induced signal from the rotating coil was often transmitted through slip rings to a frequency selective amplifier (frequency analyzer), thus providing analog harmonic analysis.

The principle of a very simple harmonic coil measurement is illustrated in Fig. 2. The radial coil extends through the length of the magnet and is rotated around the axis of the magnet. As the coil rotates, it cuts the radial flux lines. Numerous flux measurements are made between predefined angles. This permits precise and simultaneous determination of the strength, quality, and geometry of the magnetic field. A Fourier analysis of the measured flux distribution results in a precise description of the field parameters in terms of the harmonic coefficients:

$$B_{\mathrm{r}}(r,\varphi) = B_{\mathrm{o}} \sum_{n=1}^{\infty} \left(\frac{r}{r_{\mathrm{o}}}\right)^{n-1} (b_n \cos n\varphi + a_n \sin n\varphi)$$

where B_{o} is the amplitude of the main harmonic and r_{o} is a reference radius. b_n and a_n are the harmonic coefficients. In this notation b_1 will describe the normal dipole coefficient, b_2 the normal quadrupole coefficient, etc. The corresponding skew field components are described by the coefficients a_1, a_2 etc.

Due to the advent of modern digital integrators and angular encoders, harmonic coil measurements have improved considerably and are now considered the best choice for most types of particle accelerator magnets, in particular those designed with cylindrical symmetry (33). In practice, the coil is rotated one full turn in each angular direction, and the electronic integrator is triggered at the defined angles by an angular encoder connected to the axis of the coil. To speed up the calculation of the Fourier series, it is an advantage to choose n equally spaced measurement points, where n is a power of 2 (e.g., 512). A compensating coil, connected in series and rotated with the main coil, may be used to suppress the main field component and thus increase the sensitivity of the system for measuring field quality. Dynamic fields are measured with a static coil linking to selected harmonics (34). The harmonic coil measurement principle and its related equipment are described in detail in (35). A thorough description of the general theory, including detailed error analysis, can be found in (36). The practical use of the harmonic coil method for large-scale measurements in superconducting magnets is described in (37,38) and more recent developments are in (39–43)

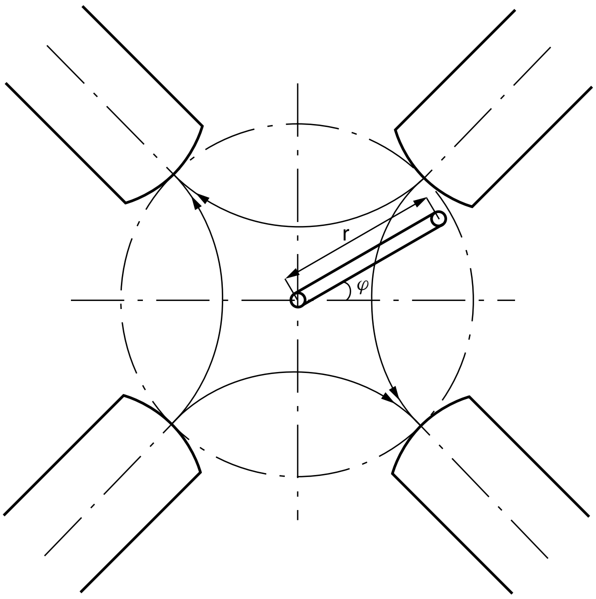

Figure 2. Harmonic coil measurement.

Another induction measurement method consists of moving a stretched wire in a magnetic field, thus integrating the flux cut by the wire (44). It is also possible to measure the flux change while varying the field and keeping the wire in a fixed position. Tungsten is often selected as a wire material, if the wire cannot be placed in a vertical position. The accuracy is determined by the mechanical positioning of the wire. Sensitivity is limited but can be improved by using a multiwire array. This method is well suited to geometric measurements, to the absolute calibration of quadrupole fields, and in particular to measurements in strong magnets that have very small apertures.

The choice of geometry and method depends on the useful aperture of the magnet. The sensitivity of the fluxmeter method depends on the coil surface and on the quality of the integrator. The coil–integrator assembly can be calibrated to an accuracy of a few tens of ppm in a homogeneous magnetic field by reference to a nuclear magnetic resonance probe, but care must be taken not to introduce thermal voltages in the related cables and connectors. Induced erratic signals from wire loops exposed to magnetic flux changes must also be avoided. One must measure the equivalent surface of the search coil and also its median plane which often differs from its geometric plane due to winding imperfections. In long measurement coils, it is important to ensure very tight tolerances on the width of the coil. If the field varies strongly over the length of the coil, it may be necessary to examine the variation of the effective width.

The main advantage of search coil techniques is the possibility of very flexible coil design. The high stability of the effective coil surface is another asset. The linearity and the wide dynamic range also play important roles. The technique can be easily adapted to measurements at cryogenic temperatures. After calibration of the coils at liquid nitrogen temperature, only a minor correction has to be applied for use at lower temperatures. On the other hand, the need for relatively large induction coils and their related mechanical apparatus, which is often complex, may be a disadvantage. Finally, measurements with moving coils are relatively slow.

Flux Measurement

Induction coils were originally used with ballistic galvanometers and later with more elaborate fluxmeters (45). The coil method was improved considerably by the development of photoelectric fluxmeters (46) which were used for a long time. The measurement accuracy was further improved by the introduction of the classic electronic integrator, the Miller integrator. It remained necessary, however, to employ difference techniques for measurements of high precision (47). Later, the advent of digital voltmeters made fast absolute measurements possible, and the Miller integrator has become the most popular fluxmeter. Due to the development of solid-state dc amplifiers, this integrator has become inexpensive and is often used in multicoil systems.

Figure 3 shows an example of such an integrator. It is based on a dc amplifier that has a very low input voltage offset and a very high open-loop gain. The thermal variation of the integrating capacitor (C) is the most critical problem. Therefore, integrating components are mounted in a temperature-controlled oven. Another problem is the decay of the output signal through the capacitor and the resetting relay. So, careful protection and shielding of these components is essential to reduce the voltages across the critical surface resistances.

The dielectric absorption of the integrating capacitor sets a limit to the integrator precision. A suitable integrating resistor is much easier to find. Most metal-film resistors have stabilities and temperature characteristics that match those of the capacitor. The sensitivity of the integrator is limited by the dc offset and the low-frequency input noise of the amplifier. A typical value is 0.5 μV, which must be multiplied by the measurement time to express the sensitivity in terms of flux. Thermally induced voltages may cause a problem, so care must be taken in choosing of cables and connectors. In tests at CERN, the overall stability of the integrator time constant proved to be better than 50 ppm during a period of three months. A few electronic fluxmeters have been developed by industry and are commercially available.

In more recent years, a new type of digital fluxmeter has been developed, which is based on a high-quality dc amplifier connected to a voltage-to-frequency converter (VFC) and a counter. The version shown in Fig. 4 was developed at CERN and is now commercially available. The input of the VFC is given an offset of 5 V to provide a true bipolar measurement. This offset is balanced by a 500-kHz signal which is subtracted from the output of the VFC. Two counters are used to measure with continuously moving coils and to provide instant readings of the integrator. One of the counters can then be read and reset while the other is active. In this way, no cumulative errors will build up. The linearity of this fluxmeter is 50 ppm. Its sensitivity is limited by the input amplifier, as in the case of an analog amplifier.

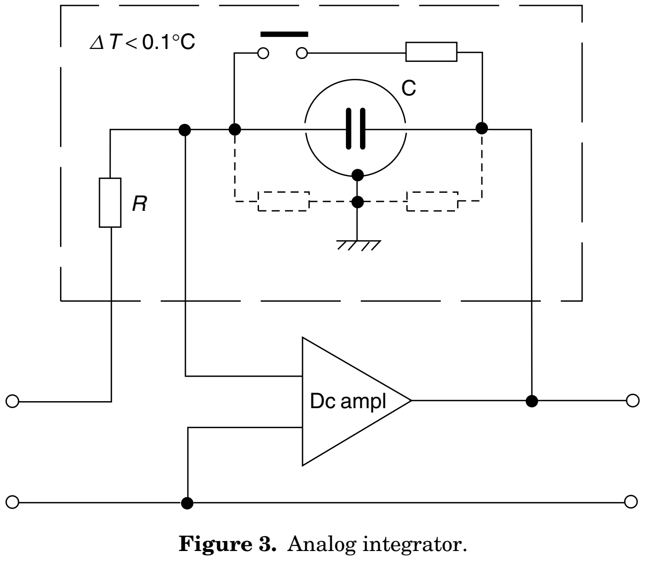

Figure 3. Analog integrator.

This system is well adapted to digital control but imposes limits on the rate of change of the flux because the input signal must never exceed the voltage level of the VFC. To obtain reasonable resolution, the minimum integration period over the full measurement range must be of the order of one second.

The Hall Generator Method

In 1879, E.H. Hall discovered that a very thin metal strip that is immersed in a transverse magnetic field and carries a current develops a voltage mutually at right angles to the current and field that opposed the Lorentz force on the electrons (48). In 1910, the first magnetic measurements were performed using this effect (49). It was, however, only around 1950 that suitable semiconductor materials were developed (50–52) and since then the method has been used extensively. It is a simple and fast measurement method, that provides relatively good accuracy, and therefore it is the most commonly used method in large-scale field mapping (53–55). The accuracy can be improved at the expense of measurement speed.

Hall Probe Measurements

The Hall generator provides an instant measurement, uses very simple electronic measurement equipment, and offers a compact probe that is suitable for point measurements. A large selection of this type of gaussmeter is now commercially available. The probes can be mounted on relatively light positioning gear (55). Considerable measurement time may be gained by mounting Hall generators in modular multiprobe arrays and applying multiplexed voltage measurement (56). Simultaneous measurements in two or three dimensions may also be carried out by using suitable probe arrays (57,58). The wide dynamic range and the possibility of static operation are other attractive features.

However, several factors set limits on the accuracy obtainable. The most serious limitation is the temperature coefficient of the Hall voltage. Temperature stabilization is usually employed to overcome this problem (59), but it increases the size of the probe assembly. The temperature coefficient may also be taken into account in the probe calibration by monitoring the temperature during measurements (60). It also depends, however, on the level of the magnetic field (60), so relatively complex calibration tables are needed. Another complication can be that of the planar Hall effect (61), which makes measuring a weak field component normal to the plane of the Hall generator problematic if a strong field component is parallel to this

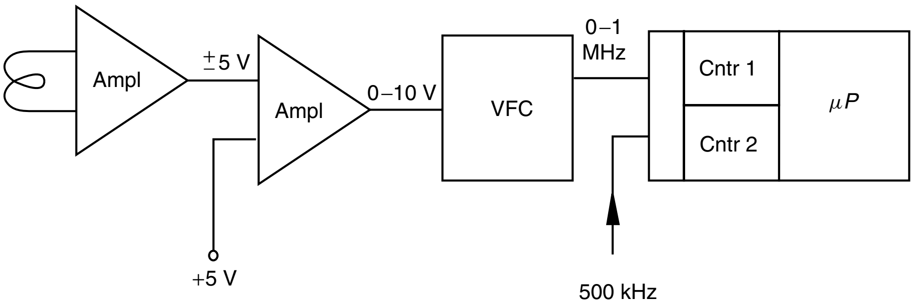

Figure 4. Digital integrator.

plane. This effect limits the use in fields of unknown geometry and in particular its use for determining of field geometry.

Last but not least is the problem of the nonlinearity of the calibration curve because the Hall coefficient is a function of the field level. The Hall generator of the cruciform type (62) has better linearity and a smaller active surface than the classical rectangular generator. Therefore, its magnetic center is better defined, so it is particularly well suited for measurements in strongly inhomogeneous fields. Special types that have smaller temperature dependence are available on the market, but they have lower sensitivity.

The measurement of the Hall voltage sets a limit of about 20 μT on the sensitivity and resolution of the measurement, if conventional dc excitation is applied to the probe. This is caused mainly by thermally induced voltages in cables and connectors. The sensitivity can be improved considerably by applying ac excitation (63,64). Good accuracy at low fields can then be achieved by employing synchronous detection techniques for measuring of the Hall voltage (65).

Special Hall generators for use at cryogenic temperatures are also commercially available. Although they have very low temperature coefficients, they unfortunately reveal an additional problem at low temperatures. The so-called "Shubnikov–de Haas effect" (66,67) shows up as a field-dependent oscillatory effect of the Hall coefficient which may amount to about 1% in high fields, depending on the type of semiconductor used for the Hall generator. This adds a serious complication to calibration. The problem may be solved by locating the Hall generator in a heated anticryostat (68). The complications related to the planar Hall effect are less important at cryogenic temperatures and are discussed in detail in (69). Altogether, the Hall generator has proved very useful for measurements at low temperature (70).

Calibration

Hall generators are usually calibrated in a magnet in which the field is measured simultaneously by the nuclear magnetic resonance technique. The calibration curve is most commonly represented as a polynomial of relatively high order (7 or 9) fitted to a sufficiently large number of calibration points. This representation has the advantage of a simple computation of magnetic induction from a relatively small table of coefficients.

A physically better representation is a piecewise cubic interpolation through a sufficient number of calibration points, which were measured with high precision. This can be done as a simple Lagrange interpolation or even better with a cubic spline function. The advantage of the spline function comes from its minimum curvature and its "best approximation" properties (71). The function adjusts itself easily to nonanalytic functions and is very well suited to interpolation from tables of experimental data. The function is defined as a piecewise polynomial of the third degree that passes through the calibration points so that the derivative of the function is continuous at these points. Very efficient algorithms can be found in the literature (72). The calculation of the polynomial coefficients may be somewhat time-consuming but need only be done once at calibration time. The coefficients (typically about 60 for the bipolar calibration of a cruciform Hall generator) can be easily stored in a microprocessor (59,65), and the subsequent field calculations are very fast. The quality of the calibration function can be verified from field values measured between the calibration points. A well-designed Hall-probe assembly can be calibrated to long-term accuracy of 100 ppm. The stability may be considerably improved by powering the Hall generator permanently and by keeping its temperature constant (56).

Fluxgate Magnetometer

The fluxgate magnetometer (73) is based on a thin linear ferromagnetic core on which detection and excitation coils are wound. The measurement principle is illustrated in Fig. 5. In its basic version, it consists of three coils

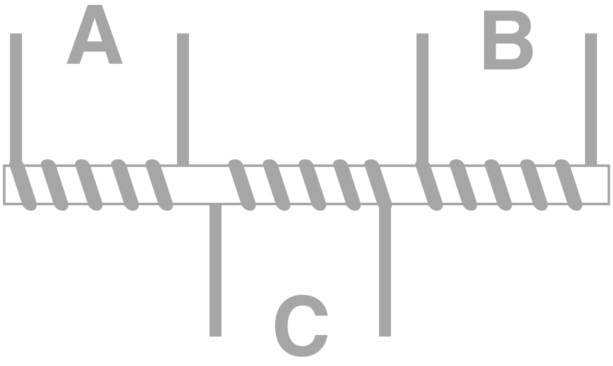

Figure 5. Fluxgate magnetometer.

wound around a ferromagnetic core: an ac excitation winding A, a detection winding B that indicates the zero field condition, and a dc bias coil C that creates and maintains the zero field. In practice, the coils are wound coaxially in successive layers. The core is made from a fine wire of Mumetal or a similar material that has an almost rectangular hysteresis curve. The method was introduced in the 1930s and was also named “peaking strip.” It is restricted to use with low fields but has the advantage of offering a linear measurement and is well suited for static operation. As a directional device of very high sensitivity, it is suitable for studying weak stray fields around magnets and mapping the earth’s magnetic field. Much more complex coil configurations are wound for precision measurements and where the measured field should not be distorted by the probe. The most interesting application is now in space research; important developments of this technique have taken place over the last decades (74–76). The use of modern materials for magnetic cores has improved the sensitivity to about 20 pT and can assure a wide dynamic range. The upper limit of the measurement range is usually of the order of a few tens of mT, but it can be extended by applying water cooling to the bias coil. Fluxgate magnetometers that have a typical range of 1 mT and a resolution of 1 nT are commercially available from several sources. They have many other practical applications, for example, in navigation equipment.

Magnetoresistivity Effect

Magnetoresistivity was discovered by W. Thomson in 1856 (77). It was exploited quite early, and a commercial instrument already existed at the end of the last century. Technical problems were, however, significant (78). Dependence on temperature and mechanical stress, combined with difficulties of manufacture and problems with electrical connections, caused a general lack of reliability in this measurement method. Similarly to the Hall generator, it was only when semiconductor materials became available that the method turned into a success. Then, inexpensive magnetoresistors came on the market and were also used for magnetic measurements (79). A more recent application for field monitoring was implemented in one of the large LEP spectrometers at CERN (80).

Visual Field Mapping

The best known visual field mapper is made by spreading iron powder on a horizontal surface placed near a magnetic source, thus providing a simple picture of the distribution of flux lines. Another very classical way of observing flux-line patterns is to place a free-moving compass needle at different points in the volume to be examined and note the direction of the needle. This compass method was applied, long before the discovery of electromagnetism, to studies of the variations in the direction of the earth’s magnetic field. Another visual effect may be obtained by observing the light transmission through a colloidal suspension of diamagnetic particles subject to the field (81,82).

Faraday Effect

The magneto-optical rotation of the plane of polarization of polarized light (Faraday effect) is a classical method for visualizing magnetic fields. A transparent container filled with a polarizing liquid and placed inside the magnet gap may reveal, for example, the field pattern in a quadrupole by observation through polarization filters placed at each end of the magnet. The rotation of the plane is proportional to the field strength and the length of the polarizing medium and may give a certain indication of the field geometry. This measurement principle has proved useful for measuring transient magnetic fields (83,84). It is less convincing when applied to the precise determination of magnet geometry, even though modern image processing techniques might improve the method substantially.

Floating Wire Method

Floating wire measurements were quite popular in the past (85). If a current-carrying conductor is stretched in a magnetic field, it will curve subject to the electromagnetic force and describe the path of a charged particle whose momentum corresponds to the current and the mechanical tension in the wire. A flexible annealed aluminium wire was used to reduce the effects of stiffness and gravity. This method has now been entirely replaced by precise field mapping and simulation of particle trajectories by computer programs.

Measurements Based on Particle Beam Observation

A method for precisely measuring the beam position with respect to the magnetic center of quadrupole magnets installed in particle accelerators has been developed during the last decade (86,87). The procedure consists of modulating the field strength in individual lattice quadrupoles while observing the resulting beam orbit oscillations. Local dc orbit distortions are applied in the search for the magnetic center. This so-called K-modulation provides perfect knowledge of the location of the particle beam with respect to the center of a quadrupole. In addition, it may provide other very useful observations for operating and adjusting of the accelerator (88). This is obviously of particular importance for superconducting accelerators (89). It is very difficult to provide a superconducting quadrupole magnet that has a direct optical reference to its magnetic center, so errors caused by changes of temperature profiles and other phenomena may build up as time passes.

The method may be further improved by synchronous detection of the oscillation, so that its phase can be identified. The sensitivity of the detection is impressive. Experience from LEP (90) showed that an absolute accuracy of 0.05 mm could be obtained in both the vertical and horizontal planes. Furthermore, it was observed that modulation of the quadrupole field by about 300 ppm could be clearly detected, which means that measurements may be carried out on colliding beams while particle physics experiments are taking place. This measurement method also played an important role for adjusting the so-called Final Focus Test Beam at Stanford Linear Accelerator Center (SLAC) (91,92).

Magnetoinductive Technology

Magnetic field sensors have been developed based on a change in inductance (L) caused by an applied magnetic field (93). These sensors, referred to as magnetoinductive sensors, contain an alloy whose permeability changes linearly over the sensors' useful range in an applied magnetic field. When the alloy is incorporated into an inductor, the inductance will change as the applied field changes. Magnetoinductive sensors contain a resonant LC circuit. As the applied field changes, so does the resonant frequency of the circuit. These devices have a dynamic range of ± 1,000 μT and an accuracy of ± 0.4 μT.

CONCLUDING REMARKS

Proven measurement methods and powerful equipment are readily available for most of the measurement tasks related to beam-guiding magnets as well as for spectrometer magnets. Therefore, it is prudent to examine existing possibilities carefully before launching the development of a more exotic measurement method. Many unnecessary costs and unpleasant surprises can be avoided by choosing commercially available instruments. The measurement methods described are complementary, and a combination of two or more of them will certainly meet most requirements.

In the field of new technologies, two methods merit consideration. Magnet resonance imaging is a promising technique, that could find lasting application. The use of superconducting quantum interference devices (SQUIDS) might also in the long run become an interesting alternative as an absolute standard and for measuring of weak fields (94,95). The complexity of these methods still prevents current laboratory use.

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