CHAPTER 1

Risk and the French Connection

PETER L. BERNSTEIN
President, Peter L. Bernstein Inc.

Abstract: What distinguishes the thousands of years of history from what we think of as modern times goes way beyond the progress of science, technology, capitalism, and democracy. The distant past was studded with brilliant scientists, mathematicians, inventors, technologists, and political scientists. Hundreds of years before the birth of Christ, the skies had been mapped, the great library of Alexandria built, and Euclid's geometry taught. Demand for technological innovation in warfare was as insatiable then as it is today. Coal, oil, iron, and copper have been at the service of human beings for millennia, and travel and communication mark the very beginnings of recorded civilization. The revolutionary idea that defines the boundary between modern times and the past is the mastery of risk: The notion that the future is more than a whim of the goods and that men and women are not passive before nature. Until human beings discovered a way across that boundary, the future was a mirror of the past or the murky domain of oracles and soothsayers who held a monopoly over knowledge of anticipated events. The remarkable vision of a group of great thinkers revealed how to put the future at the service of the present. By showing the world how to understand risk, measure it, and weigh its consequences, they converted risk taking into one of the prime catalysts that drives modern Western society. Three of these great thinkers were Frenchmen: Blaise Pascal, Pierre de Fermat, and the Chevalier de Méré.

Keywords: Blaise Pascal, Pierre de Fermat, Chevalier de Méré, theory of probability, risk, risk management, history of mathematics, gaming tables, Fermat's Last Theorem, problem of the points, Pascal's Magic Triangle, Pascal's Wager

Despite their brilliant insights, neither Cardano nor Galileo recognized that they held in their hands the most powerful tool of *risk management* ever to be invented: the laws of probability. Cardano's efforts had progressed from a series of experiments to some important generalizations, but he missed the prize because he was interested only in developing a theory of gambling, not a *theory of probability.* Galileo missed it because he had no interest in developing a theory of gambling; he went to the edge but stopped just too soon.

Galileo died in 1642. Only 12 years later, three Frenchmen took a great leap into the analysis of probability, an event that is the focus of this chapter. And less than 10 years after that, what had been just a rudimentary idea became a fully developed theory leading to significant practical applications. A Dutchman named Huygens published a widely read textbook about probability in 1657 (carefully read and noted by Newton in 1664), Leibniz was considering applying probability to legal problems, and a major work known as the Port-Royal *Logic* appeared in 1662. In 1660, an Englishman named John Graunt published the first effort to generalize demographic data from a statistical sample of mortality records kept by local churches. By the late 1660s, Dutch towns that had traditionally financed themselves by selling annuities were able to put their product on a sound actuarial footing. By 1700, the English government was financing its budget deficits by the sale of life annuities.

The story begins with an unlikely trio of Frenchmen, who saw far enough beyond the *gaming tables* to design the systematic and theoretical foundations for measuring probability. One of these men was a brilliant young dissolute who turned into a religious zealot and ended up aggressively rejecting the use of reason. The second was a successful lawyer for whom mathematics was a sideline. The third was a nobleman who combined his taste for mathematics with an irresistible urge for games of chance; his fame rests simply on having posed the question that set the other two on the road to ultimate discovery.

The dissolute and the lawyer had no need of experimentation to confirm their hypotheses. They worked inductively in pursuit of logical purity in creating for the first time a theory of probability. The theory provided a measure of probability in terms of hard numbers, a climactic break from making decisions on the basis of degrees of belief.

BLAISE PASCAL, THE NEUROTIC GENIUS

Blaise Pascal, the celebrated mathematician and occasional philosopher, was born in 1623, just about the time that Galileo was putting the finishing touches on *Sopra le Scoperte dei Dadi*. Born in the wake of the religious wars of the sixteenth century, Pascal spent half his life torn between pursuing the career of a mathematical genius and yielding to religious convictions that were anti-intellectual in their very essence. Although he was a triumphant mathematician and full of hubris about his accomplishments as a "geomaster," Pascal's religious passion ultimately came to dominate his entire existence. (The background material on Pascal is from Muir [1961, pp. 77–100], David [1962, pp. 34–79], and Hacking [1970, pp. 55–70].)

Pascal began life as a child prodigy in mathematics. He was so fascinated with shapes and figures that he discovered most of Euclidean geometry all on his own by drawing diagrams on the tiles of his playroom floor. At the age of 16, he wrote a paper on the underlying mathematics of the cone; the paper was so advanced that even the great Descartes was impressed with what Pascal had accomplished.

This enthusiasm for mathematics was a convenient asset for Pascal's father, who was a mathematician in his own right but earned a comfortable living as a tax collector, a profession known at the time as tax farmer. This individual would advance money to the monarch, thereby planting his seeds, and then go about collecting it from the citizenry, thereby gathering in his harvest. While Blaise was still in his early teens, he invented and patented a calculating machine as a response to the dreary task of adding up M. Pascal's daily accounts. This contraption was similar to the mechanical calculating machines that served for decades as the precursors to today's electronic variety, with gears and wheels going backward and forward to add and subtract. Blaise managed to multiply and divide on his machine as well, and even started to work on a method to extract square roots. Unfortunately for the clerks and bookkeepers of the next 250 years, he was unable to market his invention commercially because of prohibitively high production costs.

Recognizing his son's genius, Blaise's father introduced him at the age of 14 into a select weekly discussion group at the home of a Jesuit priest named Marin Mersenne, located near the Palais Royal in Paris. Abbé Mersenne had made himself the center of the world of science and mathematics during the first half of the 1600s. In addition to bringing the major scientific scholars together on a regular basis, he reported by post to all and sundry, in his cramped handwriting, what was new and what was significant (see David, 1962, p. 74).

In the absence of learned societies, professional journals, or other formats for the exchange of ideas and information, the abbé made a crucial contribution to the development of new scientific theories. The great academies like the Académie des Sciences in Paris or the Royal Society in London, which came into existence about 20 years after Mersenne's death, were direct descendants of Mersenne's service as the unofficial focal point for news about the latest scientific discoveries.

Although Blaise Pascal's early papers in advanced geometry and algebra impressed the high-powered mathematicians whom he met at Abbé Mersenne's, he soon developed a competing interest. In 1646, his father fell on the ice and broke his hip; the bonesetters called in to take care of M. Pascal happened to be members of a proselytizing Catholic sect called Jansenists. These people believed that the only path to salvation was through asceticism, sacrifice, and unwavering attachment to the straight and narrow. They preached that a person who fails to reach continuously for ever-higher levels of purity will be running in reverse into immorality. Emotion and faith were all that mattered; reason blocked the way to redemption.

The Jansenists not only fixed up the hip of Pascal père; they stayed around for three months and did a powerful job on the soul of Pascal fils, who swallowed their doctrine whole. This was a shocker, for Blaise as a youth had been a free-living man about town. Now math and science were left behind as well as all the fun; religion took over Pascal's full attention. All he could offer by way of explanation was to say (Muir, 1961, p. 90), "Who has placed me here? By whose order and warrant was this place and this time ordained for me? The eternal silence of these infinite spaces leaves me in terror."

The terrors became so overwhelming that in 1650, at the age of 27, he succumbed to what sounds very much like a nervous breakdown, including partial paralysis, swallowing problems, and devastating headaches. As a cure, his doctors urged him to get up, go out, and start living it up again. He lost no time in taking their cheerful advice. When his father died about that time, Pascal said to his sister (Muir, 1961, p. 93): "Let us not grieve like the pagans who have no hope." His renewed activities as a playboy exceeded even his earlier sybaritic ways and included enthusiastic participation at the gambling tables of Paris.

Luckily for science, Pascal also resumed his researches into mathematics and related subjects; he had been into them too deeply to part from them so lightly. One of his experiments in physics at this time proved that vacuums actually exist, a controversial issue ever since Aristotle had declared that nature abhors vacuums. In the process, Pascal showed that he could measure barometric pressures at varying altitudes by using mercury in a tube emptied of all air.

GAMBLING INTO MATHEMATICS

About this time, Pascal became acquainted with a wealthy nobleman named *Chevalier de Méré.* The Chevalier prided himself on his skill at mathematics as well as his ability to figure the odds at the casinos. In a letter to Pascal some time in the late 1650s, he boasted (Muir, 1961, p. 94), "I have discovered in mathematics things so rare that the most learned of ancient times have never thought of them and by which the best mathematicians in Europe have been surprised."

Leibniz must have been impressed, for he described the Chevalier as "a man of penetrating mind who was both a gambler and a philosopher." But then he must have had second thoughts, for he went on to say that (Muir, 1961, p. 95) "I almost laughed at the airs which the Chevalier de Méré takes on in his letter to Pascal."

Pascal agreed with Leibniz. "M. de Méré," he wrote to a colleague (David, 1961, p. 69), "has good intelligence but he is not a geometer and this, as you realize, is a great defect." Pascal here is the personification of the academic who rejoices in putting down practitioner friends; even worse, his snide comment underestimated de Méré. (See Huff, 1959, pp. 63–69.)

Yet Pascal himself is our source for information on de Méré's extraordinarily sharp intuition for probabilities. The Chevalier applied his talent by betting repeatedly on outcomes with just a narrow margin in his favor—outcomes that his opponents did not recognize as anything more than a random result. According to Pascal, de Méré knew that the probability of throwing a six with one die rises above 50% with four throws—in fact, to 51.77469136%. In observing the Chevalier's strategy, Pascal missed his first chance to recognize one of the features of the laws of probability, for the essence of the strategy was to win a tiny amount on a large number of throws in contrast to betting the chateau on just a few. The strategy also required large amounts of capital, because a six might fail to show up for many throws before it appeared in a cluster that would bring its average appearance over 50%. (See Hogben, 1968, p. 551; and Hacking, 1975, pp. 58–59.)

De Méré tried a variation on his system by betting that *sonnez*—the term for double-six—had a better than 50% probability of showing up on 24 throws of two dice. He lost enough money on these bets to learn the hard way that the probability of double-six was in fact only 49.14% on 24 throws. Had he bet on 25 throws, where the probability of throwing *sonnez* breaks through to 50.55%, he would have ended up richer rather than poorer. Thus, the history of risk management is written in red as well as in black.

At the time that he first met Pascal, the Chevalier was raising with a number of French mathematicians Pacioli's old *problem of the points*—how should two players in a game of *balla* share the stakes when they leave the game incomplete? Despite many intuitive guesses, no one had as yet come up with an answer.

The problem of the points fascinated Pascal, but he was reluctant to explore it on his own. In today's world, this would be the topic for a panel at the annual meeting of one of the learned societies. In Pascal's world, that vehicle did not exist. Unless there was a meeting to analyze the matter in the intimacy of Abbé Mersenne's little group of scholars, the accepted procedure was to start up a private correspondence with other mathematicians who might be able to contribute something to the subject. In 1654, Pascal turned to Pierre de Carcavi, a member of Abbé Mersenne's group, who put him in touch with a lawyer in Toulouse named *Pierre de Fermat.*

But why Fermat, a lawyer? Pascal could not have approached anyone more competent to review his proposed solution to the problem of the points. Fermat's erudition was awesome (see David, 1962, pp. 71–75). He spoke all the main European languages, even wrote poetry in foreign tongues, and was a busy commentator on the literature of the Greeks and Romans. He has long been recognized as a mathematician of rare power. He was one of the independent inventors of analytical geometry, he contributed to the early development of calculus, he did research on the weight of the earth, and he worked on light refraction and optics. In the course of what turned out to be an extended correspondence with Pascal, he made a significant contribution to the theory of probability—and mathematics was not even his primary activity.

But Fermat's crowning achievement was in the theory of numbers—the analysis of the structure that underlies the relationships of each individual number to all the others. These relationships are a hotbed of puzzles, not all of which have been resolved to this very day. The Greeks, for example, discovered what they called perfect numbers, numbers that are the sum of all their divisors other than themselves, like $6 = 1 + 2 + 3$. The next higher perfect number after 6 is $28 = 1 + 2 + 4 + 7 + 14$. The third perfect number is 496, followed by 8,128. We are moving fast, but we shall move even faster, for the fifth perfect number is 33,550,336.

Pythagoras discovered what he called amicable numbers, "One who is the other I," numbers whose divisors add up to each other. The divisors of 284, which are 1, 2, 4, 71, and 142, add up to 220; all the divisors of 220, 1, 2, 4, 5, 10, 11, 22, 44, 55, and 110, add up to 284.

No one has yet devised a rule for finding all the perfect numbers or amicable numbers that exist, nor has anyone been able to explain all the varying sequences between them. Similar difficulties arise with prime numbers, numbers like 1, 3, or 29, that are divisible only by one and by themselves. At one point, Fermat believed he might have discovered a formula that would always produce a prime number as its solution, but he warned that he could not prove theoretically that this formula would always produce a prime number. His formula produced 5, then 17, then 257, and finally 65,537, all of which were primes. That was the end of his success, for the next number to result from his formula was the imposing amount of 4,294,967,297, which turns out to be the product of 641 × 6,700,417.

Fermat is perhaps most famous for propounding what has come to be known as "*Fermat's Last Theorem.*" The notion that he scribbled on the margin of his copy of Diophantus's book *Arithmetic* is simple to describe despite the incomparable complexity of its proof.

The Greek mathematician Pythagorus first demonstrated that the square of the longest side of a right triangle, the hypotenuse, is equal to the sum of the squares of the other two sides. Diophantus, an early explorer into the wonders of quadratic equations, had written a similar expression: $x^4 + y^4 + z^4 = u^2$. "Why," asks Fermat, "did not Diophantus seek two [rather than three] fourth powers such that their sum is square? The problem is, in fact impossible, as by my method I am able to prove with all rigor."(Turnbull, 1951, p. 130). Fermat observes that Pythagorus was correct that $a^2 + b^2 = c^2$, but $a^3 + b^3$ would not be equal to c^3, nor would any integer higher than two fit the bill: The Pythagorean theorem works only for squaring.

And then he wrote (Turnbull, 1951, p. 131): "I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain." He left mathematicians dumbfounded for over 350 years with this assertion, as they struggled to find a theoretical justification for what a great deal of empirical experimentation proved to be true. In 1995, an English mathematician named Andrew Wiles solved this greatest of all puzzles.

Fermat's Last Theorem is more of a curiosity than an insight into how the world works. But the solution that he and Pascal worked out to the problem of the points has long since been paying social dividends as the cornerstone of modern insurance and other forms of risk management.

THE MAGIC OF THE MAGIC TRIANGLE

The solution to the problem of the points begins by recognizing that the player who is ahead when the game stops would obviously have the greater probability of winning if the game were to continue. But how much greater are the leading player's chances? How small are the lagging player's chances? How do these riddles ultimately translate into the science of forecasting?

The correspondence of 1654 between Pascal and Fermat on this subject comprises an epochal set of documents in the *history of mathematics* and the theory of probability. (The full text of this correspondence, translated into English, appears in Appendix 4 of David [1962].)

In response to the Chevalier de Méré's insatiable curiosity about the old problem, the neurotic scientist and the scholarly lawyer constructed a systematic method for analyzing future outcomes. When more things can happen than will happen, Pascal and Fermat give us a procedure for determining the likelihood of each of the possible results—assuming always that the outcomes submit to mathematical measurement.

The two men approached the problem from different standpoints. Fermat turned to pure algebra. Pascal was more innovative: He used a geometric format to illuminate the underlying algebraic structure. His methodology is simple and applicable to a wide variety of problems in probability.

The mathematical essence of this geometric algebra had been recognized long before Fermat and Pascal took it up. Omar Khayyam himself had considered it some 450 years earlier. In 1303, a Chinese mathematician named Chu Shih-chieh, explicitly denying originality on his part, approached the problem by means of a device that he called the "Precious Mirror of the Four Elements." Cardano had also mentioned this gadget (see Hogben, 1968, pp. 277–279; and David, 1962, p. 34).

Chu's precious mirror has since come to be known as *Pascal's Magic Triangle.* "Let no one say that I have said nothing new," boasts Pascal in his autobiography. "The arrangement of the subject is new. When we play tennis, we both play with the same ball, but one of us places it better." (See Turnbull, 1951, p. 131; and Eves, 1984, p. 6.)

$$\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1
\end{array}$$

All sorts of patterns greet the eye at the first glance at the arithmetic triangle, but the underlying structure is uncomplicated: each number is the sum of the two numbers to the right and to the left on the row above.

The analysis of probability begins with enumerating the number of different ways a particular event can come about—Cardano's "circuit"—and that is what the sequence of numbers in each of these expanding rows is designed to provide. The top row shows the probability of an event that cannot fail to happen. Here there is only one possible outcome, with zero uncertainty; it is irrelevant for probability analysis. The next row is the first row that does matter for calculating probabilities. It shows a 50–50 type of situation: the probabilities of outcomes like having a boy in a family that is planning to have only one child or of flipping a head on just one toss of a coin. Add across. With a total of only two possibilities, the result is either one way or the other, a boy or a girl, a head or a tail; the probability of having a boy instead of a girl or of flipping a head instead of a tail is 50%.

The same process applies as we move down the triangle. The third row, the second of relevance for us, shows the possible combinations of boys and girls in a family that produces two children. Now adding across shows that there are four possible results: one chance of two boys, one chance of two girls, and two chances of one each—a boy followed by a girl or a girl followed by a boy. Now at least one boy appears in three of the four outcomes, setting the probability of at least one boy in a two-child family at

75%; the probability of one boy plus one girl is 50%. The process obviously depends on combinations of numbers in a manner that Cardano had recognized but that still lay hidden and unpublished when Pascal took up the subject.

The same line of analysis with the arithmetic triangle will produce a solution for the problem of the points. Let us change the setting from Pacioli's game of *balla* to the modern game of baseball. What is the probability that your team will win the World Series after it has lost the first game? If we assume, as in a game of chance, that the two teams are evenly matched, this problem is identical to the kinds of problems of the points tackled by Fermat and Pascal. (I am grateful to Stanley Kogelman for helping me work out these examples.)

As the other team has already won a game, the Series will now be determined by the best of four out of six games instead of four out of seven. How many different sequences of six games are possible, and how many of those victories and losses would result in your team winning the four games it needs for the pennant? Your team might win the second game, lose the third, and then go on to win the last three. It might lose two in a row and win the next four. Or it could win the necessary four right away, leaving the opponents with only one game to their credit.

How many such combinations of wins and losses are there, out of six games? The triangle will tell us. All we have to do is find the appropriate row to look at.

Note that the second row of the triangle, the 50–50 row, concerns a family with an only child or a single toss of a coin and adds up to a total of two possible outcomes. The next row shows the distribution of outcomes for a two-child family, or two coin tosses, and adds up to four outcomes, or 2^2. The next row adds up to eight outcomes, or 2^3, and shows what could happen with a three-child family. With six games remaining to settle the outcome of the World Series, we would want to look at the row whose total is 2^6—or two multiplied by itself six times, where there will be 64 possible sequences of wins and losses. [Mathematicians will note that what Pascal has really provided here is the binomial expansion, or the coefficients of each successive multiplication of (a + b) by itself. For example, the first row is $(a + b)^0 = 1$, while the fourth row is $(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$.]

The sequence of numbers in that row reads:

1 6 15 20 15 6 1

Remember that your team still needs four games to win the Series, while the opposing team needs only three. There is just one way your team can win all the games—by winning all the games while the opponents win none; the number one at the beginning of the row refers to that possibility. The next number reading across is six. There are six different sequences in which your team (Y) would gain the Series while their opponents (O) win only one more game:

OYYYYY YOYYYY YYOYYY YYYOYY YYYYOY YYYYYO

And there are 15 different sequences in which your team would win four games while the opponents win two.

All the other combinations would produce at least three games for the opposing team and less than the necessary four for yours. This means that there are 1 + 6 + 15 = 22 combinations in which your team would come out on top after losing the first game in the World Series and 42 combinations in which the opposing team would become the champions. As a result, the probability is 22/64—or a tad better than one out of three—that your team will come from behind to win four games before the other team has won three.

The examples betray something odd. Why would your team play out all six remaining games in sequences where they would have won the World Series before playing six games? Or why would they play out all four games when they could win in fewer games?

Although no team in real life would extend play beyond the minimum necessary to determine the championship, a logically complete solution to the problem would be impossible without *all* of the mathematical possibilities. As Pascal put it in his correspondence with Fermat, the mathematical laws must dominate the wishes of the players themselves, who are in fact only abstractions of a general principle. He asserts that "it is absolutely equal and immaterial to them both whether they let the [match] take its natural course."

THE MORAL AND THE MORALS OF THE STORY

The correspondence between Pascal and Fermat must have been an exciting exploration of new intellectual territory for both men. More than mathematics was involved here for Pascal, the man so deeply involved with religion and morality, and for Fermat the jurist. There is a matter of moral right involved in the division of the stakes in Pacioli's unfinished game of *balla* according to the solutions of Pascal and Fermat. The players could just as easily split the stakes evenly, for example, but that solution would not have been acceptable to Pascal and Fermat because it would be unfair to the player lucky enough to be ahead when playing ceased. (This point, and the quotation from Pascal that follows, are from Guilbaud, 1968; I performed the translation from the original French.)

Pascal is explicit about the moralities involved and chooses his words with care. His essay points out that "the first thing which we must consider is that the money the players have put into the game no longer belongs to them . . . but they have received in return the right to expect that which luck will bring them, according to the rules upon which they agreed at the outset." In the event that they decide to stop playing before the game is over, they will reenter into their original ownership rights. At that point, "the rule determining that which will belong to them will be proportional to that which they had the right to expect from fortune. . . . [T]his just distribution is known as the division." The principles of probability theory determine the division because they determine the just distribution of the stakes.

Seen in these terms, the Pascal-Fermat solution is clearly colored by the notion of risk management, even though they were not thinking explicitly in those terms. Only the foolhardy take risks when the rules are unclear, whether it be *balla*, buying IBM stock, building a factory, or submitting to an appendectomy.

But beyond the moral questions, solutions proposed by Pascal and Fermat lead to precise generalizations and rules for calculating probabilities, including cases involving more than two players, two teams, two genders, two dice, or coins with two sides. Their achievement enabled them to push the limits of theoretical analysis far beyond Cardano's demonstration that two dice of six sides each (or two throws of one die) would produce 6^2 combinations or that three dice would produce 6^3 combinations.

Fermat wrote to Carcavi about Pascal that "I believe him to be capable of solving any problem that he undertakes." In one letter to Fermat, Pascal admitted that "your numerical arrangements . . . are far beyond my comprehension." Elsewhere, he also described Fermat as "a man so outstanding in intellect . . . in the highest degree of excellence . . . [that his works] will make him supreme among the geomasters of Europe."

The last letter of the series is dated October 27, 1654. Less than one month later, Pascal had some kind of mystical experience. He sewed his description of this event into his coat so that he could wear it next to his heart, claiming "Renunciation, total and sweet." He abandoned mathematics and physics, swore off high living, dropped his old friends, sold all his possessions except for his religious books, and, a short while later, took up residence in the monastery of Port-Royal in Paris.

Yet traces of the old Blaise Pascal lingered on. His innovative intuitions led him to establish the first commercial bus line in Paris, but he did not undertake this venture as an entrepreneur. All the profits went to the monastery of Port-Royal.

In July 1660, Pascal took a trip to Clermont-Ferrand, not far from Fermat's residence in Toulouse. Fermat proposed a meeting "to embrace you and talk to you for a few days," suggesting a location halfway between the two cities; Fermat claimed bad health as an excuse for not going all the way. Pascal wrote back in August, but in his case neither the spirit nor the body were willing:

> I can scarcely remember that there is such a thing as Geometry [that is, mathematics]. I recognize Geometry to be so useless that I can find little difference between a man who is a geometrician and a clever craftsman. Although I call it the best craft in the world it is, after all, nothing else but a craft. . . . It is quite possible I shall never think of it again. (David, 1962, p. 252)

PASCAL'S WAGER

Pascal put together his thoughts about life and religion while he was at Port-Royal and published them with the title *Pensées*. (All of the material that follows is from Hacking [1975, Chapter 8, pp. 63–70].) In the course of his work on this book, he filled two pieces of paper on both sides with what Ian Hacking describes as "handwriting going in all directions . . . full of erasures, corrections, and seeming afterthoughts." This fragment contains what has come to be known as *Pascal's Wager* (*le pari de Pascal*), which asks, "God is, or he is not. Which way should we incline? Reason cannot answer."

Drawing on his work in analyzing the probable outcomes of the game of *balla*, Pascal frames the issue in terms of a game of chance. He postulates a game that ends far off at an infinite distance in time. At that moment, a coin is tossed. Which way would you bet—on heads (God is) or on tails (God is not)?

Hacking asserts that Pascal's line of analysis to answer this question is the beginning of the theory of decision making. "Decision theory," as Hacking describes it, "is the theory of deciding what to do when it is uncertain what will happen" (Hacking, p. 62). The decision is the essential first step in any effort to manage *risk.*

Sometimes we make decisions on the basis of past experience, basically out of experiments we or others have conducted in the course of our lifetime. But we cannot conduct experiments that will prove either the existence or absence of God. Our only alternative is to explore the future consequences of believing in God or rejecting God. Nor can we avert the issue, for by the mere act of living we are forced to play this game.

Pascal explained that belief in God is not a decision. You cannot awaken one morning and declare, "Today I think I will decide to believe in God." You believe or you do not believe. The decision, therefore, is whether to choose to act in a manner that will lead to believing in God, like living with pious people and following a life of "holy water and sacraments." The person who follows these precepts is wagering that God is. The person who cannot be bothered with that kind of thing is wagering that God is not.

The only way to choose between a bet that God exists or that there is no God down that infinite distance of Pascal's coin-tossing game is to decide whether an outcome where God exists is preferable—more valuable in some sense—than an outcome where God does not exist, even though the probability may be only 50–50. This insight is what conducts Pascal down the path to a decision—a choice in which the value and the odds may differ because the consequences of the two outcome are different. (Note that at this point, Pascal anticipates Daniel Bernoulli's epochal breakthrough in decision analysis in 1738, which is explored in detail in Chapter VI of Bernstein [1996].)

If God is not, the way you lead your life, piously or sinfully, is immaterial. But suppose that God is. Then betting against the existence of God by refusing to live a life of piety and sacraments runs the risk of eternal damnation; the winner of the bet that God is faces the possibility of salvation. As salvation is clearly preferable to eternal damnation, the correct decision is to act on the basis that God is. "Which way should we incline?" The answer was obvious to Pascal.

THE ART OF THINKING

Pascal produced an interesting by-product when he decided to turn over the profits from his bus line to help support the Port-Royal monastery. (The material about the Port-Royal monastery is from Hacking [1975, pp. 25].) In 1662, a group of his associates at this monastery published a work of great importance, *La logique, ou l'art de penser* (*Logic, or the Art of Thinking*), a book that ran to five editions between 1662 and 1668. (The Latin title for this book was *Ars Cogitandi*. See Hacking [1975, pp. 12, 24].)

Although its authorship was not shown, the primary—but not the sole—author is believed to have been Antoine Arnauld, a man characterized by Hacking as "perhaps the most brilliant theologian of his time" (Hacking, p. 25). It was immediately translated into other languages throughout Europe and was still in use as a text in the nineteenth century.

The last part of the book contains four chapters on probability that cover in particular the process of developing a hypothesis from a limited set of facts; today, this process is called statistical inference. Among other matters, these chapters cover a "rule for the proper use of reason in determining when to accept human authority," rules for interpreting miracles, a basis of interpreting historical events, and the application of numerical measures to probability (Hacking, 1975, p. 74).

The final chapter at one point describes a game in which each of 10 players risks one coin in the hope of winning the nine coins of his fellow players. The author then points out that there are "nine degrees of probability of losing a coin for only one of gaining nine" (Hacking, 1975, p. 77). The observation is innocuous, but the sentence has earned immortality nonetheless. According to Hacking, this is the first occasion in print "where probability, so called, is measured" (Hacking, 1975, p. 77).

The passage deserves immortality for more reasons than that. The author admits the trivial character of games such as he has described, but draws an analogy to natural events. For example, the probability of being struck by lightning is tiny but "many people . . . are excessively terrified when they hear thunder" (Hacking, 1975, p. 77).

Then he adds a critically important sentence: "Fear of harm ought to be proportional not merely to the gravity of the harm, but also to the probability of the event" (Hacking, 1975, p. 77). Here is another major innovation: the idea that both gravity and probability should influence a decision. We could turn this assertion around and, at the same time, say the same thing on a more positive note by stating that a decision should involve the strength of our desire for a particular outcome as well as our degree of belief about the probability of that outcome.

The strength of our desire for something, which came to be known as utility, would soon become more than just the handmaiden of probability. Utility was about to develop into the center of all theories of decision making and risk taking. It will reappear repeatedly in subsequent chapters.

SUMMARY

Historians are fond of referring to the near misses in their field—occasions when something of enormous importance almost happened but, for one reason or another, the event failed to happen. The extended disquisition about Pascal's Triangle is a striking example. We have seen how to predict the probable number of boys or girls in a multichild family. We have gone beyond that to predict the probable outcome of the World Series (for evenly matched teams), given that part of the series has already been played.

We have been forecasting! Pascal and Fermat held in their hands the key to a systematic method for calculating the probabilities of future events. Even though they did not turn it all the way, they put the key in the lock. The significance of their work for business management, for the management of risk, and for insurance in particular, was to be developed by others—for whom the Port-Royal Logic itself was an important first step. The whole idea of forecasting economic trends or using probability to forecast economic losses was still too foreign for Pascal and Fermat to have recognized what they were missing; it is only with hindsight that we can see how close they came.

Yet Pascal and Fermat shoved the medicine men, the arbitrary gods, the oracles, and the witches, at long last, on the first steps of their march to oblivion. The inescapable uncertainty of the future will always prevent us from completely banishing the fates from our hopes and fears, but after 1654 mumbo-jumbo would no longer be the forecasting method of choice.

REFERENCES

Bernstein, P. L. (1996). *Against the Gods: The Remarkable Story of Risk*. New York: John Wiley & Sons.

David, F. N. (1962). *Games, Gods, and Gambling*. New York: Hafner Publishing Company.

Eves, H. (1984). *Great Moments in Mathematics (Before 1650)*. The Mathematical Society of America.

Guilbaud, G. Th. (1968). *Eléments de la Théorie Mathématique Des Jeux*, Paris: Dunod.

Hacking, I. (1975). *The Emergence of Proability: A Philosphical Study of Early Ideas about Probability, Induction, and Statistical Inference*. London: Cambridge University Press.

Hogben, G. (1968). *Lloyd's of London: A Reputation at Risk*. London: Allen Lane.

Huff, D. (1959). *How to Take a Chance*. New York: Norton.

Muir, J. (1961). *Of Men and Numbers: The Story of the Great Mathematicians*. New York: Dodd, Mead.

Turnbull, H. W. (1951). The great mathematicians. Reprinted in J. R. Newman (ed.) (1988), *The World of Mathematics*: A Small Library of Literature of Mathematics from A'h-mose the Scribe to Albert Einstein (pp. 73–160). Redmond, WA: Tempus Press.