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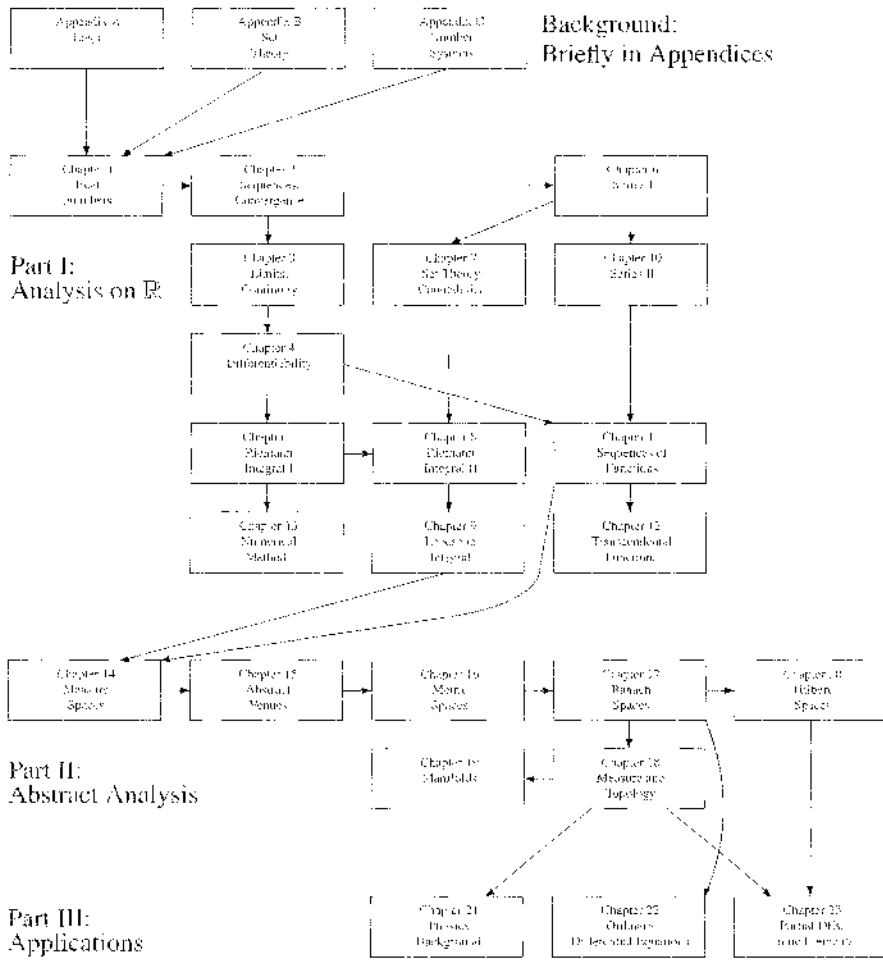


Figure 1: Content dependency chart with minimum prerequisites indicated by arrows. Some remarks, examples, and exercises in the later chapter might still depend on other earlier chapters, but this problem typically can be resolved by quoting a single result. Details about where and how the reader can “branch out” are given in boxes in the text.