CHAPTER 1

INTRODUCTION

The authors would like to share with the readers a unique experience in the systematic use of the methods of optimal control for shock isolation systems that will be applied to assorted structures intended to mitigate injuries. This book uses the theory of optimal shock isolation, which was developed in the 1950s for the protection of engineering systems from intensive shock loads, and extends the use to problems associated with reducing the risk of injuries to people who are subjected to an impact load. Impact-induced injuries may occur in vehicle accidents, at industrial or construction sites, in sports, and in military or antiterrorism activities. Depending on the situation, people may be protected from impact loads using devices such as bullet-proof vests, helmets, seat belts, or air bags. Alternatively, the source of the load can be modified to reduce the potential for injury. To be effective, these protective devices must be properly designed so that the forces and displacements experienced are below injury tolerance levels.

Within the framework of optimal shock isolation, protecting a person from injurious impact loads requires the introduction of a medium between the person and the structure that is subject to shock disturbances. This medium is known a *shock isolator*, whereas the structure to which an impact load is directly applied is called the *base*. Different structures can play the roles of the base and the isolator. For a motorcycle helmet, for example, the base is the shell of the helmet, and the isolator is the padding or the armature that separates the shell from the head. In a crashworthy automobile, the base is the body or frame of the vehicle, while seat belts and air bags are the isolators. In general, the isolators can be passive or active. Passive isolators do not involve sensors, controllers, or actuators, while active isolators may integrate these components.

When properly designed and controlled, isolators can substantially reduce the risk of severe injuries.

For injury prevention applications, injury criteria serve the role of performance criteria. The injury criteria are quantitative response measures indicating the severity of injury in terms of mechanical quantities such as displacements, velocities, accelerations, energy, and power. In addition, performance criteria may include geometric characteristics such as the maximum excursion of an occupant of a vehicle relative to the vehicle's interior in response to a crash impact load. The objective of designing injury countermeasures is to minimize the injury potential as interpreted by the injury criteria. When optimizing the design of shock isolators, it is desirable to reduce the values of all performance criteria to the greatest extent possible. However, the performance criteria are often competing and simultaneous minimization of them is impossible. There are a number of approaches to solving multicriteria optimization problems. For example, a single objective function to be minimized can be formed as a weighted sum of the performance criteria. A success or failure of this approach depends on the choice of the weighting coefficients. Another approach, which will be used in this book, involves minimizing one of the performance criteria while the other criteria are constrained.

An important stage of the optimal design of shock isolators is the evaluation of the absolute minimum of the performance index (the performance criterion to be minimized) that characterizes a hypothetically perfect (ideal) isolator that cannot be surpassed by any real isolator irrespective of its design and engineering configuration. This evaluation involves replacing the particular isolator configuration with a generic control force and solving an optimal control problem for this force. The absolute minimum of the performance index that results from the solution of this problem characterizes the *limiting performance* of the system in terms of the performance index. The evaluation of the absolute minimum of the performance index and the investigation of the behavior of this performance index relative to constraints is often called *limiting performance analysis*. By comparing the performance characteristics of a proposed design or prototype with those of the ideal isolator, an engineer can see how close his or her design comes to the ideal.

To characterize the limiting potentials for improving shock isolators, *trade-off curves* that plot the performance index against the criteria subjected to constraints can be used. A typical trade-off curve is shown in Fig. 1.1, for two performance criteria J_1 and J_2. Any design corresponding to points above this curve is feasible but is not optimal, because both

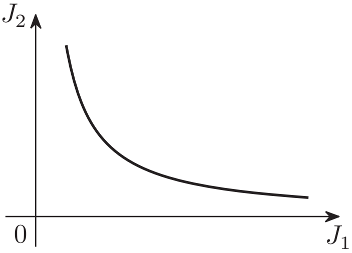

FIGURE 1.1 Trade-off curve.

criteria can be reduced against this design. Designs with the values of the performance criteria represented by points below the trade-off curve are unfeasible. Any point on the trade-off curve corresponds to a design which is optimal with respect to one of the criteria, provided that the other criterion is constrained. It is impossible to improve the design of the isolator with respect to both criteria J_1 and J_2 against any design represented by a point on the trade-off curve.

During the design process, limiting performance analysis may be followed by the *parametric synthesis* of the isolation system. At this stage, a particular design configuration of the isolation system replaces the generic control force used in the limiting performance analysis and the design parameters are determined by minimizing the performance index. If the minimum value of the performance index is close to the value that characterizes the limiting performance, this design can be recommended for practical implementation because it is near optimal. If the discrepancy between the limiting performance characteristic and the minimal value of the performance index for the isolator with the selected design configuration is large, the design configuration should be changed and the parametric optimization repeated.

The problems in this book are formulated using simple mathematical models intended to simulate the mechanical response of a human body to impact loads. Using the response characteristics of these models, quantitative measures of the injury risk are defined. Of course, simple mathematical models with only a few degrees of freedom cannot fully simulate the complex response of a body to impact. At the first stage of the analysis, use of these simple models is justified because the solutions can be obtained analytically or numerically with a minimum of effort and the findings are easy to interpret. The control strategy obtained by using a simplified model can then be verified and adjusted by using multibody or finite-element mathematical models that may have a large number of degrees of freedom and parameters. The more complicated the model, the more difficult it is to find an optimal solution.

Since many of the constitutive laws for multibody or finite-element models of the body are not currently available, much of the model development is empirical; the parameters are found by comparing the response characteristics of the model with appropriate experimental data. Since these models may be sensitive to the type of loading, the reliability of the optimization results obtained by using these complicated models is unknown for loading regimes beyond those used in the development process. In the optimization process for isolator development, a number of control laws are tested until the solution converges to an optimum. Since each trial control changes the load of the object to be protected, a more detailed and complicated model does not necessarily give more reliable optimization results. On the other hand, reasonably simple models enable the basic qualitative features of the optimal control law to be observed. These features may be taken into account when constructing a realistic impact isolator.

1.1 STRUCTURE OF THE BOOK

The book consists of two parts, one of which (Chapters 2–4) provides background information on shock isolation, control, and optimization, while the remaining part (Chapters 5–8) presents solutions of a number of topical problems related to the optimal control of shock isolation systems for protection from the injuries caused by impacts.

Chapter 2 presents the fundamentals of impact and shock isolation. In this chapter, basic concepts of the theory of shock isolation are introduced, the physical principles of shock isolation are explained, and the effectiveness of the isolation is discussed. For those with a mechanical engineering background, this chapter may be a quick review.

Chapter 3 provides a basic knowledge of the optimization of shock isolators for single-degree-of-freedom systems. The general statement of the optimal shock isolation problem is given as a problem of constrained minimization of an objective function (performance index) or an optimal control problem. The concept of the limiting performance analysis is introduced and developed in detail. A number of simple but important control problems for shock isolators are solved.

Chapter 4 presents a rigorous mathematical consideration of an optimal control problem for a shock isolation system for a two-degree-of-freedom model shown in Fig. 1.2. The object to be protected in this model consists of two bodies m_1 and m_2 connected by a spring-and-dashpot element with linear properties. This model can be used, for example, to evaluate the response of a seated person to a vertical impact load. Bodies m_1 and m_2 take into account the inertial properties of the upper and lower torso, while

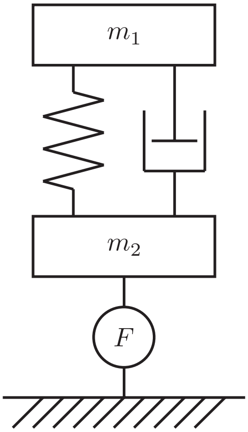

FIGURE 1.2 Two-body model with a spring-and-dashpot element.

the spring-and-dashpot element imitates the elastic and viscous properties of the vertebral column. The isolator produces the control force F between the base and the body m_2. It is required to find an optimal control force that minimizes the maximum magnitude of the displacement of body m_2 relative to the base, provided that the magnitude of the force developed in the spring-and-dashpot element does not exceed a prescribed value. The shock disturbance is specified as the time history of the absolute acceleration of the base. The presence of impulse components in the control force is established and proved. An algorithm for constructing the optimal control is described. A general concept for the limiting performance analysis is introduced for systems that involve three components: a base, a container in which the object is placed, and the object. Shock isolators separate the container from the base and the object from the container. This model can be used to represent vehicles equipped with shock isolation systems to reduce occupant injuries in a crash.

In Chapter 5, a simple model of a crashworthy helicopter seat is considered. The seat may be equipped with one or two active isolators. One isolator (a cushion) separates the lower torso of an occupant from the seat pan, and the other separates the seat pan from the helicopter's airframe. The isolators are optimized to protect an occupant from severe spinal injury for a hard landing of the helicopter. A two-degree-of-freedom model is used to simulate the response of the vertebral column to a vertical impact load. To construct the optimal controls and to evaluate the minimum displacement of the lower torso relative to the seat pan, the technique of Chapter 4 is employed. To validate the results obtained on the basis of the two-degree-of-freedom spinal injury model, a MADYMO multibody model of the occupant is used to simulate the response of an occupant in a seat to a vertical load.

Chapter 6 deals with thoracic injury control. A two-degree-of-freedom thoracic injury model is used. A frontal impact of a car against an obstacle is considered with the seat belt playing the role of a shock isolator. An optimal control is constructed for the force produced by the seat belt on the occupant's thorax. Both open-loop and feedback control modes are considered.

A limiting performance analysis for the protection of the human head from injuries caused by an impact against a fixed obstacle is performed in Chapter 7. The expected severity of the injury is evaluated using HIC (Head Injury Criterion), which is an integral criterion defined by the National Highway Traffic Safety Administration (NHTSA) in 1972. Currently, the HIC is used as a standard injury criterion in automobile crash tests and drop tests of helmets. To mitigate the impact load transmitted to the brain, the head is isolated from the surface being hit by a reasonably soft structure, for example, a liner in a helmet or a shock isolation coating on a playground surface. A minimal displacement of the head during the impact deceleration is evaluated, provided that the HIC is lower than a prescribed tolerable value and, vice versa, that the HIC is minimized, provided that the displacement of the head does not exceed a prescribed quantity.

Chapter 8 deals with the protection of a person traveling in a wheelchair who is involved in a frontal vehicular crash. To improve the protection, it is proposed to attach the wheelchair to a movable platform separated from the vehicle body by means of a shock isolator. The control of the platform is designed to reduce the occupant's injury risk, relative to what could happen if the wheelchair were attached directly to the vehicle. The isolator design is based on the minimization of the force transmitted to the wheelchair occupant, provided that the space allowed for the platform to move relative to the vehicle is constrained. Both the control without pre-action and the pre-acting control are considered.

1.2 RELATED STUDIES

The major studies carried out by the authors of the book are briefly summarized below.

1.2.1 Development of the Theory of Optimum Shock Isolation

Balandin, Bolotnik, and Pilkey (1999) considered a limiting performance problem for the shock isolation of a simple deformable system with "one and a half degrees of freedom." The system contains a mass with a Voigt element attached to it. The Voigt element consists of a spring and a dashpot with linear characteristics connected in parallel. The mass represents the inertial properties of the system, whereas the Voigt element models its elastic and viscous properties. The free end of the Voigt element is attached by means of a shock isolator to the base that is subject to a shock load. The action of the isolator is modeled by the control force acting between the base and the Voigt element. Two optimal control problems are stated. In one problem, the peak magnitude of the force developed in the Voigt element is to be minimized, provided that the displacement of the free end of the Voigt element relative to the base is constrained. In the other problem, the peak displacement of the free end of the Voigt element is to be minimized for the constraint imposed on the peak force developed in this element. It was shown that if the static deformation of the Voigt element produced by the maximum force allowed for this element is substantially less than the absolute minimum of the peak displacement of the rigid single-degree-of-freedom model of the system, then the rigid model provides an acceptable approximation to the original system. The larger the stiffness of the Voigt element, the more accurate the rigid approximation is in terms of the performance index. In this case, the optimal control for the single-degree-of-freedom system can be regarded as a near-optimal control for the original system with one and a half degrees of freedom. An estimate of the accuracy of such an approximation is given. This model was used to evaluate the potentials for the protection of the lower leg of a car driver involved in a frontal crash. Cheng and Pilkey (1999) used wavelets to discretize the control force for the numerical solution of the optimal control problem associated with the limiting performance analysis of shock isolation systems. The control force time history was approximated by a linear combination of basis wavelet functions with unknown coefficients that were employed as the design variables. A number of test problems were solved with wavelets and other functions used as the basis functions. A comparison of the convergence rates of the numerical optimization algorithms enabled the advantages and drawbacks of the wavelet approximation to be assessed.

Balandin et al. (2005) proposed a technique for impact isolation limiting performance analysis for systems that can be divided into three components: the base (subject to an impact disturbance), the container, and the object to be protected. The object is attached to the container by means of an impact isolator and the container is attached to the base by means of an additional impact isolator. Such models are typical for moving structures that involve impact-sensitive objects and can be utilized for the crashworthiness analysis of transport vehicles. The technique proposed involves the reduction of the optimal control problem for the three-component system to an auxiliary optimal control problem for a two-component system. The two-component system involves only the base and the object to be protected. In addition, the auxiliary problem involves only one control function, the control force acting on the object, whereas the primary problem has two control functions, the control force acting on the object and the control force acting on the container. This makes the auxiliary control problem easier to solve. Knowing the optimal control and the minimum of the performance index for the auxiliary problem, one can determine the optimal controls and the minimum value of the performance index for the primary problem, which is a three-component system. The solutions of these two problems are related by simple analytical equations. To solve the auxiliary problem, one, as a rule, needs to use numerical methods. However, if the object to be protected is modeled by one or two point masses, the auxiliary problem can also be solved analytically in some cases.

The concept of pre-acting control for active shock isolators was considered by Balandin, Bolotnik, and Pilkey (2005). With pre-acting control, the isolation system begins to respond to an impact before the impact has been applied to the base. The limiting performance of an isolator with pre-acting control was investigated for a single-degree-of-freedom system subject to an instantaneous impact. The isolation performance index was defined as the maximum of the absolute value of the displacement of the object to be isolated relative to the base, provided that the magnitude of the control force transmitted to the object does not exceed a prescribed value. It was shown that there is a substantial advantage in the use of pre-acting isolators over isolators without pre-action. Particular attention was given to a pre-acting isolator based on a passive elastic element (a spring) that is separating the object to be protected from the base. An example illustrated the calculation of the design parameters of such an isolator.

1.2.2 Best and Worst Disturbance Analyses

A technique to study the sensitivity of impact responses to prescribed crash test conditions was presented by Crandall et al. (1996). Motor vehicle impacts were used to illustrate the principles of this sensitivity analysis. Impact conditions were regulated by specifying either a corridor for the acceleration time history or other test parameters such as velocity change, crush distance, and pulse duration. By combining a time-domain constrained optimization method and a multibody dynamics simulator, the upper and lower bounds of occupant responses subject to the regulated corridors were obtained. It was found that these prescribed corridors may be either wide enough to allow extreme variations in occupant responses or so narrow that they are physically unrealizable in the laboratory test environment. A new corridor based on specifications for the test parameters of acceleration, velocity, crush distance, and duration for frontal vehicle impacts was given. Bai et al. (1999) investigated the best and worst possible responses of a child dummy in a child seat sled test where the sled deceleration pulse must follow a prescribed corridor. Constrained optimization techniques were applied to a two-degree-of-freedom lumped mass model of the sled test to determine the best and worst sled deceleration pulses within the prescribed corridor that produced the minimum and maximum child chest decelerations. A three-dimensional multibody model was used to predict the peak magnitudes for the absolute accelerations of the child's chest in sled tests with the best and worst sled deceleration pulses.

The philosophy of the extremal disturbance analysis for dynamical systems with uncertain inputs is described in general terms in Pilkey et al. (2006). This analysis involves solving optimal control problems in which the time histories of the inputs (external disturbances) are regarded as the control functions and a response measure of the system serves as the performance index. The performance index should be maximized or minimized over the disturbances within a prescribed class. Often this class is specified by lower and upper bounds (a corridor) between which the values of the disturbance must lie. The maximization and minimization problems are referred to as the worst disturbance and best disturbance problems, respectively. The solutions of these problems provide the extreme values between which the response measure lies for any disturbance from the specified class. The extremal disturbance analysis is important, in particular, when designing standards for testing devices for the protection of fragile objects from impact loads. This approach is illustrated for a single degree-of-freedom system that can be regarded as a simplified model of the equipment for sled tests of automobile restraint systems. A technique is proposed for constructing a corridor for inputs that provides a prescribed worst-to-best ratio for the system's response.

1.2.3 Spinal Injury Control

Cheng et al. (2001) considered the optimal performance of a helicopter seat cushion for the reduction of spinal injuries during vertical crashes. The spinal dynamic response index and the maximum spinal compression load were used as the system performance indices to be minimized. Three types of seat cushions that serve as shock isolators (passive, active, and pre-acting) were considered. A trade-off curve and the optimal control force for each type of cushion were obtained. Computational results showed that for the reduction of spinal injuries the pre-acting seat cushion is superior to the active non-pre-acting cushion, which is, in turn, superior to the passive cushion.

Cheng et al. (2005a, 2005b) used computational techniques to investigate the potential for the optimal control of an aircraft ejection seat cushion to reduce a pilot's spinal injury in an ejection event. A multibody model was used to simulate the dynamics of the biomechanical system, including the occupant, the seat pan, and the safety devices. The peak lumbar load of the occupant in the vertical direction was defined as the performance index to be minimized, while the peak acceleration of the upper torso was not allowed to exceed a prescribed tolerable value.

1.2.4 Thoracic Injury Control

Crandall, Cheng, and Pilkey (2000) used a two-mass injury model of the thorax (Lobdell et al., 1973) to study the limiting performance of seat belt systems for occupants in automobile frontal crashes. The corresponding optimal control problem was solved numerically for a specified crash deceleration pulse and the parameters of the model. The performance was measured by thoracic injury criteria, which include the maximum chest acceleration, compression, and viscous response, as well as by the maximum excursion of the occupant relative to the vehicle. It was observed that the optimal control force produced by the seat belt was not constant during the response time and that there was a substantial spike of the seat belt force at the beginning of the response.

Kent et al. (2007) proposed a concept for an active control of seat belts to mitigate the risk of thoracic injuries to automobile occupants in frontal crashes. The concept includes the determination of an optimal open-loop control that minimizes the peak excursion of the occupant in a vehicle, provided that the thoracic injury criteria remain within prescribed limits, and a feedback that sustains this control. The feedback control loop is developed based on measuring the current seat belt force and comparing the measured force with that prescribed by the optimal control. The seat belt force can be regulated, for example, by retracting and releasing the seat belts. The proposed methodology was applied to the two-mass thoracic injury model introduced by Lobdell et al. (1973).

The influence of slack in a vehicle restraint (seat belt) system on the reduction of risk of thoracic injuries in a frontal crash was studied by Kent, Purtsezov, and Pilkey (2007). The slack was modeled as a time delay in the response of the restraint system to the crash impact pulse. A limiting performance analysis was performed to determine the theoretically optimal control force–time profile generated by a vehicle restraint system with slack. The maximum chest compression was minimized subject to constraints on the chest acceleration, chest compression rate, chest viscous criterion, and excursion of the occupant in the vehicle. The two-mass injury model due to Lobdell et al. (1973) was used. For this model, regardless of the magnitude of the delay caused by the slack, the seat belt control force exhibited a short-duration period of high magnitude in the beginning of the response followed by an interval of nearly constant force. It was established that the peak magnitude of the restraining force decreases monotonically as the delay decreases.

1.2.5 Head Injury Control

Cheng et al. (1999) investigated the limiting performance of helmets for protecting the head from injury. A rigid head model and a two-mass translational head model (Rojanavanich and Stalnaker, 1988) were employed. Several head injury criteria were utilized, including head acceleration, the HIC, the energy imparted to the brain which is related to brain injury, and the power developed in the skull that is associated with skull fracture. A helmeted head hitting a rigid surface and a helmeted head hit by a moving object such as a ball were considered. The optimal characteristics of helmets and the impact responses of the helmeted head were obtained and analyzed computationally. Computational results were compared with the experimental data for bicycle helmets.

Balandin and Bolotnik (2002) and Balandin, Bolotnik, and Pilkey (2004) used analytical techniques to study the limiting capabilities of helmets for protecting the head when it is impacted against a fixed obstacle or it is hit with a nonpenetrating projectile. The translational head injury model due to Rojanavanich and Stalnaker (1988) was used to simulate the head's response to an impact. The performance index to be minimized was the peak value of the postimpact displacement of the outer shell of the helmet with respect to the head. The injury criteria subject to constraints were the peak power of the forces caused by the skull bone deformation and the peak acceleration of the brain. The action of the impact-isolating liner was modeled by a control force acting between the helmet shell and the skull. A two-sided bound for the desired minimum value of the displacement of the shell was constructed. The minimum displacement of the shell relative to the head evaluates the minimal thickness of the liner for the helmet able to provide a prescribed quality of protection from the impact.

Balandin et al. (2007) solved an optimal control problem for a rigid model of the head that hits a fixed surface covered with shock isolation padding, which mitigates the impact. The force exerted by the padding on the head was treated as a control force. An analytical expression was found for the optimal control that minimizes the path of deceleration of the head to come to a complete stop, provided that the HIC does not exceed a prescribed tolerable value. The minimum deceleration path evaluates the minimum thickness of the padding at which the risk of head injury, measured by the HIC, remains within prescribed limits for a given impact velocity. The dual problem, in which the HIC is to be minimized, while the deceleration path of the head (the padding thickness) is constrained, was also considered. The results obtained can be applied to the limiting performance analysis of the protection of the head from impacts by means of helmets. The shell of the helmet is assumed to be rigid and the impact to be instantaneous; that is, the shell comes to an instantaneous complete stop when hitting a rigid surface. The helmet's liner plays the role of the shock isolation padding. Similar problems were solved by Balandin, Belozerov, and Bolotnik (2008) for a helmeted head when the impact duration (the time of deceleration of the helmet shell to a complete stop) is finite. The impact velocity was fixed and the value of the deceleration of the helmet head was assumed to be constant. It was shown that the duration of the shock pulse substantially affects the minimal value of the HIC or the peak magnitude of the displacement of the head relative to the helmet padding achievable when the other criterion is subjected to the prescribed constraint. Accordingly, the duration of the shock pulse influences the optimal law for the control force acting between the helmet padding and the head. In the case of the minimization of the peak magnitude of the displacement of the head for the constrained HIC, the control law that is optimal for the instantaneous shock remains optimal for finite-duration impacts unless the duration does not exceed a critical value. For the minimization of the HIC subject to the constraint on the displacement of the head relative to the helmet padding, the control that is optimal for a finite-duration shock never coincides with the control that is optimal for the instantaneous shock.

1.2.6 Lower Extremity Injury Control

Cheng et al. (2004) investigated numerically the potential application of toepan padding for the prevention and reduction of lower limb injuries in an automobile crash. A two-mass lower limb injury model was developed on the basis of impact tests using postmortem human surrogates. A limiting performance analysis was used to find the best possible physical performance and characteristics of a passive or active padding for the minimization of the peak force transmitted to the tibia.

1.2.7 Injury Control of Wheelchair-Seated Occupants of Vehicles

Kang and Pilkey (1998) developed a nonlinear dynamic computer model to simulate the dynamic responses of a wheelchair–occupant system in a vehicle during a crash. The occupant, restrained by safety belts, is seated in a wheelchair that is tied down in a vehicle. The model was implemented using a version of the multibody dynamic simulator, the Articulated Total Body (ATB) program, which has been validated extensively by crash sled tests. The model was used to predict the responses of wheelchair–occupant systems in various crash environments. To evaluate the crashworthiness of different wheelchair tiedowns, the sensitivity of the dynamic responses to several design parameters, such as tiedown stiffness, wheel stiffness, and tiedown positions, was studied and optimal values of these parameters were obtained. In addition, the model was used to study the sensitivity of crash sled test results to impact pulses confined to a prescribed "corridor" in an effort to develop a sled test standard. It was found that a corridor defined by the International Organization for Standardization (ISO) allowed large variations in the responses and should be tightened.

To improve the protection of a wheelchair-seated person traveling in a vehicle from injuries in a crash, Balandin et al. (2008) proposed to attach the wheelchair to a movable platform separated from the vehicle body by means of a shock isolator. The control of the platform should be designed to reduce the occupant's injury risk compared to attaching the wheelchair directly to the vehicle. The isolator design was based on the minimization of the force transmitted to the wheelchair occupant, provided that the space allowed for the platform to move relative to the vehicle was constrained. The possibility of pre-acting control when the isolator is engaged for a time prior to the crash was discussed. A multibody model of the platform-based occupied wheelchair was utilized for full-scale simulation of the response of the system to a crash pulse. The simulation showed a noticeable reduction in the injury risk due to the platform and an even greater reduction of injury with pre-acting control.

1.2.8 Reviews

The first review related to the development of the theory of optimum shock and vibration isolation was compiled by Balandin, Bolotnik, and Pilkey (1998) and covers the period from the beginning of active studies on that subject matter in the late 1950s to 1997. It contains a concise outline of the basic concepts and an annotated bibliography of relevant publications.

Later, these authors published a more detailed review (Balandin, Bolotnik, and Pilkey, 2000) that characterized the historical perspectives and state of the art of the theory of optimal shock isolation. Mathematical statements of basic problems were given and solutions of these problems for single-degree-of-freedom models were presented. Separate sections were devoted to the limiting performance analysis and the parametric optimization of shock isolation systems. Some topical issues, essential for the further development of optimization techniques for shock isolators, were indicated. An extensive bibliography was included.

In 2001, they published a paper that characterized the trends and first results of using optimal control techniques to evaluate the limiting capabilities of the protection from impact-induced injuries by means of shock isolators (Balandin, Bolotnik, and Pilkey, 2001). At that time, studies in this field had just been started at the University of Virginia and some other research institutions. Problems related to the optimal design of a crashworthy helicopter seat, automobile seat belts, and sporting helmets were discussed.

Cheng et al. (2005a and 2005b) gave an overview of optimization techniques to investigate the limiting performance and to identify the design variables of biomechanical systems for injury control. A biomechanical system involves a vehicle, an occupant (or dummies in crash tests), and safety equipment such as seat belts, padding, or air bags. Basic approaches, such as limiting performance analysis, parametric optimization of injury control devises, best and worst disturbance analyses to evaluate the range of responses of the occupant to impact pulses of the vehicle from a prescribed class, and sensitivity analysis of the response criteria to variations in the control parameters, are characterized. Basic injury models of different degrees of complexity, from few-degree-of-freedom analytical models to multibody and finite-element models, are outlined and software packages that implement these models are indicated. Computational techniques for simulation and optimization of biomechanical systems are discussed.

REFERENCES

Bai, C. Q., Morris, R. A., Sieveka, E. M., Crandall, J. R., Cheng, Z. Q., and Pilkey, W. D., 1999, A study of standards for child restraint systems for impact tests, *International Journal of Crashworthiness*, Vol. 4, No. 1, pp. 59–70.

Balandin, D. V., Belozerov, A. S., and Bolotnik, N. N., 2008, Limiting performance analysis of the protection of the human head from impact-induced injuries taking into account the duration of the shock pulse, *Journal of Computer and Systems Science International*, Vol. 47, No. 1, pp. 355–370.

Balandin, D. V., and Bolotnik, N. N., 2002, Limiting performance analysis of helmets for head injury prevention, *Journal of Computer and Systems Sciences International*, Vol. 47, No. 1, pp. 149–156.

Balandin, D. V., Bolotnik, N. N., Crandall J. R., Pilkey, W. D., and Purtsezov, S. V., 2007, Optimal impact isolation for injury prevention evaluated by the head injury criterion, *Shock and Vibration*, Vol. 14, pp. 355–370.

Balandin, D. V., Bolotnik, N. N., and Pilkey, W. D., 1998, Review: Optimal shock and vibration isolation, *Shock and Vibration*, Vol. 5, pp. 73–87.

Balandin, D. V., Bolotnik, N. N., and Pilkey, W. D., 1999, On the optimal shock isolation of a system with one and a half degrees of freedom, *Shock and Vibration*, Vol. 6, pp. 159–167.

Balandin, D. V., Bolotnik, N. N., and Pilkey, W. D., 2000, Optimal protection from impact and shock: Theory and methods, *Applied Mechanics Reviews*, Vol. 53, pp. 237–264.

Balandin, D. V., Bolotnik, N. N., and Pilkey, W. D., 2001, Limiting performance analysis of impact isolation systems for injury prevention, *Shock and Vibration Digest*, Vol. 33, No. 6, pp. 453–472.

Balandin, D. V., Bolotnik, N. N., and Pilkey, W. D., 2004, Capabilities of helmets for preventing head injuries induced by ballistic impacts, *Shock and Vibration*, Vol. 11, pp. 547–562.

Balandin, D. V., Bolotnik, N. N., and Pilkey, W. D., 2005, Preacting control for shock and impact isolation systems, *Shock and Vibration*, Vol. 12, pp. 49–66.

Balandin, D. V., Bolotnik, N. N., Pilkey, W. D., and Purtsezov, S. V., 2005, Impact isolation limiting performance analysis for three-component models, *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, Vol. 127, No. 3, pp. 463–471.

Balandin, D. V., Bolotnik, N. N., Pilkey, W. D., Purtsezov, S. V., and Shaw, C. G., 2008, Concept of a platform-based impact isolation system for protection of wheelchair occupants from injuries in vehicle crashes, *Medical Engineering and Physics*, Vol. 30, No. 2, pp. 258–267.

Cheng, Z. Q., Crandall, J. R., Darvish, K., and Pilkey, W. D., 2004, Limiting performance analysis of toepan padding for mitigating lower limb injuries, *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, Vol. 218, pp. 619–628.

Cheng, Z. Q., and Pilkey, W. D., 1999, Wavelet-based limiting performance analysis of mechanical system subject to transient disturbances, *Finite Elements in Analysis and Design*, Vol. 33, pp. 233–245.

Cheng, Z. Q., Pilkey, W. D., Balandin, D. V., Bolotnik, N. N., Crandall, J. R., and Shaw, C. G., 2001, Optimal control of helicopter seat cushions for the reduction of spinal injuries, *International Journal of Crashworthiness*, Vol. 6, pp. 321–338.

Cheng, Z. Q., Pilkey, W. D., Crandall, J. R., Bass, C. R., and Darvish, K., 1999, Limiting performance of helmets for the prevention of head injury, *Shock and Vibration*, Vol. 6, pp. 299–320.

Cheng, Z. Q., Pilkey, W. D., Pellettiere, J. A., and Rizer, A. L., 2005a, Limiting performance analysis of biomechanical systems for optimal injury control—Part 1: Theory and methodology, *International Journal of Crashworthiness*, Vol. 10, No. 6, pp. 567–577.

Cheng, Z. Q., Rizer, A. L., Pellettiere, J. A., and Pilkey, W. D., 2005b, Limiting performance analysis of biomechanical systems for optimal injury control—Part 2: Applications, *International Journal of Crashworthiness*, Vol. 10, No. 6, pp. 579–587.

Crandall, J. R., Cheng, Z. Q., and Pilkey, W. D., 2000, Limiting performance of seat belt systems for the prevention of thoracic injuries, *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, Vol. 214, pp. 127–139.

Crandall, J. R., Pilkey, W. D., Kang, W., and Bass, C. R., 1996, Sensitivity of occupant response subject to prescribed corridors for impact testing, *Shock and Vibration*, Vol. 3, pp. 435–450.

Kang, W., and Pilkey, W. D., 1998, Crash simulations of wheelchair occupant systems in transport, *Journal of Rehabilitation Research and Development*, Vol. 35, No. 1, pp. 73–84.

Kent, R. W., Balandin, D. V., Bolotnik, N. N., Pilkey, W. D., and Purtsezov, S. V., 2007, Optimal control of restraint forces in an automobile impact, *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, Vol. 129, No. 4, pp. 415–424.

Kent, R. W., Purtsezov, S. V., and Pilkey, W. D., 2007, Limiting performance analysis of a seat belt system with slack, *International Journal of Impact Engineering*, Vol. 34, pp. 1382–1395.

Lobdell, T. E., Kroell, C. K., Schneider, D. C., Hering, W. E. and Nahum, A. M., 1973, “Impact Response of the Human Thorax,” *Human Impact Response Measurement and Simulation*, W. F. King, H. J. Mertz, eds., Plenum Press, London, pp. 201–245.

Pilkey, W. D., Balandin, D. V., Bolotnik, N. N., and Purtsezov, S. V., 2006, Extremal disturbance analysis for dynamical systems with uncertain input, *Journal of the Brazilian Society of Mechanical Science and Engineering*, Vol. 28, No. 4, pp. 512–518.

Rojanavanich, V., and Stalnaker, R. L., 1988, Parametric Studies of the Translational Head Injury Model, *Proceedings of IRCOBI*, International Research Council on the Biomechanics of Impact, 181–194, Bron, France.