Chapter 1

Assembling Your Tools

In This Chapter

- Nailing down the basics: Numbers
- Recognizing the players: Variables and signs
- Grouping terms and operations together
- Playing the game and following the rules

You probably have heard the word *algebra* on many occasions and knew that it had something to do with mathematics. Perhaps you remember that algebra has enough information to require taking two separate high school algebra classes — Algebra I and Algebra II. But what exactly *is* algebra? What is it *really* used for?

This chapter answers these questions and more, providing the straight scoop on some of the contributions to algebra's development, what it's good for, how algebra is used, and what tools you need to make it happen.

In a nutshell, *algebra* is a way of generalizing arithmetic. Through the use of variables that can generally represent *any* value in a given formula, general formulas can be applied to *all* numbers. Algebra uses positive and negative numbers, integers, fractions, operations, and symbols to analyze the relationships between values. It's a systematic study of numbers and their relationship, and it uses specific rules.

For example, the formula $a \times 0 = 0$ shows that any real number, represented here by the *a,* multiplied by zero always equals zero. (For more information on the multiplication property of zero, see Chapter 14.)

In algebra, by using an x to represent the number two, for example in $x + x + x = 6$, you can generalize with the formula $3x = 6$.

You may be thinking, "That's great and all, but come on. Is it really *necessary* to do that — to plop in letters in place of numbers and stuff?" Well, yes. Early mathematicians found that using letters to represent quantities simplified problems. In fact, that's what algebra is all about — simplifying problems.

The basic purpose of algebra has been the same for thousands of years: to allow people to solve problems with unknown answers.

FUN FACT

Aha algebra

Dating back to about 2000 B.C. with the Babylonians, algebra seems to have developed in slightly different ways in different cultures. The Babylonians were solving three-term quadratic equations, while the Egyptians were more concerned with linear equations. The Hindus made further advances in about the sixth century A.D. In the seventh century, Brahmagupta of India provided general solutions to quadratic equations and had interesting takes on zero. The Hindus regarded irrational numbers as actual numbers — although everybody didn't hold to that belief at that time.

The sophisticated communication technology that exists in the world now was not available then, but early civilizations still managed to exchange information over the centuries. In A.D. 825, al-Khowarizmi of Bagdad wrote the first algebra textbook. One of the first solutions to an algebra problem, however, is on an Egyptian papyrus that is about 3,500 years old. Known as the Rhind Papyrus after the Scotsman who purchased the 1-foot-wide, 18-foot-long papyrus in Egypt in 1858, the artifact is preserved in the British Museum — with a piece of it in the Brooklyn Museum. Scholars determined that in 1650 B.C., the Egyptian scribe Ahmes copied some earlier mathematical works onto the Rhind papyrus.

One of the problems reads, "Aha, its whole, its seventh, it makes 19." The "aha" isn't an exclamation. The word "aha" designated the unknown. Can you solve this early Egyptian problem? It would be translated, using current algebra symbols, as: $x + \frac{x}{7} = 19$. The unknown is represented by the x, and the solution is $x = 16\frac{5}{8}$. It's not hard; it's just messy.

Beginning with the Basics: Numbers

Where would mathematics and algebra be without numbers? A part of everyday life, numbers are the basic building blocks of algebra. Numbers give you a value to work with.

Where would civilization be today if not for numbers? Without numbers to figure the total cubits, Noah couldn't have built his ark. Without numbers to figure the distances, slants, heights, and directions, the pyramids would never have been built. Without numbers to figure out navigational points, the Vikings would never have left Scandinavia. Without numbers to examine distance in space, humankind could not have landed on the moon.

Even the simple tasks and the most common of circumstances require a knowledge of numbers. Suppose that you wanted to figure the amount of gasoline it takes to get from home to work and back each day. You need a number for the total miles between your home and business and another number for the total miles your car can run on one gallon of gasoline.

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The different sets of numbers are important because what they look like and how they behave can set the scene for particular situations or help to solve particular problems. It's sometimes really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers do not include fractions. This may happen if you're working through a problem that involves a number of cars. Who wants half a car?

Algebra uses different sets of numbers, such as whole numbers and those that follow here, to solve different problems.

Really real numbers

Real numbers are just what the name implies. In contrast to imaginary numbers, they represent *real* values — no pretend or make-believe. Real numbers, the most inclusive set of numbers, comprise the full spectrum of numbers; they cover the gamut and can take on any form — fractions or whole numbers, decimal points or no decimal points. The full range of real numbers includes decimals that can go on forever and ever without end. The variations on the theme are endless.

For the purposes of this book, I always refer to real numbers.

Counting on natural numbers

A *natural* number is a number that comes naturally. What numbers did you first use? Remember someone asking, "How old are you?" You proudly held up four fingers and said, "Four!" The natural numbers are also *counting* numbers: 1, 2, 3, 4, 5, 6, 7, and so on into infinity.

You use natural numbers to count items. Sometimes the task is to count how many people there are. A half-person won't be considered (and it's a rather grisly thought). You use natural numbers to make lists.

Wholly whole numbers

Whole numbers aren't a whole lot different from the natural numbers. The *whole* numbers are just all the natural numbers plus a zero: 0, 1, 2, 3, 4, 5, and so on into infinity.

Whole numbers act like natural numbers and are used when whole amounts (no fractions) are required. Zero can also indicate none. Algebraic problems often require you to round the answer to the nearest whole number. This makes perfect sense when the problem involves people, cars, animals, houses, or anything that shouldn't be cut into pieces.

Integrating integers

Integers allow you to broaden your horizons a bit. Integers incorporate all the qualities of whole numbers and their opposites, or additive inverses of the whole numbers (refer to the "Operating with opposites" section in this chapter for information on additive inverses). *Integers* can be described as being positive and negative whole numbers: $\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$.

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, it's not a fraction! This doesn't mean that answers in algebra can't be fractions or decimals. It's just that most textbooks and reference books try to stick with nice answers to increase the comfort level and avoid confusion. This is the plan in this book, too. After all, who wants a messy answer, even though, in real life, that's more often the case.

Being reasonable: Rational numbers

Rational numbers act rationally! What does that mean? In this case, acting rationally means that the decimal equivalent of the rational number behaves. The decimal ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving." Some rational numbers have decimals that end in 2, 3.4, 5.77623, –4.5. Other rational numbers have decimals that repeat the same pattern, such as $3.164164164\ldots = 3.\overline{164}$, or $.666666666\ldots = .\overline{6}$. The horizontal bar over the 164 and the 6 lets you know that these numbers repeat forever.

In *all* cases, rational numbers can be written as a fraction. They all have a fraction that they are equal to. So one definition of a *rational number* is any number that can be written as a fraction.

Restraining irrational numbers

Irrational numbers are just what you may expect from their name — the opposite of rational numbers. An *irrational* number *cannot* be written as a fraction, and decimal values for irrationals never end and never have a nice pattern to them. Whew! Talk about irrational! For example, pi, with its never-ending decimal places, is irrational.

Evening out even and odd numbers

An *even* number is one that divides evenly by two. "Two, four, six, eight. Who do we appreciate?"

Digits, fingers, and toes through history

The Hindu-Arabic numerals, such as 1, 2, 3, 4, 5, 6, 7, 8, 9, originated with the Hindus and were created to go along with a decimal system. The word *decimal* comes from the Latin word meaning *tenth* or *tithe*. The Hindu-Arabic system is a *positional* system, which means that the order you write digits in matters. The number 35 is different from the number 53 because in 35, the 3 stands for three tens and in 53, the three stands for three ones.

The main reason humans developed a decimal, or base-ten, system is because humans usually have ten fingers and ten toes. It could have been a base-twenty system or a base-five system — like the Babylonians had. From about 1700 B.C. until about A.D. 500, however, most scientists used a base-sixty system. Using sixty as a base came about because the number of days in a year was estimated to be roughly 360 days, and sixty was one of the nice divisors of 360. Remnants of the early base-sixty system are found in our minutes and seconds. Can you imagine having to remember sixty different digits instead of just ten?

The symbols used in the early systems all stood for something: one of these, two of those, and so on. For a long while, there was no digit or symbol for nothing or zero. The first symbol for zero (something like an upside-down W) wasn't introduced until about 300 B.C. Before then, to indicate that there was nothing, the writer left an empty space, which wasn't very efficient. Sometimes the writer forgot to leave a space, and a careless writer may not have left enough of a space. Plus, there was no clear way to indicate more than one zero.

An *odd* number is one that *does not* divide evenly by two. The even and odd numbers alternate when you list all the integers.

Varying Variables

Variable is the most general word for a letter that represents the unknown, or what you're solving for in an algebra problem. A variable *always* represents a number.

Algebra uses letters, called *variables,* to represent numbers that correspond to specific values. Usually, if you see letters toward the beginning of the alphabet in a problem, such as a, b, or c, they represent known or set values, and the letters toward the end of the alphabet, such as x, y, or z, represent the unknowns, things that can change, or what you're solving for.

The following list goes through some of the more commonly used variables.

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- An *n* doesn't really fall at the beginning or end of the alphabet, but it's used frequently in algebra, often representing some unknown quantity or number — probably because *n* is the first letter in *number*.
- The letter *x* is often the variable you solve for, maybe because it's a letter of mystery: X marks the spot, the x-factor, *The X Files.* Whatever the reason *x* is so popular as a variable, the letter also is used to indicate multiplication. You have to be clear, when you use an *x*, that it isn't taken to mean multiply.
- *C* and *k* are two of the more popular letters used for representing known amounts or constants. The letters that represent variables and numbers are usually small case: *a, b, c,* and so on. Capitalized letters are used most commonly to represent the answer in a formula, such as the capital *A* for area of a circle equals pi times the radius squared, $A = \pi r^2$. (You can find more information on the area of a circle in Chapter 17.) The letter *C,* mentioned previously as being a popular choice for a constant, is used frequently in calculus and physics, and it's capitalized there — probably more due to tradition than any good reason.

Speaking in Algebra

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. It's important to know the vocabulary in a foreign language; it's just as important in algebra.

- An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. $2x^2 + 4x$ is an example of an expression.
- A *term,* such as $4xy$, is a grouping together of one or more factors (variables and/or numbers). Multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another. For example, the expression $3xy + 5x - 6$ has three *terms*.
- An *equation* uses a sign to show a relationship — that two things are equal. By using an equation, tough problems can be reduced to easier problems and simpler answers. An example of an equation is $2x^2 + 4x = 7$. See the chapters in Part III for more information on equations.
- An *operation* is an action performed upon one or two numbers to produce a resulting number. Operations are addition, subtraction, multiplication, division, square roots, and so on. See Chapter 6 for more on operations.

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- A *variable* is a letter that always represents a number, but it varies until it's written in an equation or inequality. (An *inequality* is a comparison of two values. See more on inequalities in Chapter 16.) Then the fate of the variable is set — it can be solved for, and its value becomes the solution of the equation.
- A *constant* is a value or number that never changes in an equation — it's constantly the same. Five (5) is a constant because it is what it is. A variable can be a constant if it is assigned a definite value. Usually, a variable representing a constant is one of the first letters in the alphabet. In the equation $ax^2 + bx + c = 0$, *a, b,* and *c* are constants and the *x* is the variable. The value of *x* depends on what *a, b,* and *c* are assigned to be.
- An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2. It's used to show repeated multiplication. An exponent is also called the *power* of the value. For more on exponents, see Chapter 4.

Taking Aim at Algebra Operations

In algebra today, a variable represents the unknown (see more on variables in the "Speaking in Algebra" section earlier in this chapter). Before the use of symbols caught on, problems were written out in long, wordy expressions. Actually, using signs and operations was a huge breakthrough. First, a few operations were used, and then algebra became fully symbolic. Nowadays, you may see some words alongside the operations to explain and help you understand, like having subtitles in a movie. Look at this example to see what I mean. Which would you rather write out:

> The number of quarts of water multiplied by six and then that value added to three
>
> *or*
>
> $6x + 3$?

I'd go for the second option. Wouldn't you?

By doing what early mathematicians did — letting a variable represent a value, then throwing in some operations (addition, subtraction, multiplication, and division), and then using some specific rules that have been established over the years — you have a solid, organized system for simplifying, solving, comparing, or confirming an equation. That's what algebra is all about: That's what algebra's good for.

FUN FACT

What's in a word?

The word *algebra* is a variation on the word *al-jabr*, an Arabic word, which roughly means a reunion or joining together of parts. This word was changed further when the Moors brought the word *algebrista,* meaning *bonesetter* (someone who joins or puts together bones), to Spain in the the Middle Ages. Signs over barber shops in Spain saying *Algebrista y Sangradoe* indicated that the shop offered a bonesetter *and* bloodletter. At that time, and for centuries, barbers performed minor medical procedures to supplement their income. The traditional red-and-white-striped barber pole symbolized blood and bandages. Maybe that's why the word *algebra,* which comes from *algebrista,* has a reputation for being painful at times.

Deciphering the symbols

The basics of algebra involve symbols. Algebra uses symbols for quantities, operations, relations, or grouping. The symbols are shorthand and are much more efficient than writing out the words or meanings. But you need to know what the symbols represent, and the following list shares some of that info.

TECHNICAL STUFF

- $+$ means *add* or *find the sum, more than,* or *increased by;* the result of addition is the *sum.*
- $-$ means *subtract* or *minus* or *decreased by* or *less;* the result is the *difference*.
- $\times$ means *multiply* or *times.* The values being multiplied together are the *multipliers* or *factors;* the result is the *product.* Some other symbols meaning multiply can be grouping symbols: ( ), [ ], { } •, *. In algebra, the $\times$ symbol is used infrequently because it can be confused with the variable *x*. The dot is popular because it's easy to write. The grouping symbols are used when you need to contain many terms or a messy expression. By themselves, the grouping symbols don't mean to multiply, but if you put a value in front of a grouping symbol, it means to multiply. For more on the grouping symbols, skip ahead to the "Grouping" section.
- $\div$ means divide. The number that's going into the *dividend* is the *divisor.* The result is the *quotient.* Other signs that indicate division are the fraction line and slash, /.
- $\sqrt{}$ means to take the *square root* of something — to find the number, which multiplied by itself gives you the number under the sign (see more on square roots in Chapter 4).
- $|\ \ |$ means to find the *absolute value* of a number, which is the number itself or its distance from zero on the number line (see more on *absolute value* in Chapter 2).

- **. . .** means *et cetera, and so on,* or *in the same pattern.* You use an ellipsis in algebra when you have a long list of numbers and don't want to have to write all of them. For instance, if you want to list numbers starting with 1 and going up by 1 forever and ever, write: "1, 2, 3, 4, **. . .** ". Or you can write the list of numbers from 600 through 1,000 as, "600, 601, 602, **. . .** , 1,000."
- π is the Greek letter pi that refers to the irrational number: 3.14159 . . . . It represents the relationship between the diameter and circumference of a circle. For more information on this relationship, see Chapter 17.

Grouping

When a car manufacturer puts together a car, several different things have to be done first. The engine experts have to construct the engine with all its parts. The body of the car has to be mounted onto the chassis and secured, too. Other car specialists have to perform the tasks that they specialize in as well. When these tasks are all accomplished in order, then the car can be put together. The same thing is true in algebra. You have to do what's inside the *grouping* symbol before you can use the result in the rest of the equation.

Grouping symbols tell you that you have to deal with the *terms* inside the grouping symbols *before* you deal with the larger problem.

The main grouping symbols are

- **()** Parenthesis (This one is used most often.)
- **[ ]** Brackets
- **{ }** Braces

For example, 8 – (4 – 2) says to do what's in the parenthesis first. This is different from (8 – 4) – 2. The first expression works out to be 6, and the second expression to 2.

These three grouping symbols — the parenthesis, bracket, and brace — are used both alone and with each other. When used together, the symbols organize a more complicated problem.

Defining relationships

Algebra is all about relationships — not the he-loves-me-he-loves-me-not kind of relationship — but the relationships between numbers or among the terms of an equation. Although algebraic relationships can be just as complicated as romantic ones, you have a better chance of understanding an algebraic relationship. The symbols for the relationships are given here.

The beginning of the equal sign

Robert Recorde first used the equal sign (=) in the mid-1500s. He wrote, "I will sette as I doe often in woorke use, a paire of parrallels, or Gemowe lines of one lengthe, thus ==, because noe 2 thynges can be moare equalle." However, not all mathematicians immediately accepted the equal sign. Some preferred two upright parallel lines. A symbol resembling α (with longer "tails") was also popular for quite a while. The equal sign seemed to have been generally accepted in the mid-1600s.

- = means that the first value *is equal to* or the same as the value that follows.
- ≠ means that the first value *is not equal to* the value that follows.
- ≈ means that one value is *approximately the same* or *about the same* as the value that follows; this is used when rounding numbers.
- ≤ means that the first value is *less than or equal to* the value that follows.
- < means that the first value is *less than* the value that follows.
- ≥ means that the first value is *greater than or equal to* the value that follows.
- > means that the first value is *greater than* the value that follows.

Operating with opposites

When solving equations in algebra, doing the *opposite* to work your way toward the answer comes up often. You have to *undo* operations that have been *done* to the variable. The opposite of an operation is another operation that gets you back where you started. This is used primarily to get rid of numbers that are combined with a variable so you can solve for the variable in an equation.

Being contrary: Doing opposite operations

The opposite of adding three is subtracting three. If you add three to 100, you get 103. If you then subtract three from 103, you're back where you started.

- The opposite of addition is subtraction.
- The opposite of subtraction is addition.
- The opposite of multiplication is division.
- The opposite of division is multiplication.

- The opposite of taking a square root is *squaring* (multiplying a value by itself).
- The opposite of squaring is taking the square root.
- The opposite of cubing is taking the cube root.

Dealing with the opposites of numbers

A number actually has two *opposites:* the *additive inverse* and the *multiplicative inverse:*

- The *additive inverse* is the number with the opposite sign. So −3 is the additive inverse of 3, and 16 is the additive inverse of −16. Use these if 3 or 16 is being added to a variable, and you want to get the variable alone; this is used when solving an equation for the value of the variable.
- The *multiplicative inverse* is also called the reciprocal. The *reciprocal* is the original number written as the bottom of a fraction with a one on the top. So $\frac{1}{2}$ is the reciprocal of 2, and 25 is the reciprocal of $\frac{1}{25}$. If a number starts out as a fraction, its reciprocal is just that number written upside-down. So the reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$. Use this if a number multiplies or divides a variable; it gets the variable alone so it can be solved for.

Playing by the Rules

The basics of algebra also involve rules, like the rules to follow when you're driving. If everyone follows the same rules, accidents and chaos are less likely. The same goes for algebra. You have to observe the rules of algebra when you work with variables, numbers, and symbols. Following the rules is especially important when you solve problems because you don't know what number a variable stands for. The rules were developed, and everyone uses the same ones as everyone else, which is why the language of algebra is so universal.

Algebra involves symbols, such as variables and operation signs, which are the tools that you can use to make algebraic expressions more useable and readable. These things go hand in hand with simplifying, factoring, and solving problems, which are easier to solve if broken down into basic parts. Using symbols is actually much easier than wading through a bunch of words.

- To *simplify* means to combine all that can be combined, cut down on the number of terms, and put an expression in an easily understandable form. To find more on simplifying, see Chapter 13.
- To *factor* means to change two or more terms to just one term. See Part II for more on factoring.
- *Solve* means to find the answer. In algebra, it means to figure out what the variable stands for.

Equation solving is fun because there's a point to it. You solve for something (often a variable, such as *x*) and get an answer that you can check to see whether you're right or wrong. It's like a puzzle. It's enough for some people to say, "Give me an *x*." What more could you want? But solving these equations is just a means to an end. The real beauty of algebra shines when you solve some problem in real life — a practical application. Are you ready for these two words: *story problems?* Story problems are the whole point of doing algebra. Why do algebra unless there's a good reason? Oh, I'm sorry. Some of you may just like to solve algebra equations for the fun alone. Yes, there are folks like that. But some folks love to see the way a complicated paragraph in the English language can be turned into a neat, concise expression, such as, "The answer is three bananas."

Going through each step and using each tool to play this game is entirely possible. *Simplify, factor, solve, check*. That's good! Lucky you. It's time to dig in!