SECTION 1

Dealing with 'Real-World' Problems

Probability theory is a branch of mathematics that has been developed for more effective mathematical treatment of those situations in the real world that involve uncertainty, or randomness, in some sense. Most nontrivial real-world problems incorporate just such situations, so that the practical importance of probability theory hardly needs to be stressed. It turns out, however, that intensive study of the theory alone can still leave the would-be practitioner peculiarly inept in its application. Why should this be so?

It is not inappropriate to ask this question right at the outset. After all, when commencing the exploration of new territory, it helps to be alerted about the nature of the obstacles that lie ahead. A few reflections can lead to an answer to our question.

Example 1.1

A young man working as a sales clerk in a furniture store is asked to count all the chairs that are on display in the store. These chairs may differ from each other in design and material. Nevertheless, the sales clerk automatically adds to his count each time his gaze comes to rest on an object that, in fact, anyone else would also regard as an instance of 'chair'. This is because the *concept* 'chair' is firmly established in his mind. Possession of the concept 'chair' permits the clerk in this situation to carry out the simple mathematical operation of *counting*. Furthermore, the widely shared agreement regarding the concept 'chair' assures that the count will be the same as that obtained by anyone else, barring oversights or counting errors. We may say that with the aid of the concept 'chair', the clerk avoids *conceptual errors* in his task, although not necessarily mathematical errors, or procedural errors.

We can imagine how the clerk in this Example has his store environment modeled in terms of a large variety of concepts, some as simple and common as 'chair'. These concepts serve to classify the environment, and act as discriminators in simple tasks such as discussed in the Example. On the other hand, it is important to realize that the applicability of a concept is not always clear-cut; concepts have a certain 'fuzziness'. Surely we can devise an object that neither obviously is a chair, nor obviously isn't a chair.

Probability Concepts and Theory for Engineers, First Edition. Harry Schwarzlander.

In Example 1.1, the sales clerk is performing a '*real-world*' task. He is concerned with the immediate experience of his store environment, to which he is applying his counting ability. This is the sense in which we will use 'real world'. A person who picks up two dice and throws them acts in the real world, in contrast to another person who merely talks about throwing two dice.

Whenever mathematics is applied to a real-world problem, an important step is the establishment of a suitable model, or idealization, of the physical problem under consideration. This makes possible the application of the logical rules of mathematics to the real world. When we have a very simple problem, the modeling seems to take place rather automatically, so that we may not even be aware of it, as is surely the case with the sales clerk in Example 1.1. In more sophisticated situations, more effort generally goes into the modeling process. Because this modeling process always involves 'conceptualization'—fitting the real-world problem at hand into our way of thinking about things—we will call such models *conceptual models*.

Example 1.2

An electronic engineer is handed a small cylindrical capsule with a short wire protruding from each end. He tries to 'identify' this object as a particular kind of electronic component, such as a resistor, or a capacitor. Actually, 'resistor', 'capacitor', etc., are conceptual models for real-world objects—the engineer's way of viewing the great variety of electronic components he encounters. His task is, therefore, to determine which of these models is most appropriate for the object at hand.

The model called 'resistor', for instance, pertains to real-world objects having two (or more) distinct places for electrical connections. Furthermore, the applicability of this model depends on the effect produced by the component in question when incorporated into an electric circuit. This effect depends on the internal construction of the component.

We noted before that a conceptual model makes possible the application of mathematics to the real world. In the situation just described it is the mathematics of electrical network theory. For instance, the conceptual model 'resistor' implies to the engineer that under suitable conditions the relationship

$$i = ke \tag{1.1}$$

adequately characterizes the current i through the device in question, at an instant when the voltage across the terminals is e, where k is a proportionality constant. The conceptual model 'resistor' can therefore be considered as a bridge between a real-world object and the *mathematical model* that is expressed by the relation (1.1).

Of course, for a given real-world problem, the choice of a suitable model need not be unique, and the mathematical result obtained may depend on the particular model used. The appropriateness of any one model can be judged by how well the results obtained with it agree with, or predict, the actual physical nature or behavior of the situation being considered.

'Networks' are the conceptual models to which Network Theory is applied. Network Theory is not used in *building* the model, when some real-world electrical circuit is analyzed. But, once a model has been decided on, then Network Theory allows various conclusions to be drawn because it provides rules for applying abstract mathematical techniques to the model. The same process can be discerned whenever mathematics is applied to real-world problems. *The application of Probability Theory to situations of randomness is also made through suitable conceptual models.* However, as we will see in Section 2, the conceptual models required for probability problems are considerably more complicated than those needed in connection with most other branches of applied mathematics.

Just as Network Theory cannot be applied where there are no network models, so is Probability Theory useless without appropriate models. The establishment of an adequate model is therefore an essential ingredient in tackling any real-world problem by means of Probability Theory. Neglect of this important point easily results in confused and erroneous applications of the theory. Here lies the answer to the question posed at the beginning of this Section.[1]

As we pursue our study of Probability in this book, we will try to pay attention to the connection between real-world problems and their mathematical formulation from the start. This will be possible with the help of a standard scheme for building conceptual models—a scheme that can be applied to all probability problems.

[1] For further reading on the role of models in Applied Mathematics and the Sciences, see for example [Fr1], [Ri1], [MM1].