Centrifuges, Sedimenting

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Symbols

a	area, m^2
d	diameter, m
D	inner diameter of test tube, m
D	inner diameter of decanter bowl, m
g	acceleration due to gravity (9.81 m/s^2)
k	constant
KQ	semi-empirical measure of centrifuge size (Eq. 15)
L	length of decanter bowl, m
n	bowl speed, min^{-1}
N	number of disks
p	pressure, Pa
Q	flow rate, m^3/s
r	radius, m
R	outer radius of rotor, m
s	settling distance, m
t	distance between disks, m
T	spin time, s
u	velocity, m/s
V	bowl volume, m^3
z	number of nozzles
Z	acceleration factor or g number
α	half-cone angle of disk
β	angle of tube in test-tube centrifuge
χ	angle of nozzle to the tangent of bowl
η	dynamic viscosity of fluid, mPa · s
λ	dimensionless number (Eq. 18)
μ	separation efficiency factor (Eq. 13)
ν	kinematic viscosity of fluid, m^2/s
ϱ	density, kg/m^3
ω	angular velocity, rad/s
Σ	area equivalent (Eq. 10), m^2

Subscripts

act	actual
c	centrifugal
d	drag
e	equivalent

f	fluid
g	gravity
i	interface
in	inner
h	heavy liquid
l	light liquid
N	nozzle
out	outer, outlet
p	particle
theor	theoretical
1	inner
2	outer

1. Introduction

Sedimenting centrifuges are used for the separation of two or three phases: liquid – liquid, liquid – solid and liquid – liquid – solid. Sedimenting centrifuges use centrifugal force to accelerate the sedimentation process. By rotating the process fluid, the sedimentation rate can be increased by a factor of several thousands compared to static sedimentation or settling. Other types of equipment also use centrifugal force for phase separation. This article deals only with sedimenting centrifuges. For filtration centrifuges and hydrocyclones, see → Hydrocyclone Separation and → Centrifuges, Filtering.

In *liquid – liquid separation* the most common alternative to sedimenting centrifuges are static settlers. The advantage of the centrifugal separator compared to the settler is the small equipment volume and/or short residence time.

The choice between a centrifugal filter and a sedimenting centrifuge in *solid – liquid separation* depends on the particle size and the permeability of the cake of solids in the centrifugal field [15].

Cross-flow filtration may be an alternative to centrifugation for small particle sizes and mostly for small flow volumes.

2. Fundamentals

2.1. Centrifugal Sedimentation

General. The sedimentation rate of solid particles or droplets in the gravity field is a function of the particle (or droplet) size, the density difference, and the viscosity of the suspension (or emulsion). In applications in which centrifuges are used, the static sedimentation velocity is between 10^{-9} and 10^{-4} m/s. To accelerate the process, centrifuges apply centrifugal forces of 500 – 30 000 g.

At lower sedimentation rates, special ultracentrifuges with up to 900 000 g are used to separate macromolecules, virus particles, and microbial cell constituents. Sedimentation velocities are expressed as sedimentation coefficients in Svedberg units (S), have the dimension seconds, and are defined as 10^{13} times the settling velocity per unit acceleration (in m/s^2). They range from 4 to 10^6 S [4].

A third class of centrifugal separators are gas centrifuges, today used primarily to separate isotopes (→ Isotopes, Natural). Uranium enrichment is the best known application. The centrifugal force is greater than 150 000 g. The enrichment in a single centrifuge is very small, so a large number of machines have to be arranged in series to obtain a sufficient degree of enrichment.

Settling Velocity. The settling velocity of the particles or droplets is a fundamental property of a suspension or emulsion and determines the flow rate through or size of the centrifuge together with the residence time and settling length. In most centrifugations the settling velocity u_c is so small that Stokes law applies:

$$u_c = \frac{\left(\varrho_p - \varrho_l\right) d_p^2}{18\,\eta} \cdot r\,\omega^2 \text{ [m/s]} \tag{1}$$

where ϱ_p and ϱ_l are the densities of particle and surrounding medium, d_p is the particle diameter, η is the viscosity of the suspension, r is the distance from the center of rotation, and ω is the angular velocity, which is related to the bowl speed n in rpm according to the relation

$$\omega = \frac{2\,\pi\,n}{60} \left[\text{s}^{-1}\right] \tag{2}$$

The acceleration factor Z (the g number) is

$$Z = \frac{r\,\omega^2}{g} \tag{3}$$

where g is the acceleration due to gravity. The settling velocity u_g in the gravity field is

$$u_g = u_c \cdot \frac{g}{r\,\omega^2} \text{ [s/m]} \tag{4}$$

The Equations (1) and (4) are valid under the conditions that

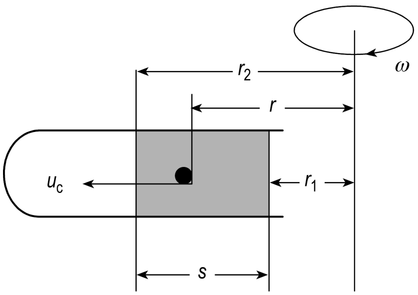

Figure 1. Bottle swing-out centrifuge

1) The flow round the particle is laminar (Reynolds number $Re < 0.4$)
2) The settling of a particle is not influenced by other particles, i.e., the concentration is very low

For particle $Re > 0.4$ alternative equations have been derived [2], [4]. For high particle concentrations, a phenomenon called hindered settling sets in, decreasing the settling rate. Equations for correction factors < 1 for the settling rates u_c and u_g in Equations (1) and (4) are summarized in [8]. At 20 vol % of particles, the settling rates decrease by a factor > 2.

Residence Time and Settling Length. A high g number (Z) is not the only factor that determines the flow rate at acceptable separation efficiency through a centrifuge. The residence time in the centrifugal force field, i.e., the rotor volume, can compensate for a small Z. A small settling length, i.e., the distance a particle must travel to be captured, will also improve the flow rate at acceptable separation efficiency.

The Bottle Centrifuge. The settling velocity of particles that are separated in a centrifuge is often so small that its measurement is not practical without application of centrifugal force. The bottle (swing-out) centrifuge, shown schematically in Figure 1, is often a suitable tool for measurement of the settling velocity.

The following equations describe the particle motion:

$$u_c = \frac{dr}{dt} \tag{5}$$

$$u_c = u_g \cdot \frac{r\,\omega^2}{g} \tag{6}$$

The aim is for all particles to move to a radius $> r_2$. In the worst case, the particle starts at the liquid surface at r_1. Insertion of Equation (6) into Equation (5), rearranging, and integrating gives

$$\int_0^T dt = \frac{g}{u_g\,\omega^2} \int_{r_2}^{r_1} \frac{dr}{r} \tag{7}$$

$$T = \frac{g}{u_g\,\omega^2} \cdot \ln \frac{r_2}{r_1} \tag{8}$$

where T is the spinning time necessary for all particles to leave the volume between levels r_1 and r_2. By rearranging Equation (8) one obtains

$$u_g = \frac{g}{T\,\omega^2} \cdot \ln \frac{r_2}{r_1} \tag{9}$$

It is clear that a small settling velocity necessitates a long spinning time, i.e., residence time, and/or a short sedimentation distance at a high angular velocity. This is also valid for centrifuges with continuous feed and effluent flow. Because of mechanical and hydraulic limitations, these requirements are in conflict with each other and can not be met in industrial centrifuges without compromises. This has led to the development of a broad range of machines in which different requirements have been given priority.

2.2. Centrifugal Sedimentation with Continuous Flow

The Σ Theory. *The Generalized Σ Formula.* The most used quantity to characterize centrifuges, the Σ concept, was presented by AMBLER [33], [34]. It is the calculated equivalent surface area of a static settling tank with the same theoretical performance. In its derivation he considered particles with a critical diameter d_c that were separated to 50 %. Today, however, the most widely used definition of the critical particle is that which is separated to 100 %. This does not influence the formulas for the Σ value, but the value for the feed flow rate Q is halved.

Σ is defined as

$$Q_{\text{theor}} = u_g\,\Sigma \tag{10}$$

where u_g is the Stokes settling velocity (Eq. 1). Σ is given by the general expression

$$\Sigma = \frac{V}{s_e} \cdot \frac{\omega^2 \cdot r_e}{g} \tag{11}$$

where V is the liquid volume in the bowl, s_e is the effective settling distance, and r_e is the effective distance from the center of rotation.

With Equations (1) and (4) the equation for the critical diameter becomes

$$d_c = \left(\frac{18\,\eta\, Q_{\mathrm{theor}}}{\left(\varrho_p - \varrho_f\right)} \cdot \frac{s_e}{V \cdot \omega^2 \cdot r_e} \right)^{\frac{1}{2}}$$

$$= \left(\frac{18\,\eta\, Q_{\mathrm{theor}}}{\Sigma \left(\varrho_p - \varrho_f\right) g} \right)^{\frac{1}{2}} \qquad (12)$$

The derivation above is based on the assumption that

- Viscous drag determines the particle movement
- The flow in disk bowls between the disks is laminar and symmetrical
- The liquid rotates at the same speed as the bowl
- The particle concentration is low (no hindered settling)
- The particle at all times moves with its final settling velocity u_c
- The settling velocity u_c is proportional to the g force.

AMBLER uses the critical particle in his formula; therefore, his analysis is a special case of Svarovsky's grade efficiency function [6], in which for each particle size the ratio of sedimented mass to the mass in the feed material is determined and integrated over all particle sizes to give a more realistic measure of the separation efficiency. The terminology, definitions and theory of particle classification is dealt with comprehensively in [8].

Σ for Various Centrifuges. The flow system in between two conical disks in a disk-bowl centrifuge is shown in Figure 24. The particle has a radial velocity component u_c because of the centrifugal force, which increases with increasing r. It has also a drag dependent velocity component u_d which can have any size or direction depending on the flow situation round the particle. In the Σ theory it is assumed that it depends on uniform radial plug flow parallel to the disks. The derivation of the Σ for various types of centrifuge is found in [34], and the results are summarized in the following:

Disk-bowl [33], [34]

$$\frac{\pi \cdot \omega^2}{g} \cdot \frac{2}{3} \cdot N \cdot \left(r_2^3 - r_1^3\right) \cdot \cot\alpha$$

r_2 = max. radius of disk,
r_1 = min. radius of disk,
N = number of disk,
α = half-cone angle of disk

Decanter [6], [34]

$$\frac{\pi \cdot \omega^2}{g} \left[L_1 \left(\frac{3}{2} \cdot r_2^2 + \frac{1}{2} \cdot r_1^2 \right) + L_2 \left(\frac{r_2^2 + 3\,r_2\,r_1 + 4\,r_1^2}{4} \right) \right]$$

L_1 = length of cylindrical part,
L_2 = length of conical part,
r_1 = inner radius of liquid,
r_2 = inner radius of bowl,

Tubular-bowl, chamber-bowl [34]

$$\frac{\pi \cdot \omega^2}{g} L \frac{r_2^2 - r_1^2}{\ln\left(\dfrac{2\,r_2^2}{r_2^2 + r_1^2} \right)}$$

r_1 = inner radius of liquid,
r_2 = inner radius of bowl,
L = inner length of bowl,

Multichamber-bowl [2]

$$\frac{\pi \cdot \omega^2}{g} \frac{L}{3} \sum_{i=0}^{i=n} \frac{r_{2i+1}^3 - r_{2i+2}^3}{r_{2i+1} - r_{2i+2}}$$

indices with even numbers: inner radius of chamber,
indices with odd numbers: outer radius of chamber,
$n+1$ = number of chambers,
L = height of chambers

Separation Efficiency. The assumptions in the Σ theory are not fulfilled in reality. Therefore, Equation (10) should include an efficiency factor μ, so that in practice the flow rate is lowered for the required separation performance. Estimated efficiency factors are listed in the following [5]:

Disk-bowl machines	45 – 73 %
Decanter centrifuges	54 – 67 %
Tubular bowls	90 – 98 %

By careful measurement of the density difference and particle dimensions in one case an efficiency factor of max. 16 % was found to fit the experimental data in a pilot-scale disk bowl machine [35]. However, this probably also included effects of hindered settling.

In the absence of hindered settling, the equation describing the relationship between the feed flow rate, the Σ value, and the settling rate, is thus

$$Q_{act} = \mu u_g \Sigma \tag{13}$$

The KQ Formula for Disk-Bowl Centrifuges. A semi-empirical equation for the "separation area" [36] is based on the finding that separation data fit the relation

$$\frac{u_c}{u_g} = \left(\frac{\omega^2 r}{g}\right)^k \tag{14}$$

with $k = 0.75$ better than the classical analysis (Eq. 4) with $k = 1$. This is a consequence of the increasing shear forces in the Ekman layers (see Section 5.2). For practical purposes the following relation is used:

$$\mathrm{KQ} = 280 \cdot \left(\frac{n}{1000}\right)^{1.5} \cdot N \cot\alpha \left(r_{out}^{2.75} - r_{in}^{2.75}\right) \tag{15}$$

where n is the bowl speed in rpm and r_{out} and r_{in} are the radii of the disk in centimeters. This form of the KQ formula is dimensionally incorrect, but it is mathematically possible to make it correct and to express the KQ value as an entity with the dimension of area. In practice the difference between KQ and Σ for scale-up purposes is negligible, compared to other inaccuracies.

Hindered Settling. One of the few experimental investigations into hindered settling of cells [35] shows that complex phenomena occur at high concentrations. A model with several experimental constants was developed. It showed that settling rates decrease rapidly with increasing solids contents, by a factor of about 2 at 20 vol % and by 15 – 20 at 50 vol %. The classical model used in [36] gave similar results. The decreased settling velocity can be attributed to increased viscosity of the suspension at high solids concentrations [8]. In the tests in [35] it was also found that cells may settle as flocs at higher concentration. Hindered settling is one further example of the difficulty to calculate theoretically the performance of centrifuges.

Gas Centrifuges. The theory for gas centrifugation is too complicated to be included here. The performance of a gas centrifuge is, besides peripheral velocity and rotor length, very much depending on the flow pattern in the rotor and the location of entry and exit points of the streams. The reader is referred to [4] and [12].

3. Centrifugal Separators with Continuous Feed

3.1. General

The different types of continuous centrifuge were developed from different principles of solids handling. The first centrifuges were developed for separating a process fluid (e.g., milk, fermented yeast broth) into two fluid streams. Heavy solids that could not flow out were collected at the periphery and removed by periodic manual cleaning. These were followed by centrifuges with peripheral nozzles that discharge solids as a slurry. In many processes these replaced machines needing expensive manual cleaning. For processes in which the solids must be collected in a more concentrated, nonfluid, form, machines with internal conveyors (decanter centrifuges) were developed. Intermittently discharging machines were developed to fill the gap between the three above-mentioned types and are now the most common continuous centrifuges. A number of discharging mechanisms have been developed over the years. A bottle centrifuge with continuous feed and separated effluent (centrate) is available.

The most comprehensive description of centrifuges can be found in [2]. Brochures and technical publications from centrifuge manufacturers provide the most modern information about machine types, sizes, and capacities.

3.2. Disk-Bowl Machines

General. In disk-bowl machines the sedimentation path is made short by introduction of conical discs, separated by spacers. The distance is between 0.4 and 3 mm, depending on the viscosity and solids content. The half-cone angle is in the range 35 – 45° and is smaller for solids with a small angle of repose.

The spacers between the disks have different designs, depending on the separation duty. The most common spacers are long ribs placed radially along the cone; these help the liquid retain its rotation velocity. The majority of disk-bowl machines are used for separating two liquid phases. In many cases, droplets of the dispersed phase in the feed liquid coalesce at an interface inside the bowl, and the newly formed liquid leaves the bowl continuously through a second outlet, separated from the original continuous phase. The position of the interface in the bowl is important and can be

Table 1. Characteristics of continous-feed centrifuges [a]

Type	Mode of solids discharge	Feed flow rate, L/h	Feed solids content, vol %	Solids flow rate, L/h	Σ value, m^2	Max. Z developed	Consistency of solids
Disk solid bowl	manual	20 – 100 000	< 1	0	1000 – 300 000	10 000	firm paste
Solids-ejecting, radial	intermittent	20 – 100 000	< 25	< 3000	1000 – 170 000	14 500	thick flowing slurry
Solids-ejecting, axial	intermittent	1000 – 150 000	< 15	< 1000	110 000 – 220 000	15 000	thick flowing slurry
Nozzle, pressurized-discharge [b]	continuous	1000 – 180 000	4 – 30	> 150 < 40 000	69 000 – 180 000	15 000	thick flowing slurry
Peripheral nozzle	continuous	300 – 310 000	2 – 30	> 3000 < 140 000	35 000 – 180 000	11 000	thin slurry
Decanter centrifuge	continous with scroll	300 – 200 000	5 – 50	< 50 000	400 – 25 000	10 000	thick slurry to semisolid
Tubular bowl	manual	20 – 7000	< 1	0	1400 – 4500	31 000	firm paste
Multichamber bowl	manual	100 – 20 000	< 5	0		9000	firm paste
Centritech machines [c]	intermittent	5 – 100	< 1	< 15	–	100	very thin slurry
Inverted solid bowl	intermittent with scraper	10 – 6000	1 – 30	< 1000	800 – 20 000	20 000	firm paste

[a] Low-speed imperforate tubular and basket machines not included. [b] Only for slurry of single cells (see text). [c] Only for mammalian cells.

controlled by, for example, adjusting the radius of one of the weirs, usually that for the heavy phase. Its radius r_h (see Fig. 2) is given by

$$r_h = \sqrt{r_i^2\left(1 - \frac{\varrho_l}{\varrho_h}\right) + r_l^2 \cdot \frac{\varrho_l}{\varrho_h}} \tag{16}$$

where r_i is the radius of the interface, r_l the outlet radius of the light liquid phase, and ϱ_l and ϱ_h are the densities of the light and heavy liquid phases. The equation is derived from a pressure balance but is a simplification because it neglects pressure drops and weir heights. It is adequate for a first approximation. It is also applicable to chamber bowls, tube centrifuges and decanters (Sections 3.3, 3.4 and 3.5).

If the objective of the separation is to obtain a very clean light phase, the interface should be placed at a large radius to avoid heavy droplets' contaminating the light phase. For obtaining a clean heavy phase, the interface should have a small radius.

Several important applications of liquid – liquid separations in disk bowls do not involve coalescence at an interface. The most common examples are separation of cream from milk and alkali refining of vegetable oil. The split between the outlet streams is then controlled by counterpressures that can be applied in several ways.

Solid-Bowl Centrifuge. Figure 2 shows a solid-bowl centrifuge requiring manual cleaning. Heavy solids are collected at the periphery; the machine must be dismantled and the bowl opened to remove the solids, often by lifting up a collecting basket, fitted inside the bowl. The feed enters the bowl in the center through a still-standing pipe. The version of bowl shown is suitable for liquid – liquid – solid separation and therefore has two liquid outlets. The light liquid flows through the disk stack and out of the bowl through a still-standing centripetal pump known as a paring disk. The heavy liquid passes over a disk near the bowl hood and is removed through another paring disk. The paring disk converts the kinetic energy of the rotating liquid, and discharges it under pressure.

This type of bowl has diameters between 140 and 750 mm. The solids space has a volume of up to 34 L. Further data are listed in Table 1. The principles illustrated in Figure 2 are also applicable to other bowl types. Instead of paring disk(s),