



# Chapter 1

## Measurement and Units of Measurement

At the core of any science is measurement. Being able to measure volumes, pressures, masses, and temperatures as well as the ability to count atoms and molecules allows chemists to understand nature more precisely. Modern science uses the International System of Units (SI) that was adopted worldwide in 1960. The metric system of measurement, which is consistent with the International System, is widely used in chemistry and is the principal system used in this book.

Chemists often have to work with numbers that are very, very small or very, very large. It is more convenient to express numbers of this kind in scientific notation, so that is the first topic to look at in this chapter.

### Writing Numbers in Scientific Notation

It is likely that you have already seen numbers expressed in scientific notation on your calculator. With only 8 or 9 spaces to display numbers, calculators must resort to scientific notation to show very small or very large numbers. In scientific notation a number is expressed in this form

$$a \times 10^p$$

where “*a*” is a number between 1 and 10 (often a decimal number) and “*p*” is a positive or negative whole number written as an exponent on 10, often called the *power* of 10. The average distance from the earth to the sun is 93,000,000 miles, a very large number. In scientific notation this would be  $9.3 \times 10^7$  miles. The power of 7 equals the number of places a decimal point would be moved from the right end of 93,000,000 to the left to get an “*a*” value between 1 and 10 (9.3). The  $10^7$  term equals 10,000,000 ( $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ ) and when multiplied by 9.3 would restate the original number in conventional form.

$$9.3 \times 10^7 \text{ miles} = 9.3 \times 10,000,000 \text{ miles} = 93,000,000 \text{ miles}$$

Likewise, the year 1492 would be written  $1.492 \times 10^3$  in scientific notation.

Small numbers, those less than 1, are handled in a similar way, except the decimal point has to be moved to the *right* to get an “*a*” value between 1 and 10, and the “*p*” exponent is a *negative number*. For example, an atom of gold has a diameter of 0.000000342 meter. The decimal must be moved 7 places to the right to get 3.42, and the number is stated as  $3.42 \times 10^{-7}$  meter in scientific notation. The  $10^{-7}$  term equals 1 divided by 10,000,000.

$$3.42 \times 10^{-7} \text{ meter} = 3.42 \times 1/10,000,000 \text{ meter} = 0.000000342 \text{ meter}$$

Similarly, the number 0.000045 would be written  $4.5 \times 10^{-5}$ .

One last thing: If you are given a number between 1 and 10 and need to write it in scientific notation, the power on 10 would be zero. In scientific notation, the number 8 would be written  $8 \times 10^0$ . In mathematics,  $10^0$  equals 1.

### Example Problems

1. Express these numbers in scientific notation.

(a) 22,500,000

**Answer:**  $2.25 \times 10^7$

This is a large number, so the decimal is moved 7 places to the left to form 2.25, and 7 is written as a positive exponent on 10.

(b) 0.0006

**Answer:**  $6 \times 10^{-4}$

This is a small number, so the decimal must be moved 4 places to the right to form 6, and 4 is written as a negative exponent on 10.

(c) 602,200,000,000,000,000,000

**Answer:**  $6.022 \times 10^{23}$

This is a very large number, so the decimal is moved 23 places to the left to form 6.022, and 23 is written as a positive exponent on 10.

2. Express these numbers in conventional form.

(a)  $6.35 \times 10^5$

**Answer:** 635,000

The power of 10 is 5, a positive number, so the decimal point is moved 5 places to the right.

(b)  $2.4 \times 10^{-3}$

**Answer:** 0.0024

The power of 10 is  $-3$ , a negative number, so the decimal point is moved 3 places to the left.

### Work Problems

1. Express these numbers in scientific notation.

(a) 1945   (b) 0.00000255   (c) 388000000000   (d) 0.023

2. Express these numbers in conventional form: (a)  $7.55 \times 10^{-4}$    (b)  $8.80 \times 10^2$

## Worked Solutions

- (a)  $1.945 \times 10^3$ . Because this is a large number, the decimal is moved 3 places to the left to form 1.945. The exponent on 10 is 3.

(b)  $2.55 \times 10^{-6}$ . This is a small number, so the decimal is moved 6 places to the right to form 2.55. The exponent on 10 is  $-6$ .

(c)  $3.88 \times 10^{11}$ . This is a large number, so the decimal is moved 11 places to the left to form 3.88. The exponent on 10 is 11.

(d)  $2.3 \times 10^{-2}$ . Because this is a small number, the decimal is moved 2 places to the right to form 2.3. The exponent on 10 is  $-2$ .
- (a) **0.000755**. The power of 10 is  $-4$ , a negative number, so the decimal point is moved 4 places to the left.

(b) **880**. The power of 10 is 2, a positive number, so the decimal point is moved 2 places to the right.

## Significant Figures and Rounding Off Numbers

It is not possible to measure anything exactly; there will always be some amount of uncertainty. In many cases, the tool used to do the measurement causes the uncertainty. An inexpensive laboratory balance, for example, measures the mass of a gold ring to be 2.83 grams, while a more expensive analytical balance measures the mass to a greater accuracy, 2.8275 grams. The greater accuracy of the analytical balance is reflected in the larger number of digits in the numerical value of the mass. In either number, 2.83 or 2.8275, the right-most digit is the only digit that is not known with certainty. The mass of the ring is *closer* to 2.83 grams than to 2.82 or 2.84 grams on the first balance ( $2.83 \pm 0.01$ ), and *closer* to 2.8275 grams than to 2.8274 or 2.8276 grams on the second ( $2.8275 \pm 0.0001$ ). In both cases, all the digits are certain except the last one. The number of digits shown in a measured value (the certain digits + the one uncertain digit) indicates the accuracy of that value. These digits are referred to as significant digits or, more commonly, **significant figures (sig. figs.)**.

## Counting Significant Figures

You need to know how to count the number of significant figures in a number, because they affect the way answers are stated in calculations. Zeros can be a problem. A zero may or may not be significant depending on how it is used. To handle this “zero problem,” follow this set of six rules:

- Rule 1.** All nonzero digits (1, 2, 3, 4, 5, 6, 7, 8, and 9) are always significant and must be counted.
- Rule 2.** A zero standing alone to the left of a decimal point is not significant. For example, in 0.63 and 0.0055, the 0 to the left of the decimal only helps you see the decimal point. It has no other use.
- Rule 3.** For a number less than 1, any zeros between the decimal point and the first nonzero digit are not significant. These zeros are simply placing the decimal point. The zeros in bold type in **0.00457** and **0.0000864** are not significant. Both numbers have three significant figures: 457 in the first number and 864 in the second.

**Rule 4.** A zero between two nonzero digits is significant. In 2.0056 and 0.0040558, both numbers have 5 significant figures. Because the second number is less than 1, only the 4, 0, 5, 5, and 8 are significant.

**Rule 5.** If the number has a decimal point, any zeros at the end of the number are significant. Both of these numbers have 4 significant figures: 4.500 and 0.01380.

**Rule 6.** If the number does not have a decimal point, like 1,500, the zeros at the end of the number may or may not be significant. If they are significant, place a decimal after the last zero, as in 1,500., or the number could be written in scientific notation,  $1.500 \times 10^3$ . Otherwise, 1,500 means the value is  $1,500 \pm 100$  with 2 significant figures, the 1 and 5. For any number written in scientific notation, all digits in the first part of the number are significant. Both 1,500. and  $1.500 \times 10^3$  show 4 significant figures.

### Example Problems

Count the number of significant figures in each of these numbers.

1. 2.054

**Answer:** 4

The zero between two nonzero digits is counted (Rule 4) along with the 2, 5, and 4.

2. 0.00399

**Answer:** 3

The zeros between the decimal and the 3 are not counted (Rule 3), so only the 3, 9, and 9 are significant.

3. 0.99800

**Answer:** 5

The two zeros following the 8 are counted (Rule 5), so five digits are significant.

4.  $6.014 \times 10^{-3}$

**Answer:** 4

The zero is counted (Rule 4), so all four digits are significant.

5. 6,500

**Answer:** 2

At most, 2 significant figures. 6,500 without a decimal indicates  $6,500 \pm 100$  (Rule 6).

## Work Problems

Count the number of significant figures in these numbers.

- (a) 93.082   (b) 0.00059   (c) 4.520   (d)  $1.0 \times 10^6$    (e) 120,000.

## Worked Solutions

- (a) **5.** All digits are significant; the zero is counted (Rule 4).
- (b) **2.** The 5 and 9 are significant; the zeros place the decimal (Rule 3).
- (c) **4.** All digits are significant; the zero is counted (Rule 5).
- (d) **2.** All digits in the non-exponential part of a number written in scientific notation are significant (Rule 1 and Rule 5).
- (e) **6.** The decimal tells us all digits are significant (Rule 6).

## Rounding Off Numbers

In calculations, we often obtain answers that have more digits than can be justified considering significant figures. This is a major problem when using calculators. Removing the digits that are not significant from the answer is called **rounding off**. Here are three rules to guide you:

**Rule 1.** If the next digit after those you want to retain is 4 or less, drop that digit and all that follow and keep the digits that remain.

Rounding to three figures:  $3.253 \rightarrow 3.25 \mid 3 \rightarrow 3.25$

**Rule 2.** If the next digit after those you want to retain is 5 or greater, drop that digit and all that follow, and then increase the last retained digit by 1. This is sometimes called rounding up.

Rounding to three figures:  $6.3466 \rightarrow 6.34 \mid 66 \rightarrow 6.35$

**Rule 3.** If the number to be rounded is less than 1 with zeros between the decimal point and the first nonzero digit (like 0.0004638), consider only the numerals that follow the zeros when counting digits.

Rounding to three figures:  $0.0004638 \rightarrow 0.000463 \mid 8 \rightarrow 0.000464$

## Example Problems

1. Round off 45,317 to 2 digits and express the answer in scientific notation.

**Answer:**  $4.5 \times 10^4$

Separating the first 2 digits ( $45 \mid 317$ ) shows the next digit to be 3. It and the following digits are dropped (Rule 1), leaving 45,000.

2. Round off 368 to 2 digits and express the answer in scientific notation.

**Answer:**  $3.7 \times 10^2$

The digit following the first 2 digits is greater than 5 (36 | 8). It is dropped, and 6 is increased to 7.

3. Round off 0.00941 to 1 digit and express the answer in scientific notation.

**Answer:**  $9 \times 10^{-3}$

Consider only the digits following the zeros. Separating 1 digit from the rest (0.009 | 41) shows the next digit, 4, and all that follow can be dropped.

### Work Problems

Round off each number to the indicated number of significant figures, shown in parentheses, and express the answer in scientific notation.

- (a) 55,583 (4)   (b) 38,953 (2)   (c) 0.007665 (2)

### Worked Solutions

(a)  $5.558 \times 10^4$

55,58 | 3; Rule 1 applies; drop the 3, keep the rest.

(b)  $3.9 \times 10^4$

38 | 953; Rule 2 applies; 38 is rounded up to 39 since the next digit is greater than 4.

(c)  $7.7 \times 10^{-3}$

0.0076 | 65; Both Rules 2 and 3 apply, and 0.0076 is rounded up to 0.0077.

## Significant Figures in Calculations

Keeping track of the number of significant figures in calculations depends on the kind of calculation you are doing.

**Multiplication and division:** The product or quotient can have no more significant figures than the number with the smallest number of significant figures used in the calculation.

**Addition and subtraction:** The sum or difference can have no more places after the decimal than there are in the number with the smallest number of digits after the decimal.

In addition and subtraction, those places after the decimal that are of unknown value (?) negate the numerals in those same places in the other numbers used in the calculation.

Example:  $3.45762$  ← five places after the decimal are known  
 $+ 4.32???$  ← only 2 places after the decimal are known

Here is how these two numbers are added.  $3.45762$  is rounded off to two places after the decimal:  $3.45762 \rightarrow 3.45 \mid 762 \rightarrow 3.46$

It is then added to the other number.

$$\begin{array}{r} 3.46 \\ + 4.32 \\ \hline 7.78 \end{array}$$

### Example Problems

1.  $1,949 \div 6.33 =$

**Answer:** 308

There are 4 significant figures in 1,949 and 3 in 6.33, so the answer can have no more than 3 significant figures.

2.  $34.238442 + 9.35 =$

**Answer:** 43.59

The answer can only have 2 digits after the decimal.  $34.238442$  is rounded up to 34.24 and added to 9.35.

3.  $19.42 - (8.5 \times 10^{-3}) =$

**Answer:** 19.41

Start by writing  $8.5 \times 10^{-3}$  in conventional form, 0.0085. Only 2 digits after the decimal are allowed. 0.0085 is rounded up to 0.01 and then subtracted from 19.42.

### Work Problems

1.  $64.33 \times 2.1416 =$

2.  $64.362 - 4.2 =$

3.  $19 - (3.3 \times 10^{-2}) =$

## Worked Solutions

### 1. 138.0

The 4 significant figures in 64.33 limit the answer to 4 figures.

### 2. 60.2

Answer is limited to 1 digit after the decimal. 64.362 is rounded up to 64.4, and 4.2 is subtracted from it.

### 3. 19

$3.3 \times 10^{-2} = 0.033$ . There are no digits after the decimal in 19, so subtracting 0.033 has no effect on 19.

## Calculators and Significant Figures

You use a calculator to do your math chores, but calculators know nothing about significant figures. Calculators blindly grind out all the digits they can show, so it is up to you to get rid of the unnecessary digits and round off the number. If you divide 1.0 by 3.0 (both have 2 significant figures), the calculator gives 0.333333333. You need to round off this answer to 2 significant figures: 0.33 | ~~3333333~~ becomes 0.33, the correct answer with 2 significant figures.

## Example Problems

Use your calculator to do each calculation and state the answer in scientific notation with the correct number of significant figures.

1.  $600.3 \div 0.22 =$

**Answer:**  $2.7 \times 10^3$

Use 2 significant figures. The calculator display, 27 | ~~27.272727~~, becomes 2,700 when rounded to 2 significant figures. The decimal is moved 3 places left to write the answer in scientific notation.

2.  $3.1416 \times 6.52 =$

**Answer:**  $2.05 \times 10^1$

Use 3 significant figures.  $2.04 | 83232 \times 10^1$  is rounded up to the correct answer.

## Work Problems

1.  $0.443 \div 9 =$

2.  $33.0 \div 10.466 =$

### Worked Solutions

Use your calculator to do each calculation and state the answer in scientific notation with the correct number of significant figures.

1.  $5 \times 10^{-2}$

Use 1 significant figure.  $0.04 | 92222222$  is rounded up to 0.05, and the decimal moved 2 places right.

2.  $3.15 \times 10^0$

Use 3 significant figures.  $3.15 | 306707$  could be stated correctly as 3.15, forgoing scientific notation.

## The Metric System

The metric system is a system of measurement using units based on the decimal system. Today, in English, it is formally called the International System, abbreviated SI from the original French, *Système International*. The base units of the modern metric system used in general chemistry are given in the following table. From these, you can derive all other units of measure.

The Base Units of the Metric System		
Quantity	Base Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
temperature	Kelvin	K
amount of substance	mole	mol

In this chapter, we are concerned only with the base units for length, mass, and temperature and those derived from them.

### Length

The base unit of length is the **meter**, a length a little longer than the English yard; 39.37 inches to be exact. It isn't a convenient length for describing the length of things that are very small, like molecules, or for larger measures, like the distances between cities. To increase the usefulness of the meter, its length can be subdivided or multiplied by the use of the metric prefixes, which indicate different powers of 10. The most common metric prefixes, their abbreviations, and the mathematical meaning of each are listed in the following table.

<b>The Metric Prefixes</b>			
<b>Prefix</b>	<b>Abbreviation</b>	<b>Numerical Value</b>	<b>Equivalent Value in Power of Ten</b>
giga	G	1,000,000,000	$10^9$
mega	M	1,000,000	$10^6$
kilo	k	1,000	$10^3$
base unit	—	1	$10^0$
deci	d	0.1	$10^{-1}$
centi	c	0.01	$10^{-2}$
milli	m	0.001	$10^{-3}$
micro	$\mu^*$	0.000001	$10^{-6}$
nano	n	0.000000001	$10^{-9}$
pico	p	0.000000000001	$10^{-12}$

\* $\mu$  is the Greek letter mu.

The prefixes are used to modify the base unit, such as the meter (m), to make larger or smaller units that are more appropriate for a particular use. The most often used units of length in the metric system are shown in the following table.

<b>The Metric Units of Length</b>			
<b>Unit of Length</b>	<b>Abbreviation</b>	<b>Meter Equivalent</b>	<b>Power Equivalent</b>
kilometer	km	1,000 m	$1 \times 10^3$ m
meter (base)	m	1 m	1 m
decimeter	dm	0.1 m	$1 \times 10^{-1}$ m
centimeter	cm	0.01 m	$1 \times 10^{-2}$ m
millimeter	mm	0.001 m	$1 \times 10^{-3}$ m
micrometer	$\mu\text{m}$	0.000001 m	$1 \times 10^{-6}$ m
nanometer	nm	0.000000001 m	$1 \times 10^{-9}$ m

All the relationships between the unit and the meter in the preceding table are exact by definition; for example, 1 kilometer is *exactly* 1,000 meters *by definition*.

Relating each to 1 meter:

$$1 \text{ m} = 0.001 \text{ km} = 1 \times 10^{-3} \text{ km}$$

$$1 \text{ m} = 10 \text{ dm} = 1 \times 10^1 \text{ dm}$$

$$1 \text{ m} = 100 \text{ cm} = 1 \times 10^2 \text{ cm}$$

$$1 \text{ m} = 1,000 \text{ mm} = 1 \times 10^3 \text{ mm}$$

$$1 \text{ m} = 1,000,000 \mu\text{m} = 1 \times 10^6 \mu\text{m}$$

Because the inch–centimeter relationship is defined to be exact, all English–metric unit comparisons of length are also exact when used with the number of digits shown below. This exactness is only true for length comparisons.

$$1 \text{ inch (in)} = 2.54 \text{ cm (exactly)}$$

$$1 \text{ mile (mi)} = 1.609 \text{ km (exactly)}$$

$$1 \text{ m} = 39.37 \text{ in (exactly)}$$

## Mass

The base unit of mass is the **kilogram** (kg), which, because of the *kilo* prefix (kilo = 1,000), indicates that 1 kilogram is 1,000 grams. Although the base is the kilogram, the more commonly used unit is the gram. The units of mass in the metric system are given in the following table.

The Metric Units of Mass			
<i>Unit of Mass</i>	<i>Abbreviation</i>	<i>In Terms of Grams</i>	<i>Power Equivalent</i>
kilogram (base)	kg	1,000 g	$1 \times 10^3 \text{ g}$
gram	g	1 g	1 g
decigram	dg	0.1 g	$1 \times 10^{-1} \text{ g}$
centigram	cg	0.01 g	$1 \times 10^{-2} \text{ g}$
milligram	mg	0.001 g	$1 \times 10^{-3} \text{ g}$
microgram	$\mu\text{g}$	0.000001 g	$1 \times 10^{-6} \text{ g}$

Relating each to 1 gram:

$$1 \text{ g} = 0.001 \text{ kg}$$

$$1 \text{ g} = 10 \text{ dg}$$

$$1 \text{ g} = 100 \text{ cg}$$

$$1 \text{ g} = 1,000 \text{ mg}$$

$$1 \text{ g} = 1,000,000 \mu\text{g}$$

Comparing English and metric units of mass:

$$1 \text{ kg} = 2.20 \text{ pounds (lbs) (3 sig. figs.)}$$

$$1 \text{ lb} = 454 \text{ g (3 sig. figs.)}$$

## Volume

There is no base unit for volume in the SI system since volume is derived from the base unit of length (volume = length  $\times$  length  $\times$  length). The derived unit of volume in the SI system is the cubic meter,  $\text{m}^3$ , a volume that is too large for most laboratory work, so a smaller volume has been adopted, the liter (L); note the uppercase “L.” The liter is a little larger than the English quart and is exactly 1 cubic decimeter,  $\text{dm}^3$ . The most often used units of volume are given in the following table.

### The Metric Units of Volume

<i>Unit of Volume</i>	<i>Abbreviation</i>	<i>In Terms of the Liter</i>	<i>Power Equivalent</i>
kiloliter	kL	1,000 L	$1 \times 10^3$ L
liter	L	1 L	1 L
deciliter	dL	0.1 L	$1 \times 10^{-1}$ L
centiliter	cL	0.01 L	$1 \times 10^{-2}$ L
milliliter	mL	0.001 L	$1 \times 10^{-3}$ L
microliter	$\mu$ L	0.000001 L	$1 \times 10^{-6}$ L

Relating each to 1 liter:

$$\begin{aligned}
 1 \text{ L} &= 0.001 \text{ kL} \\
 1 \text{ L} &= 10 \text{ dL} \\
 1 \text{ L} &= 100 \text{ cL} \\
 1 \text{ L} &= 1,000 \text{ mL} \\
 1 \text{ L} &= 1,000,000 \text{ } \mu\text{L}
 \end{aligned}$$

The most common unit of volume used in laboratory work is the milliliter (mL), which is the volume of exactly 1 cubic centimeter,  $\text{cm}^3$ , or cc. The units of cc,  $\text{cm}^3$ , and mL are interchangeable.

$$1 \text{ mL} = 1 \text{ cm}^3 = 1 \text{ cc (exactly)}$$

Comparing English and metric units of volume:

$$\begin{aligned}
 1 \text{ L} &= 1.06 \text{ quart (qt) (3 sig. figs.)} \\
 1 \text{ qt} &= 946 \text{ mL (3 sig. figs.)}
 \end{aligned}$$

## Metric-Metric Conversions

It is often necessary to convert one metric unit to another. Suppose you needed to convert 4.5 cm into the equivalent length in millimeters. A conversion factor is used to do this. Conversion factor statements always have the form:

$$\begin{aligned}
 \text{quantity being SOUGHT} &= \text{quantity KNOWN} \times (\text{conversion factor}) \\
 \text{quantity KNOWN} &= 4.5 \text{ cm} \\
 \text{quantity being SOUGHT} &= \text{length in mm} \\
 \text{length in mm} &= \text{length in cm} \times (\text{conversion factor})
 \end{aligned}$$

The conversion factor is a fraction that relates centimeters to millimeters. The factor needed to convert centimeters to millimeters can be developed like this:

$$100 \text{ cm} = 1 \text{ m} = 1,000 \text{ mm}$$

Since both equal 1 meter, they equal each other:  $100 \text{ cm} = 1,000 \text{ mm}$

Dividing both sides by 100 gives:  $1 \text{ cm} = 10 \text{ mm}$

Two conversion factors (fractions) can be developed from this equality:  $1 \text{ cm} = 10 \text{ mm}$

$$\frac{1 \text{ cm}}{10 \text{ mm}} \text{ and } \frac{10 \text{ mm}}{1 \text{ cm}}$$

The conversion factor chosen to change cm to mm is the one that will allow cm to be canceled (divided out) and replaced with mm. Units can be canceled in fractions just as numbers can:

$$\text{length in mm} = 4.5 \text{ cm} \left( \frac{10 \text{ mm}}{1 \text{ cm}} \right) = 45 \text{ mm}$$

Conversion factors can also be linked together to make a series of unit changes. Converting 25 kg to the equivalent mass in centigrams can be done using two conversion factors.

$$25 \text{ kg} = ? \text{ cg}$$

The needed relationships between kg, g, and cg, are found in the table of metric mass units.

$$\begin{aligned} \text{mass in cg} &= 25 \text{ kg} \left( \frac{1,000 \text{ g}}{1 \text{ kg}} \right) \left( \frac{100 \text{ cg}}{1 \text{ g}} \right) = 2,500,000 \text{ cg} = 2.5 \times 10^6 \text{ cg} \\ 25 \text{ kg} &= 2.5 \times 10^6 \text{ cg} \text{ (2 sig. figs.)} \end{aligned}$$

The first conversion factor changes kg to g, and the second changes g to cg. Notice that two significant figures in 25 kg dictates 2 significant figures in the final answer. Remember, the metric-to-metric conversion factors are exact and don't alter the usual rules of significant figures.

### Example Problems

1. 9.35 mm = ? cm

**Answer:** 0.935 cm

Earlier it was shown that 1 cm = 10 mm. The answer is stated to 3 significant figures.

$$\text{length in cm} = 9.35 \text{ mm} \left( \frac{1 \text{ cm}}{10 \text{ mm}} \right) = 0.935 \text{ cm}$$

2. 1.50 km = ? cm

**Answer:**  $1.50 \times 10^5$  cm

Two conversion factors are used here, the first converting km to m, the second, m to cm. The answer is stated to 3 significant figures.

$$\text{length in cm} = 1.50 \text{ km} \left( \frac{1,000 \text{ m}}{1 \text{ km}} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) = 150,000 \text{ cm} = 1.50 \times 10^5 \text{ cm}$$

3. 0.0045 dg = ? mg

**Answer:** 0.45 mg

The first factor converts dg to g; the second converts g to mg. The answer is stated to 2 significant figures.

$$\text{mass in mg} = 0.0045 \text{ dg} \left( \frac{0.1 \text{ g}}{1 \text{ dg}} \right) \left( \frac{1,000 \text{ mg}}{1 \text{ g}} \right) = 0.45 \text{ mg}$$

## Work Problems

Make the required conversions and state the answers in the proper number of significant figures.

- 0.025 g = \_\_\_\_\_ kg
400. m = \_\_\_\_\_ cm
- 12  $\mu\text{L}$  = \_\_\_\_\_ mL

## Worked Solutions

- $2.5 \times 10^{-5}$  kg**

The answer requires 2 significant figures.

$$\text{mass in kg} = 0.025 \text{ g} \left( \frac{1 \text{ kg}}{1,000 \text{ g}} \right) = 0.000025 \text{ kg} = 2.5 \times 10^{-5} \text{ kg}$$

- $4.00 \times 10^4$  cm**

The answer requires 3 significant figures.

$$\text{length in cm} = 400. \text{ m} \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) = 40,000 \text{ cm} = 4.00 \times 10^4 \text{ cm}$$

- 0.012 mL**

The answer requires 2 significant figures.

$$\text{volume in mL} = 0.012 \text{ } \mu\text{L} \left( \frac{1 \text{ L}}{1,000,000 \text{ } \mu\text{L}} \right) \left( \frac{1,000 \text{ mL}}{1 \text{ L}} \right) = 1.2 \times 10^{-5} \text{ mL}$$

## English-Metric Conversions—More Conversion Factors

Summarizing the English-metric conversions presented up to now:

$$1 \text{ in} = 2.54 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in}$$

$$1 \text{ mi} = 1.609 \text{ km}$$

$$1 \text{ L} = 1.06 \text{ qt}$$

$$1 \text{ qt} = 946 \text{ mL}$$

$$1 \text{ lb} = 454 \text{ g}$$

$$1 \text{ kg} = 2.20 \text{ lb}$$

Conversions between the metric and English systems are done exactly the same way as converting only metric terms, but with one difference. The English-metric conversion factors, with the exception of length, are *not exact* and can affect significant figures. You will need to pay attention to this difference. Depending how pounds to grams or quarts to liters are related, there can be 3 or 4 significant figures:

$$1 \text{ lb} = 454 \text{ g} \text{ or } 1 \text{ lb} = 453.6 \text{ g}$$

$$1 \text{ qt} = 0.946 \text{ L} \text{ or } 1 \text{ qt} = 0.9464 \text{ L}$$

Unlike mass and volume, the inch-centimeter relationship is exact:

$$1 \text{ inch} = 2.54 \text{ cm (exactly)}$$

Within the English system of units, relationships between length, mass, and volume are exact:

$$1 \text{ mi} = 5,280 \text{ ft}$$

$$1 \text{ lb} = 16 \text{ ounces (oz)}$$

$$1 \text{ gallon (gal)} = 4 \text{ qts}$$

$$1 \text{ ft} = 12 \text{ in}$$

$$1 \text{ ton}^* (\text{T}) = 2,000 \text{ lbs}$$

$$1 \text{ yd} = 36 \text{ in}$$

*\*This is the ton used in the United States and Canada, not the metric ton.*

### Example Problems

Make the following conversions between the English and metric systems, stating the answers in the correct number of significant figures.

1.  $6.25 \text{ yd} = \underline{\hspace{2cm}} \text{ cm}$

**Answer:** 572 cm

The answer is limited to 3 significant figures by the 3 digits in 6.25.

$$\text{length in cm} = 6.25 \text{ yd} \left( \frac{36 \text{ in}}{1 \text{ yd}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) = 571.5 \text{ cm} = 572 \text{ cm}$$

2.  $3.0 \text{ gal} = \underline{\hspace{2cm}} \text{ L}$

**Answer:** 11 L

The volume in L can only have 2 significant figures.

$$\text{volume in L} = 3.0 \text{ gal} \left( \frac{4 \text{ qt}}{1 \text{ gal}} \right) \left( \frac{946 \text{ mL}}{1 \text{ qt}} \right) \left( \frac{1 \text{ L}}{1,000 \text{ mL}} \right) = 11.352 \text{ L} = 11 \text{ L}$$

3.  $1.25 \text{ lb} = \underline{\hspace{2cm}} \text{ mg}$

**Answer:**  $5.68 \times 10^5 \text{ mg}$

State the answer to 3 significant figures.

$$\text{mass in mg} = 1.25 \text{ lb} \left( \frac{454 \text{ g}}{1 \text{ lb}} \right) \left( \frac{1,000 \text{ mg}}{1 \text{ g}} \right) = 567,500 \text{ mg} = 5.68 \times 10^5 \text{ mg}$$

### Work Problems

Make the following conversions between English and metric units, stating the answers in the correct number of significant figures.

1.  $2.25 \text{ ft} = \underline{\hspace{2cm}} \text{ mm}$

2.  $20.5 \text{ mL} = \underline{\hspace{2cm}} \text{ qt}$

3.  $750. \text{ kg} = \underline{\hspace{2cm}} \text{ oz}$

## Worked Solutions

### 1. 686 mm

$$\text{length in mm} = 2.25 \text{ ft} \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) \left( \frac{10 \text{ mm}}{1 \text{ cm}} \right) = 685.8 \text{ mm} = 686 \text{ mm}$$

### 2. $2.17 \times 10^{-2}$ qt

$$\text{volume in qt} = 20.5 \text{ mL} \left( \frac{1 \text{ qt}}{946 \text{ mL}} \right) = 0.02167 \text{ qt} = 2.17 \times 10^{-2} \text{ qt}$$

### 3. $2.64 \times 10^4$ oz

$$\text{mass in oz} = 750. \text{ kg} \left( \frac{1,000 \text{ g}}{1 \text{ kg}} \right) \left( \frac{1 \text{ lb}}{454 \text{ g}} \right) \left( \frac{16 \text{ oz}}{1 \text{ lb}} \right) = 26,431.7 \text{ oz} = 2.64 \times 10^4 \text{ oz}$$

## Density

Density equals the mass of a sample divided by its volume and is stated as mass per *unit* volume (1 mL, 1 cc, 1 qt, and so on).

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

The density of water is 1.00 g/mL. This means that 1.00 mL of water has a mass of 1.00 g. The density of gold, a solid, is 19.3 g/cc; one cubic centimeter of gold (1 cc = 1 mL) has a mass of 19.3 grams. A person might say by mistake that gold is “heavier” than water, although the correct comparison would be that the density of gold is greater than that of water. Anything with a density greater than water will sink in water. If its density is less, it will float. In general, densities for liquids are expressed as g/mL, while for solids, g/cc or g/cm<sup>3</sup>.

## Example Problems

1. 5.00 mL of ethyl alcohol has a mass of 3.95 g. What is the density of ethyl alcohol?

**Answer:** 0.790 g/mL

$$\text{density} = \frac{3.95 \text{ g}}{5.00 \text{ mL}} = 0.790 \text{ g/mL}$$

2. 100. lbs of sulfuric acid has a volume of 24.5 L. What is the density of sulfuric acid in pounds per quart, lb/qt?

**Answer:** 3.85 lb/qt

The first term calculates the density in lb/L and the second converts L to qt.

$$\text{density} = \left( \frac{100. \text{ lb}}{24.5 \text{ L}} \right) \left( \frac{1.06 \text{ qt}}{1 \text{ L}} \right) = 3.85 \text{ lb/qt}$$

**Work Problems**

1. What is the density of table salt, sodium chloride, if 50.0 cc of salt has a mass of 108 g?
2. 1.00 gallon of gasoline has a mass of 5.85 pounds. What is the density of gasoline in pounds per quart?

**Worked Solutions**

1. **2.16 g/cc**

$$\text{density} = \frac{108 \text{ g}}{50.0 \text{ cc}} = 2.16 \text{ g/cc}$$

2. **1.46 lb/qt**

$$\text{density} = \left( \frac{5.85 \text{ lbs}}{1.00 \text{ gal}} \right) \left( \frac{1 \text{ gal}}{4 \text{ qt}} \right) = 1.46 \text{ lb/qt}$$

Densities relate the mass of a substance to its volume and for that reason can be used as conversion factors. The density of iron is 7.86 g/cc. This can be written as two conversion factors:

$$\left( \frac{7.86 \text{ g}}{1.00 \text{ cc}} \right) \text{ or inverted to be } \left( \frac{1.00 \text{ cc}}{7.86 \text{ g}} \right)$$

Volume is written to 3 significant figures, such as 1.00 cc, to match the 3 significant figures in the mass. As conversion factors, densities can be used to convert a known volume to mass or a known mass to volume.

**Example Problems**

1. The density of iron is 7.86 g/cc. What would be the mass of 950. cc of iron?

**Answer:**  $7.47 \times 10^3 \text{ g}$

$$\text{mass of iron} = 950. \text{ cc} \left( \frac{7.86 \text{ g}}{1.00 \text{ cc}} \right) = 7,467 \text{ g} = 7.47 \times 10^3 \text{ g}$$

2. What volume, in cubic centimeters (cc), would be occupied by 855 g of iron? The density of iron is 7.86 g/cc.

**Answer:** 109 cc

$$\text{volume} = 855 \text{ g} \left( \frac{1.00 \text{ cc}}{7.86 \text{ g}} \right) = 108.78 \text{ cc} = 109 \text{ cc}$$

**Work Problems**

1. What is the mass of 775 mL of ethyl alcohol, a colorless liquid that has a density of 0.789 g/mL?
2. What volume would 800. g of vegetable oil occupy if the density of the oil is 0.933 g/mL?

## Worked Solutions

### 1. 611 g

$$\text{mass} = 775 \text{ mL} \left( \frac{0.789 \text{ g}}{1.00 \text{ mL}} \right) = 611.47 \text{ g} = 611 \text{ g}$$

### 2. 857 mL

$$\text{volume} = 800. \text{ g} \left( \frac{1.00 \text{ mL}}{0.933 \text{ g}} \right) = 857.45 \text{ mL} = 857 \text{ mL}$$

## Measuring Temperature

Temperature is a measure of the intensity of heat energy in a sample of matter. Temperature is not heat. Heat energy is related to the motion of the particles that make up a sample. The higher the temperature, the more rapid the motion of particles.

You need to be familiar with three temperature scales, two of which are commonly used in science: the *Celsius* scale and the *Kelvin* scale, also known as the *Absolute* scale. The third is the *Fahrenheit* scale that is in everyday use in commerce in the United States but not in science. The three temperature scales are summarized in the following table.

The Temperature Scales			
	<i>Celsius Scale</i>	<i>Kelvin Scale</i>	<i>Fahrenheit Scale</i>
Symbol of temperature	C	K	F
Unit of temperature and its symbol	degree (°)	degree, no symbol	degree (°)
Freezing point of water	0°C	273 K*	32°F
Boiling point of water	100°C	373 K*	212°F
Number of degrees between freezing and boiling points of water	100°	100	180°

\*273.15 K and 373.15 K to be exact, although 273 K and 373 K will be used here.

The number of degree units between the freezing and boiling points of water are identical in the Celsius and Kelvin scales, 100 degrees. This means the size of the Celsius degree and the Kelvin degree is the same.

A change of 1°C = a change of 1 K

There are 180 degrees between the freezing and boiling points on the Fahrenheit scale. Compared to the Celsius and Kelvin scales:

A change of 1°C or a change of 1 K = a change of 1.8°F (exactly)

Using the freezing point of water to compare the three temperature scales (0°C, 273 K, and 32°F) and knowing how the size of the degree compares, it is possible to develop equations to convert a temperature on one scale to the corresponding temperature on another.

A Celsius temperature can be converted to a Kelvin temperature simply by adding the number of the Celsius temperature to 273 and attaching “K” (the symbol for Kelvin temperature).

$$\begin{aligned}\text{Kelvin temperature} &= (\text{temperature in } ^\circ\text{C}) + 273 \\ \text{K} &= ^\circ\text{C} + 273\end{aligned}$$

Changing a Celsius temperature to one in Fahrenheit must take into account the two different values for the freezing point of water and the different size of the Celsius and Fahrenheit degree.

$$\begin{aligned}\text{Fahrenheit temperature} &= 1.8 (\text{Celsius temperature}) + 32 \\ ^\circ\text{F} &= 1.8 (^\circ\text{C}) + 32\end{aligned}$$

The equation tells you that the numerical value of the Celsius temperature is multiplied by 1.8, and then 32 is added to the product. The symbol for degree Fahrenheit,  $^\circ\text{F}$ , is attached to the sum.

This equation can be rearranged to ease conversion of Fahrenheit temperatures to Celsius.

$$\begin{aligned}\text{Celsius temperature} &= \frac{(\text{Fahrenheit temperature} - 32)}{1.8} \\ ^\circ\text{C} &= \frac{(^{\circ}\text{F} - 32)}{1.8}\end{aligned}$$

This equation tells you to first subtract 32 from the numerical value of the Fahrenheit temperature, then divide the difference by 1.8. The symbol for degree Celsius,  $^\circ\text{C}$ , is added to the answer.

### Example Problems

Make the following temperature conversions.

1.  $125^\circ\text{F} = ? ^\circ\text{C}$

**Answer:**  $51.7^\circ\text{C}$  Note that only the numerical value of the Fahrenheit temperature is used.  $125^\circ\text{F}$  is written as 125.

$$^\circ\text{C} = \frac{(125 - 32)}{1.8} = 51.6666^\circ\text{C} = 51.7^\circ\text{C} \text{ (3 sig. figs.)}$$

2.  $-18^\circ\text{C} = ? \text{K}$

**Answer:** 255 K

$$\text{K} = -18 + 273 = 255\text{K}$$

3.  $45^\circ\text{F} = ? \text{K}$

**Answer:** 280 K First convert Fahrenheit to Celsius; then convert Celsius to Kelvin.

$$^\circ\text{C} = \frac{(45 - 32)}{1.8} = 7.2^\circ\text{C}$$

$\text{K} = 7.2 + 273 = 280\text{K}$  In terms of significant figures, 7.2 is rounded to 7, and then added to 273.

## Work Problems

Make the following temperature conversions.

1.  $25^{\circ}\text{C} = \underline{\hspace{2cm}}^{\circ}\text{F}$
2.  $450\text{ K} = \underline{\hspace{2cm}}^{\circ}\text{C}$
3.  $70^{\circ}\text{F} = \underline{\hspace{2cm}}\text{ K}$

## Worked Solutions

1. **77°F**

$$^{\circ}\text{F} = (25 \times 1.8) + 32 = 77^{\circ}\text{F}$$

2. **177°C**

$$^{\circ}\text{C} = 450 - 273 = 177^{\circ}\text{C}$$

3. **294 K**

$$^{\circ}\text{C} = \frac{(70 - 32)}{1.8} = 21^{\circ}\text{C} \text{ then } \text{K} = 21 + 273 = 294\text{ K}$$

## Energy and the Measurement of Heat

A body possesses energy if it has the ability to move another object. So, for example, a rapidly moving atom possesses energy, since it can crash into another atom, causing it to move. The energy possessed by a body by virtue of its motion is called *kinetic energy*. A sample of TNT also has the ability to move objects. It possesses energy stored in its chemical composition—energy that has the potential to move an object. Stored energy is called *potential energy*. Both an arrow in a drawn bow and a wound clock spring possess potential energy.

There are many forms of energy: light energy, acoustic energy, electrical energy, heat energy, and nuclear energy, to name a few. Energy can change from one kind into another. The electrical energy of a battery can heat the filament of a light bulb, generating both heat and light energy. Although energy is changing form, it is important to realize that no new energy is being created nor is any energy being destroyed. This fact is summed up in the *Law of Conservation of Energy*, which states that energy can be neither created nor destroyed, although it may change form.

It is more difficult to measure energy than it is to measure length, mass, volume, or temperature. But when measured, energy is quantitatively given in the SI unit of the Joule (J) or in the calorie (cal), an older unit used for measuring heat energy. By definition:

$$4.184\text{ Joules} = 1\text{ calorie (exactly)}$$

If larger units of energy are needed, the kilojoule (kJ) and the kilocalorie (kcal) can be used.

$$1\text{ kilojoule} = 1,000\text{ Joules}$$

$$1\text{ kilocalorie} = 1,000\text{ calories}$$

The amount of heat needed to raise the temperature of 1 gram of a substance 1°C is called the *specific heat* (S.H.) of that substance. It requires 4.184 Joules (or 1 calorie) of heat to raise the temperature of 1 gram of water 1°C. It follows, then, that the specific heat of water is 4.184 J/g°C. If you know the amount of heat needed to raise the temperature of 1 g of water 1°C, you can calculate the amount of heat (symbolized,  $q$ ) needed to raise the temperature of 10 g of water 10°C. The formula for doing just this is:

$$\text{amount of heat energy} = q = (\text{S.H. of water})(\text{mass of water})(\text{temperature change of water})$$

So, how many Joules of heat energy are required to raise the temperature of 10.0 g of water 10.0°C?

$$q = \left( \frac{4.184 \text{ J}}{\text{g}^\circ\text{C}} \right) (10.0 \text{ g}) (10.0^\circ\text{C}) = 418 \text{ Joules}$$

Notice that the units of gram and °C cancel, leaving Joule, the unit of energy. This equation can be rewritten in a general way that applies to all substances.  $\Delta T$  symbolizes a change in temperature. This equation is referred to as the **heat equation**.

$$q = (\text{S.H.})(\text{mass})(\Delta T)$$

Read  $\Delta$  as “a change in.” If the temperature rises from 25°C to 40°C,  $\Delta T$  equals 15°C, the difference between the final temperature and the initial temperature,  $\Delta T = (T_{\text{final}} - T_{\text{initial}}) = (40^\circ\text{C} - 25^\circ\text{C}) = 15^\circ\text{C}$ .

The heat equation can be rearranged to solve for specific heat, S.H., mass,  $m$ , or  $\Delta T$ .

$$\text{S.H.} = \frac{q}{(\text{mass})(\Delta T)} \quad \text{mass} = \frac{q}{(\Delta T)(\text{S.H.})} \quad \Delta T = \frac{q}{(\text{mass})(\text{S.H.})}$$

### Example Problems

- 125 cal = ? J; 500. J = ? cal

**Answer:** 523 J; 120 cal

The conversion factor is derived from: 4.184 J = 1 cal (exactly)

$$125 \text{ cal} \left( \frac{4.184 \text{ J}}{1 \text{ cal}} \right) = 523 \text{ J}; \quad 500 \text{ J} \left( \frac{1 \text{ cal}}{4.184 \text{ J}} \right) = 119.50 \text{ cal} = 120 \text{ cal}$$

- The specific heat of water is 4.184 J/g°C. How many Joules of heat energy would be required to raise the temperature of 150. g of water from 20.0°C to 40.0°C?

**Answer:**  $1.26 \times 10^4 \text{ J}$

$$\Delta T = 40.0^\circ\text{C} - 20.0^\circ\text{C} = 20.0^\circ\text{C}$$

$$q = \left( \frac{4.184 \text{ J}}{\text{g}^\circ\text{C}} \right) (150. \text{ g}) (20.0^\circ\text{C}) = 12,552 \text{ J} = 1.26 \times 10^4 \text{ J}$$

3. 55.1 g of aluminum absorbed 1,500 Joules of heat. Its temperature increased from 20.0°C to 50.2°C. What is the specific heat of aluminum?

**Answer:** 0.901 J/g°C

$$\Delta T = 50.2^{\circ}\text{C} - 20.0^{\circ}\text{C} = 30.2^{\circ}\text{C}$$

Rearranging the heat equation to solve for specific heat:

$$\text{S.H.} = \frac{q}{(\text{mass})(\Delta T)}$$

$$\text{S.H.} = \frac{1,500. \text{ J}}{(55.1 \text{ g})(30.2^{\circ}\text{C})} = 0.901 \frac{\text{J}}{\text{g}^{\circ}\text{C}}$$

### Work Problems

- 1,750 cal = ? kJ
- How many Joules of heat energy are required to raise the temperature of 500. g of water from 23.5°C to 75.0°C? The specific heat of water is 4.184 J/g-°C. State the answer in scientific notation.
- The addition of 1,800. J of heat energy to a 200. g sample of copper raised the temperature of the sample 23.4°C. What is the specific heat of copper?

### Worked Solutions

1. **7.322 kJ**

$$1,750. \text{ cal} \left( \frac{4.184 \text{ J}}{1 \text{ cal}} \right) = 7,322 \text{ J}$$

$$7,322 \text{ J} \left( \frac{1 \text{ kJ}}{1,000 \text{ J}} \right) = 7.322 \text{ kJ}$$

2. **Amount of heat energy = 1.08 × 10<sup>5</sup> J**

$$\Delta T = 75.0^{\circ}\text{C} - 23.5^{\circ}\text{C} = 51.5^{\circ}\text{C}$$

$$q = \left( \frac{4.184 \text{ J}}{\text{g}^{\circ}\text{C}} \right) (500. \text{ g})(51.5^{\circ}\text{C}) = 107,738 \text{ J}$$

$$q = 107,738 \text{ J} = 1.08 \times 10^5 \text{ J} \text{ (3 sig. figs.)}$$

3. **S.H. = 0.385 J/g-°C**

$$\text{S.H.} = \frac{q}{(\text{mass})(\Delta T)}$$

$$\text{S.H.} = \frac{1,800. \text{ J}}{(200. \text{ g})(23.4^{\circ}\text{C})} = 0.385 \frac{\text{J}}{\text{g}^{\circ}\text{C}}$$