Chapter 1

Basic Geometric Ideas

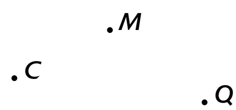
The word *geometry* comes from two ancient Greek words, *ge*, meaning earth, and *metria*, meaning measure. So, literally, geometry means to measure the earth. It was the first branch of math that began with certain assumptions and used them to draw more complicated conclusions. Over time, geometry has become a body of knowledge that helps us to logically create chains of conclusions that let us go from knowing certain things about a figure to predicting other things about it with certainty. Although a little arithmetic and a little algebra are used in building an understanding of geometry, this branch of math really can stand on its own, as a way of constructing techniques and insights that may help you to better understand later mathematical ideas, and that, believe it or not, may help you to live a more fulfilling life.

Naming Basic Forms

The bulk of this book deals with plane geometry—that is, geometry on a perfectly flat surface. Many different types of plane figures exist, but all of them are made up of a few basic parts. The most elementary of those parts are points, lines, and planes.

Points

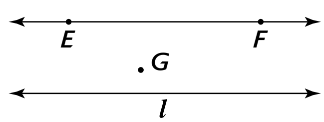
A **point** is the simplest and yet most important building block in geometry. It is a location and occupies no space. Because a point has no height, length, or width, we can't actually draw one. This is true of many geometric parts. We can, however, *represent* a point, and we use a dot to do that. We name points with single uppercase letters.



This diagram shows three dots that represent points *C*, *M*, and *Q*.

Lines

Lines are infinite series of points. *Infinite* means without end. A line extends infinitely in two opposite directions, but has no width and no height. Just to be clear, in geometry, *line* and *straight line* mean the same thing. Contrary to the popular notion, a line is *not* the shortest distance between two points. (We'll come back to this later.)

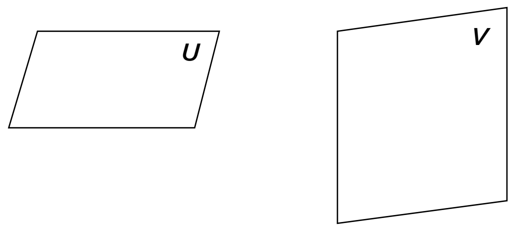


A line may be named by any two points on it, as is line *EF*, represented by the symbol $\overleftrightarrow{EF}$ or $\overleftrightarrow{FE}$. It may also be named by a single lowercase letter, as is line l.

Points that are on the same line are said to be **collinear points**. Point *E* and Point *F* in the preceding diagram are collinear points. Point *G* is not collinear with *E* and *F*. Taken altogether, it may be said that *E*, *F*, and *G* are **noncollinear points**. You'll see why this distinction is important a little later in this chapter.

Planes

A **plane** is an infinite set of points extending in all directions along a perfectly flat surface. It is infinitely long and infinitely wide. A plane has a thickness (or height) of zero.



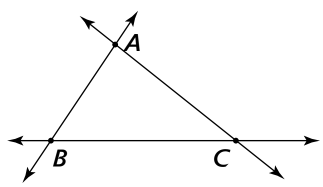
A plane is named by a single uppercase letter and is often represented as a four-sided figure, as in planes *U* and *V* in the preceding diagram.

Example Problems

These problems show the answers and solutions.

1. What is the maximum number of lines in a plane that can contain two of the points *A*, *B*, and *C*?

   ***Answer:* 3** Consider that two points name a line. It is possible to make three sets of two points from the three letters: *AB*, *AC*, and *BC*. That means it's possible to form three unique lines: $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{BC}$. See the following figure.



2. What kind of geometric form is the one named *H*?

   ***Answer:* not enough information** A single uppercase *H* could be used to designate a point or a plane.

Postulates and Theorems

As noted at the very beginning of the chapter, geometry begins with assumptions about certain things that are very difficult, if not impossible, to prove and flows on to things that can be proven. The assumptions that geometry's logic is based upon are called **postulates**. Sometimes, you may see them referred to as **axioms**. The two words mean essentially the same thing, Here are the first six of them, numbered so that we can refer back to them easily:

Postulate 1: A line contains at least two points.

Postulate 2: A plane contains a minimum of three noncollinear points.

Postulate 3: Through any two points there can be exactly one line.

Postulate 4: Through any three noncollinear points there can be exactly one plane.

Postulate 5: If two points lie in a plane, then the line they lie on is in the same plane.

Postulate 6: Where two planes intersect, their intersection is a line.

From these six postulates it is possible to prove these **theorems,** numbered for the same reason:

Theorem 1: If two lines intersect, they intersect in exactly one point.

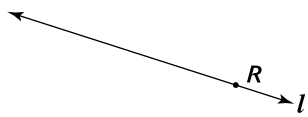
Theorem 2: If a point lies outside a line, then exactly one plane contains the line and the point.

Theorem 3: If two lines intersect, then exactly one plane contains both lines.

Example Problems

These problems show the answers and solutions. State the postulate or theorem that may be used to support the statement made about each diagram.

1. There is another point on line *l* in addition to *R*.



***Answer*: A line contains at least two points.** (Postulate 1)

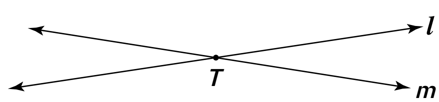
2. Only one line contains point *M* and point *N*.

***Answer*: Through any two points there can be exactly one line.** (Postulate 3)

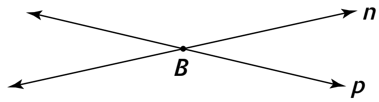
Work Problems

Use these problems to give yourself additional practice. State the postulate or theorem that may be used to support the statement made about each diagram.

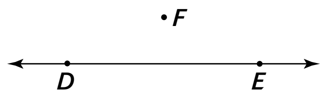
1. Lines *m* and *l* are in the same plane.



2. There is no other intersection for n and p other than B.

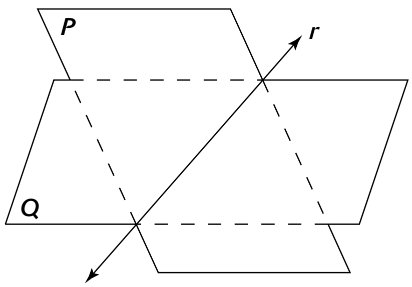


3. Point F and $\overleftrightarrow{DE}$ are in the same plane.

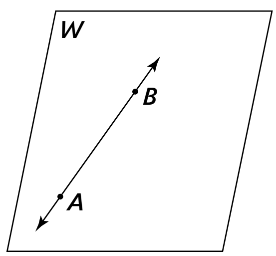


4. Points J, K, and L are all in the same plane.

5. The intersection of planes P and Q is line r.



6. $\overleftrightarrow{AB}$ lies in plane W.



Worked Solutions

1. The figure shows two intersecting lines, and the statement mentions a plane. That relationship is dealt with by Theorem 3: If two lines intersect, then exactly one plane contains both lines.

2. The figure shows two intersecting lines, and the statement mentions the point of intersection. That's covered in Theorem 1: If two lines intersect, they intersect in exactly one point.

3. This figure concerns a line and a noncollinear point, and the statement mentions a plane. That's Theorem 2: If a point lies outside a line, then exactly one plane contains the line and the point.

4. We are shown three noncollinear points, and a plane is mentioned. That's Postulate 4: Through any three noncollinear points there can be exactly one plane.

5. Here, we have two intersecting planes and line *r*. That's Postulate 6: Where two planes intersect, their intersection is a line.

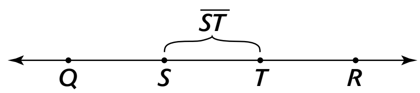
6. The diagram shows a line in a plane, but two points on that line are clearly marked. That should lead us straight to Postulate 5: If two points lie in a plane, then the line they lie on is in the same plane.

Finding Segments, Midpoints, and Rays

You'll recall that geometry means earth measure. We've already dealt with the concept of lines, but because lines are infinite, they can't be measured. Much of geometry deals with parts of lines. Some of those parts are very special—so much so that they have their own special names and symbols. The first such part is the **line segment**.

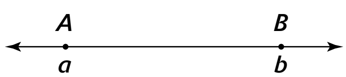
Line Segments

A line segment is a finite portion of a line and is named for its two endpoints.



In the preceding diagram is segment $\overline{ST}$. Notice the bar above the segment's name. Technically, $\overline{ST}$ refers to points *S* and *T* and all the points in between them. *ST*, without the bar, refers to the distance from *S* to *T*. You'll notice that $\overline{ST}$ is a portion of $\overleftrightarrow{QR}$.

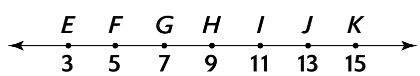
Each point on a line or a segment can be paired with a single real number, which is known as that point's **coordinate**. The distance between two points is the absolute value of the difference of their coordinates.



If $b > a$, then $AB = b - a$. This postulate, number 7, is known as the *Ruler Postulate*.

Example Problems

These problems show the answers and solutions.



1. Find the length of $\overline{EH}$, or, put more simply, find *EH*.

   ***Answer:* 6** To find the length of $\overline{EH}$, first find the coordinates of point *E* and point *H*.

E's coordinate is 3, and point *H*'s coordinate is 9.

$$EH = 9 - 3$$
$$EH = 6$$

2. Find the length of $\overline{FK}$, or, put more simply, find *FK*.

***Answer:* 10** To find *FK*, first find the coordinates of point *F* and point *K*.

F's coordinate is 5, and point *K*'s coordinate is 15.

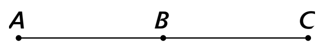
$$FK = 15 - 5$$
$$FK = 10$$

Segment Addition and Midpoint

Postulate 8 is known as the Segment Addition Postulate. It goes like this:

Postulate 8 (Segment Addition Postulate): If *N* lies between *M* and *P* on a line, then *MN* + *NP* = *MP*. This is, in fact, one of many postulates and theorems that can be restated in general terms as the whole is the sum of its parts.

The midpoint of a line segment is the point that's an equal distance from both endpoints. B is the midpoint of $\overline{AC}$ because $\overline{AB} = \overline{BC}$.

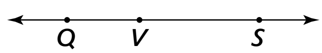


This brings us to the obvious fact stated by Theorem 4:

Theorem 4: A segment has exactly one midpoint.

Example Problems

These problems show the answers and solutions.

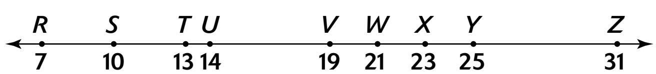


1. *V* lies between *Q* and *S*. Find *QS* if *QV* = 6 and *VS* = 10.

***Answer:* 16** Since *V* lies between *Q* and *S*, Postulate 8 tells us that

$$QV + VS = QS$$
$$6 + 10 = 16$$
$$QS = 16$$

2. Find the midpoint of $\overline{RZ}$.



***Answer:* 19** We can solve this in two ways. First consider the coordinates of the endpoints, *R* (7) and *Z* (31). Their difference is 24 (31 − 7 = 24). That means that the segment is 24 units long, so its midpoint must be half of 24, or 12 units from either endpoint. Adding 12 to the 7 (*R*'s coordinate) gives us 19, the coordinate of *V*. *V* is the midpoint.

The other method is to take the average of the coordinates of the two endpoints, which is done by adding them together and dividing by 2.

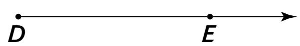
$$\frac{7+31}{2}=\frac{38}{2}$$

$$\frac{38}{2}=19$$

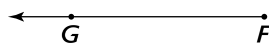
And 19, of course, is the coordinate of midpoint, *V*.

Rays

Geometric rays are like the sun's rays. They have a beginning point (or endpoint), and they go on and on without end in a single direction. The sun's rays don't have names (as far as we know). A geometric ray is named by its endpoint and any other point on it.



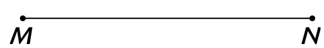
Above is $\overrightarrow{DE}$ (read ray *DE). The* arrow above the letters not only indicates that the figure is a ray. It also indicates the direction in which the ray is pointing. The arrow's head is over the non-endpoint.



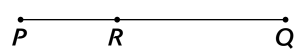
This is ray *FG*. It may be written $\overrightarrow{FG}$, or $\overleftarrow{GF}$.

Work Problems

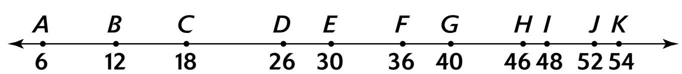
Use these problems to give yourself additional practice.



1. Is the preceding figure more accurately named $\overline{MN}$ or $\overline{NM}$?

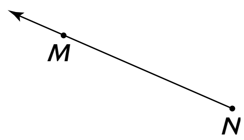


2. *PR* is 8, and *PQ* is 22. What is the length of $\overline{RQ}$?

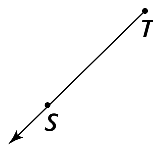


3. Find the midpoint of $\overline{AE}$.

4. Find the midpoint of $\overline{AK}$.



5. Name the ray here.



6. Name the ray here.

Worked Solutions

1. **Both names are suitable.** A segment does not have direction, so the order in which its endpoints are named is not relevant.

2. **14** By the Segment Addition Postulate,

$$PR + RQ = PQ$$
$$\text{Then } 8 + RQ = 22$$
$$\text{So } RQ = 22 - 8$$
$$RQ = 14$$

3. **C** $\overline{AE}$'s endpoints are *A* and *E* with coordinates of 6 and 30. That means the length

$$AE = 30 - 6$$
$$AE = 24$$

The midpoint is half of 24, or 12 from either endpoint. 12 from the starting coordinate of 6 is 18, the coordinate of point *C*.

4. **E** This is essentially the same problem as 3, but we'll use the alternate method to solve it. Let's average the endpoints' coordinates.

First, add them together: $54 + 6 = 60$

Then, divide by 2: $\frac{60}{2} = 30$

30 is the coordinate of point *E*.

5. $\overrightarrow{NM}$ **or** $\overleftarrow{MN}$ The only thing that matters in the naming of a ray is which end has the endpoint and which point is elsewhere on the ray. The endpoint will be under the blunt end of the arrow, and the point that is elsewhere on the ray goes under the arrowhead.

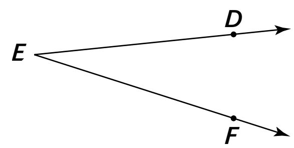
6. $\overrightarrow{TS}$ **or** $\overleftarrow{ST}$ See the explanation for problem 5.

Angles and Angle Pairs

Angles are as important as line segments when it comes to forming geometric figures. Without them, there would be no plane figures, with the possible exception of circles.

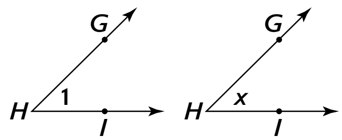
Forming and Naming Angles

An angle is formed by two rays that have a common (or shared) endpoint. The rays form the **sides** of the angle, and their endpoint forms its **vertex**. The measurement of the opening of an angle is expressed in degrees. The smallest angle practical has a degree measure of 0°. Imagine the angle formed by the hands of an analog clock at noon. That is a 0° angle. There is no real limit to the upper number of degrees in an angle, but no unique angle contains more than 359°, since an angle of 360° is indistinguishable from one of 0°.



Rays $\overrightarrow{ED}$ and $\overrightarrow{EF}$ form the sides of this angle, shown by the symbol $\angle DEF$, whose vertex is at E.

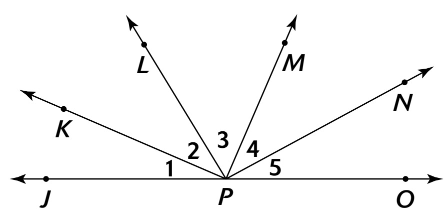
Here we have two pictures of the same angle.



We need two pictures in order to cover all the different ways there are to name this angle. It may be named by three letters—one from each ray and the vertex, with the name of the vertex always being in the middle. Thus, it is $\angle GHI$ or $\angle IHG$. If there is no chance of ambiguity (more than one angle at the vertex), it may be named by the vertex alone, hence $\angle H$. It may also be named by a number or a lowercase letter inside the angle, so it is also $\angle 1$, on the left, or $\angle x$, on the right.

Example Problems

These problems show the answers and solutions.



1. State another name for $\angle 3$.

   Answer: $\angle LPM$ **or** $\angle MPL$ Use the name of one point on each ray with the designation of the vertex in the middle. $\angle P$ would not do, since there are 5 separate angles at vertex P.

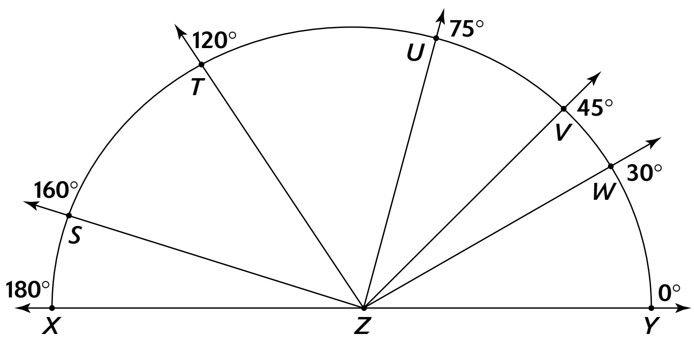
2. What is another name for $\angle KPL$?

***Answer:* $\angle LPK$ or $\angle 2$** The first is the reverse of the order of the letters in the question (always legitimate), or second, the number that is written in the opening of that angle.

3. What is another name for the sum of angles 4 and 5?

***Answer:* $\angle OPM$ or $\angle MPO$** This wasn't really a fair question, since we haven't yet discussed angle addition, still it is a natural application of the already noted fact that a whole is the sum of its parts. The two angles combined into one are bounded by $\overrightarrow{PM}$ and $\overrightarrow{PO}$, hence the two choices given.

The Protractor Postulate and Addition of Angles



The Protractor Postulate, Postulate 9, supposes that a point, Z, exists on line XY. Think of all rays with endpoint Z that exist on one side of line XY. Each of those rays may be paired with exactly one number between 0° and 180°, as you can see in the preceding figure. The positive difference between two numbers representing two different rays is the degree measure of the angle with those rays as its sides. So, the measure of $\angle VZW$ (represented $m\angle VZW$) = 45° – 30° = 15°.

Postulate 10 is the Addition of Angles postulate. Simply put, if $\overrightarrow{SZ}$ lies between $\overrightarrow{XZ}$ and $\overrightarrow{TZ}$, then $m\angle XZT = m\angle XZS + m\angle SZT$. It's just another statement of the whole equaling the sum of its parts, as already used in the previous problems.

Example Problems

These problems show the answers and solutions. Use the preceding diagram to solve these problems.

1. Find $m\angle TZU$.

***Answer:* 45°**

$$m\angle TZU = 120° - 75°$$
$$m\angle TZU = 45°$$

2. Find $m\angle UZY$.

***Answer:* 75°**

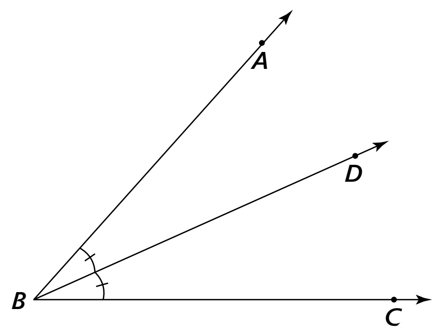
$$m\angle UZY = 75° - 0°$$
$$m\angle UZY = 75°$$

3. Find $m\angle SZV$.

Answer: **115°**

$$m\angle SZV = 160° - 45°$$
$$m\angle SZV = 115°$$

Angle Bisector



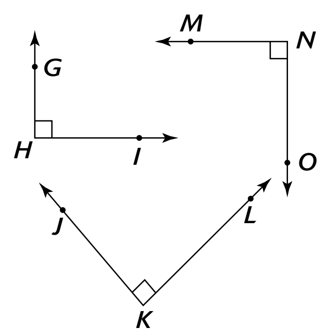
An **angle bisector** is a ray that divides an angle into two angles of equal degree measure. In the figure immediately preceding, $m\angle ABD$ is marked as being equal to $m\angle DBC$, therefore $\overrightarrow{DB}$ is the angle bisector of $\angle ABC$.

Certain angles have special names, and we'll begin looking at them next, but first, consider the following:

> **Theorem 5:** An angle that is not a straight angle has only one angle bisector.

Think about why that is.

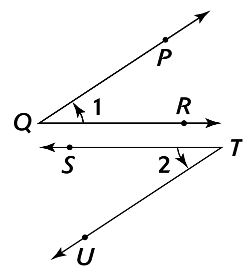
Right Angles



A **right angle** is an angle whose measure is 90°. Angles *GHI*, *JKL*, and *MNO* are all right angles. That little corner in each of those right angles should serve as a reminder that they have the same shape as the corners of a book. That brings us to the following theorem:

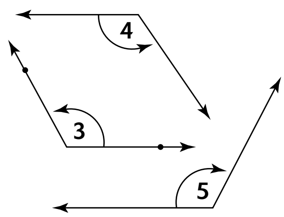
> **Theorem 6:** All right angles are equal.

Acute Angles



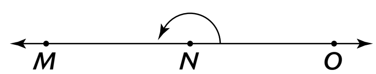
An **acute angle** is an angle whose degree measure is greater than 0° and less than 90°. The word *acute* means sharp, and you'll notice that at the endpoint of the rays that form $\angle 1$ and $\angle 2$ is a sharp point. Note that the way any angle faces never has anything to do with the kind of angle it is. Only the degree measure counts.

Obtuse Angles



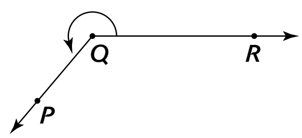
An **obtuse angle** has a degree measure greater than 90° and less than 180°. Angles 3, 4, and 5 are three examples of obtuse angles. *Obtuse* means dull or blunt. Compare the shapes to that of a sharp acute angle.

Straight Angles



$\angle MNO$ is a **straight angle**. It's not really very difficult to see why. The degree measure of a straight angle is exactly 180°, or two right angles. Any angle that contains exactly 180° is a straight angle and indistinguishable, save for its vertex, from a straight line.

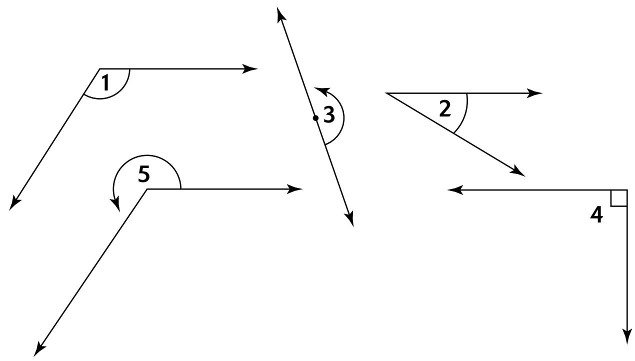
Reflex Angles



A **reflex angle** is formed by a pair of rays where one of them has remained stationary and the other one has rotated through an angle of greater than 180° and less than 360°. You are not likely to encounter many reflex angles in your study of geometry, but you should be aware of their existence.

Example Problems

These problems show the answers and solutions.



1. Which of the angles shown is a right angle?

   Answer: **$\angle 4$** The square inside the angle tells us that $\angle 4$ is a right, or 90° angle.

2. Which of the angles shown is an acute angle?

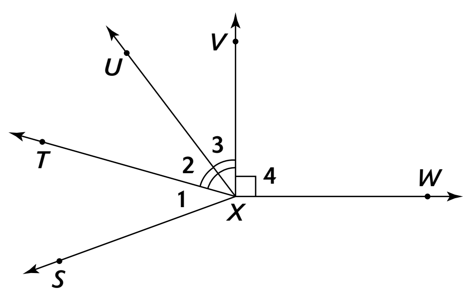
   Answer: **$\angle 2$** An acute angle must measure fewer than 90°. The fact that $\angle 2$ comes to an acute (sharp) point is the tip-off.

3. Which of the angles shown is an obtuse angle?

   Answer: **$\angle 1$** The measures of three of the angles are greater than 90°, but only $\angle 1$'s measure is both greater than 90° and smaller than 180°—the definition of an obtuse angle.

Work Problems

Use these problems to give yourself additional practice. All of the work problems in this section refer to this figure.



1. Name a reflex angle shown in the diagram.
2. Name two obtuse angles shown in the figure.
3. Give the letter name of a right angle in the diagram.
4. Name as many acute angles as you can find in the diagram, using letter designations.
5. Name the angle bisector.

Worked Solutions

1. ***∠WXS.*** By angle addition of angles 1 through 4, we are able to form reflex angle *WXS*. We can also form several other reflex angles by combining the lower ∠*WXS* with ∠ 1, ∠1 +∠2, and ∠1 + ∠2 + ∠3.

2. ***∠VXS, ∠SXV, ∠TXW, ∠WXT, ∠UXW,* and *∠WXU.*** By addition of angles 1 through 3, we can form ∠*VXS*, or ∠*SXV*; by addition of angles 2 through 4, we can form ∠*TXW*, or ∠*WXT*; by addition of angles 3 and 4, we can form ∠*UXW*, or ∠*WXU*. Each of those is greater than 90° and less than 180°.

3. ***∠VXW* or *∠WXV.*** The square marking designates ∠4 as a right angle.

4. ***∠TXS, ∠SXT, ∠TXU, ∠UXT, ∠VXU, ∠UXV, ∠UXS, ∠SXU, ∠VXT,* and *∠TXV.***

   ∠1 is ∠*TXS*, or ∠*SXT*, ∠2 is ∠*TXU*, or ∠*UXT*, and ∠3 is ∠*VXU*, or ∠*UXV*.

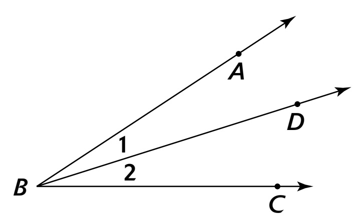
   ∠1 + ∠2 make ∠*UXS*, ∠*SXU*; ∠2 + ∠3 make ∠*VXT*, and ∠*TXV*. All are clearly <90°.

5. $\overrightarrow{XU}$ Angles 2 and 3 are marked as being of equal measure. An angle bisector is defined as a line that separates an angle into two angles of equal measure.

Special Angle Pairs

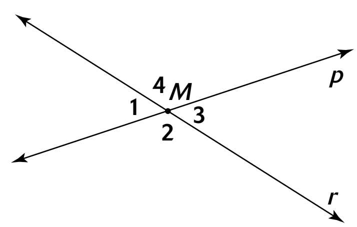
Certain pairs of angles have special names. Those names are in some cases based upon their positions relative to one another. In other cases they are based upon the angles' degree measures adding up to a certain amount.

Adjacent Angles



Two angles that share a vertex and share a common side that separates them are known as **adjacent angles**. Angles 1 and 2 are examples of adjacent angles.

Vertical Angles

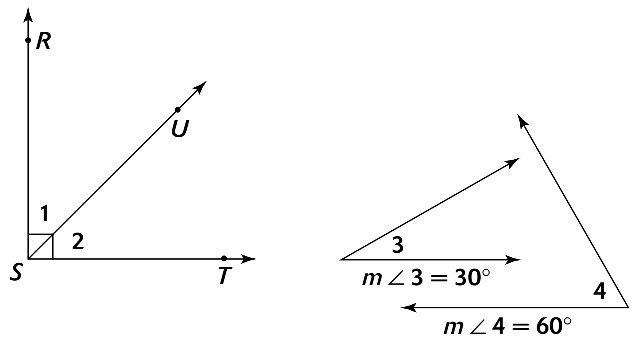


When two lines intersect so as to form four angles, the angles on opposite sides of the common vertex are known as **vertical angles**. Of the four angles formed at *M*, ∠1 and ∠3 are vertical angles. So are ∠2 and ∠4.

Theorem 7 tells us that vertical angles are equal in measure. That means $m\angle 1 = m\angle 3$, and $m\angle 2 = m\angle 4$.

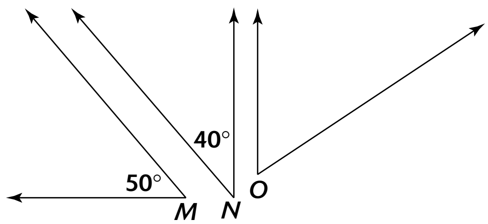
This pair of lines also forms four pairs of adjacent angles: $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 4$ and $\angle 1$.

Complementary Angles



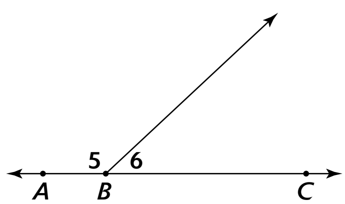
Any two angles that add up to 90° are called **complementary angles**. $\angle 1$ and $\angle 2$ are adjacent complementary angles. $\angle 1$ is said to be the **complement** of $\angle 2$. $\angle 2$ is said to be the complement of $\angle 1$. Notice that the word has no "i" in it. It's complement, not compl**i**ment. Angles 3 and 4 are **nonadjacent** complementary angles.

That brings us to **Theorem 8,** which states: If two angles are complements of the same or equal angles, they are equal to each other.



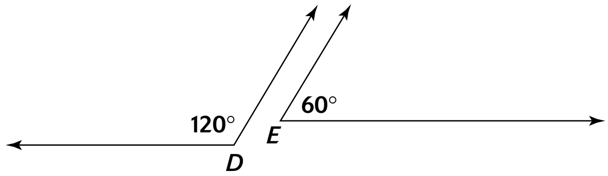
$m\angle M = 50°$. $\angle M$ is complementary to $\angle N$. $\angle O$ is also complementary to $\angle N$. That means that $m\angle O$ must be 50°, since both $\angle M$ and $\angle O$ are complements of the same angle.

Supplementary Angles



When two angles add up to a total of 180°, they're called **supplementary angles**. $\angle 5$ and $\angle 6$ are adjacent supplementary angles. That gives us:

Theorem 9: If two adjacent angles have their noncommon sides lying on a line, then they are supplementary angles. That's pretty much a no-brainer.

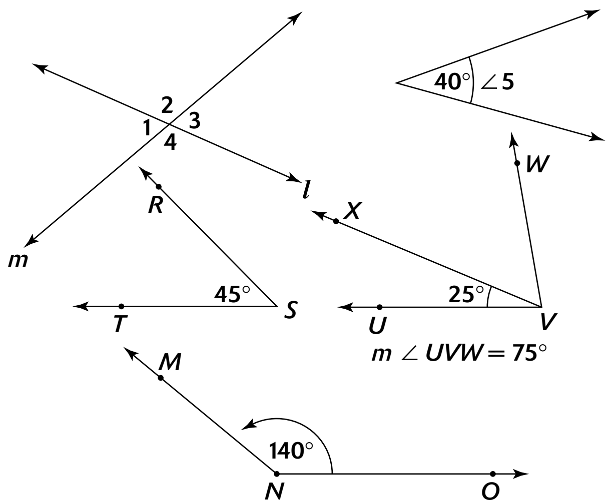


∠*D* and ∠*E* are nonadjacent supplementary angles.

Theorem 10, does for supplementary angles what Theorem 8 did for complementary ones: If two angles are supplements of the same or equal angles, they are equal to each other.

Example Problems

These problems show the answers and solutions. They all refer to the following diagram.



1. Identify a pair of supplementary angles.

   Answer**: ∠1 and ∠2, ∠2 and ∠3, ∠3 and ∠4, ∠4 and ∠1** When two lines intersect, each pair of adjacent angles is also a pair of supplementary angles.

2. Identify a pair of vertical angles.

   Answer: **∠1 and ∠3, ∠2 and ∠4** Vertical angles are nonadjacent angles formed by a pair of intersecting lines.

3. Identify a pair of complementary angles.

   Answer: **∠5 and ∠*XVW* or ∠*WVX*** (From here on, we'll name each angle only once.) ∠*WVU* has angle measure of 75°, and *m*∠*XVU* is 25°. By the Angle Addition Postulate,

$$m\angle WVU = m\angle XVW + m\angle XVU$$

$$\text{That means } 75° = m\angle XVW + 25°$$

$$\text{So } 50° = m\angle XVW$$
$$\text{Or } m\angle XVW = 50°.$$

Complementary angles total 90°, so the complement of a 50° $\angle XVW$ is the 40° $\angle 5$.

Work Problems

Use these problems to give yourself additional practice

1. Can an angle be its own complement? Give an example.
2. Can an angle be its own supplement? Give an example.
3. Is it possible to have more than two pairs of vertical angles at a given point? Explain.
4. Can two adjacent angles formed by intersecting lines at a single point not be supplementary? Explain.
5. If two angles are supplementary and one of them is acute, what must be true of the second angle?

Worked Solutions

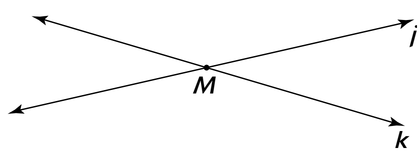
1. **Yes** Since there are 90° in a right angle, half of 90, or 45° is its own complement.
2. **Yes** Since there are 180° in a straight angle, half of 180, or 90° is its own supplement.
3. **Yes** If 3 lines intersect, there'll be three pairs, 4 lines, 4 pairs, and so on.Actually you can have more than that if you double up the angles or even triple them up, but why bother?
4. **Yes** The adjacent angles are supplementary if and only if exactly two lines cross. If three or more lines cross, the measures of any two adjacent angles would sum to less than 180°.
5. **It must be obtuse.** Since an acute angle's measure is less than 90°, its supplement's measure must be more than 90° for them to sum to 180°.

Special Lines and Segments

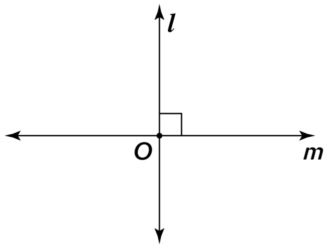
You've probably stood in line at an ice cream parlor or waiting to get in to a concert. If you live in a city, lines are a part of everyday life; in a small town, not so much so. In geometry, there are three types of line that are of interest, and it is essential for students to understand each of them.

Intersecting Lines and Segments

Two or more lines that meet or cross at a point are called **intersecting lines**. The point at which they meet is considered to be a part of both lines. Here we see line *j* intersecting line *k* at *M*.

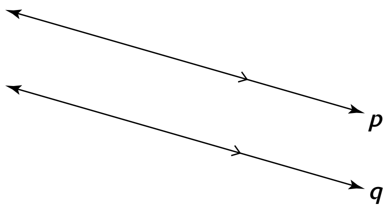


Perpendicular Lines and Segments



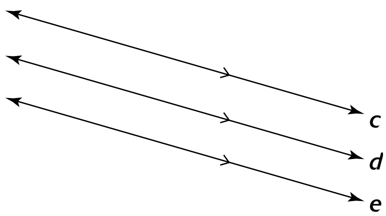
Two intersecting lines that form right angles are known as perpendicular lines. They are denoted by the symbol, ⊥. In the preceding figure, line $l \perp$ line *m*.

Parallel Lines and Segments



Two lines in the same plane that never intersect are called parallel lines. We'll discuss them in detail in the next chapter. The symbol // means is parallel to and is used as a shorthand for parallel lines. So, in this diagram, line *p* // line *q*. Notice that arrow heads mark the lines as being parallel.

Theorem 11: If two lines are parallel to a third line, they are parallel to each other.



Line *c* // line *d*. Line *e* // line *d*. Therefore, by Theorem 11, line *c* // line *e*.

Example Problems

These problems show the answers and solutions.

1. Lines *v* and *w* intersect at a non-right angle. What kind of lines are they?

   ***Answer:* Intersecting lines** Parallel lines never cross, and perpendicular lines cross at a right angle. Only intersecting lines remain.

2. Lines *q* and *r* are in the same plane and never meet. Lines *r* and *s* are in the same plane and never meet. What must be true of lines *q* and *s*?

   ***Answer:* They must be parallel.** By Theorem 11, if two lines are parallel to a third line, they are parallel to each other.

Chapter Problems

Problems

Solve these problems for more practice applying the skills from this chapter. Worked out solutions follow.

1. What do we call the intersection of two planes?
2. Line segment *LM* has endpoints with coordinates 7 and 35. Where is its midpoint?
3. Which geometric form is infinite in length but contains only a single endpoint?
4. How does $\overrightarrow{EF}$ differ from $\overleftarrow{FE}$?
5. How does a segment differ from a ray or a line?
6. An angle is named $\angle BFE$. What three things can we tell about the angle?
7. What is the geometric name for the ray that separates an angle into two smaller angles of equal degree measure?
8. What kind of angle contains more than 90° but less than 180°?
9. How many degrees is the measure of a straight angle?
10. What is the degree measure of the supplement of a 35° angle?
11. What is the degree measure of the complement of a 15° angle?
12. What is the name given to angles that share a vertex and a common side that separates them?
13. Two lines intersect at *M* to form four angles. One of those angles has an angle measure of 55°. What is the measure of its nonadjacent angle?
14. Refer to the description in problem 20. What is the degree measure of the adjacent angle?
15. The angle bisector is drawn inside a straight angle. What is the relationship between the angle bisector and the line it intersects?
16. Two angles are supplementary, and one of them is obtuse. What must be true of the other angle?
17. Two line segments are everywhere an equal distance apart. What can you conclude about them?
18. What is the shortest distance between two points?

Answers and Solutions

1. ***Answer:* A line** Two planes intersect in exactly one line (Postulate 6).

2. ***Answer:* At the point with coordinate 21** Add the coordinates of the endpoints (7 + 35 = 42), and divide by 2 to find their average: $\frac{42}{2} = 21$.

3. ***Answer:* A ray** A ray has a single endpoint and goes on forever away from that endpoint.

4. ***Answer:* It does not.** Both $\overrightarrow{EF}$ and $\overleftarrow{FE}$ have endpoint at *E* with *F* being a point lying toward the arrowhead part of the ray.

5. ***Answer:* A segment has two endpoints and is finite in length. A ray has one endpoint and a line has none. Both line and ray are infinite in length.**

6. ***Answer:* The sides of the angle are formed by $\overrightarrow{FB}$ and $\overrightarrow{FE}$, and its vertex is at *B*.** Remember, the middle letter in the name of an angle is always the name of the vertex, and the common endpoint of both rays forming the sides of the angle.

7. ***Answer:* Angle bisector** Angle bisector is the name for the ray that separates an angle into two angles of equal degree measure.

8. ***Answer:* Obtuse** An obtuse angle contains more than 90° but less than 180°.

9. ***Answer:* 180°** A straight angle consists of two right angles back to back. That's 90° + 90° = 180°.

10. ***Answer:* 145°** Supplementary angles total to 180°. 180° − 35° = 145°.

11. ***Answer:* 75°** Complementary angles total to 90°. 90° − 15° = 75°.

12. ***Answer:* Adjacent angles** Adjacent angles are defined as two angles that share a vertex and a common side that separates them.

13. ***Answer:* 55°** The nonadjacent angle is the vertical angle to the 55° one. By Theorem 7, vertical angles are equal in measure.

14. ***Answer:* 125°** When two lines intersect at a point, all pairs of adjacent angles are supplementary. 180° − 55° = 125°.

15. ***Answer:* Perpendicular** The angle bisector of a straight angle forms two 90° angles. Lines, or in this case the line and the ray, that meet at right angles are perpendicular.

16. ***Answer:* It's acute.** Since an obtuse angle must be greater than 90°, in order for the two angles to sum to 180°, the second must be less than 90°. That's the definition of an acute angle.

17. ***Answer:* They're parallel.** Two segments that are everywhere equidistant is one definition of parallel segments.

18. ***Answer:* A segment** Remember, a straight line has no endpoints, a segment does, so a segment is the shortest distance between two points.

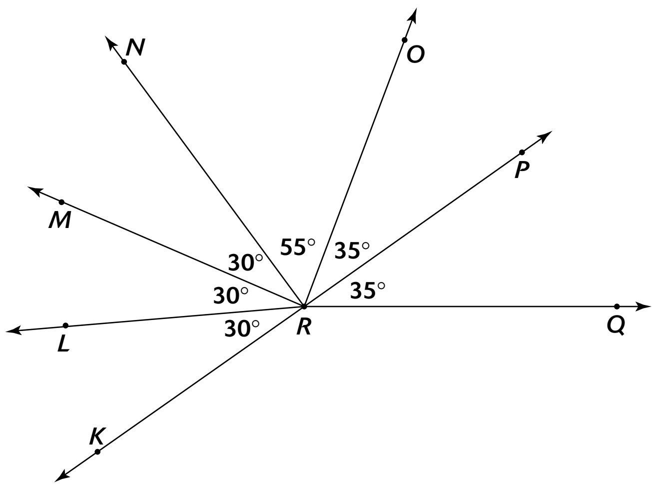
Supplemental Chapter Problems

Problems

Solve these problems for even more practice applying the skills from this chapter. The answer section will direct you to where you need to review.

1. What makes points collinear?
2. How many lines may pass through 2 points?
3. S lies between Q and W on $\overline{QW}$. The length of $\overline{QW}$ is 27. The length of $\overline{QS}$ is 13. Find the length of $\overline{SW}$.
4. $\overline{FG}$ has endpoints with coordinates 12 and 65. Where is its midpoint?
5. What is another name for $\angle JKL$?
6. How does $\overrightarrow{PQ}$ differ from $\overrightarrow{QP}$?
7. What's the name given to the common endpoint of two rays that form an angle?
8. $\overrightarrow{RT}$ is between $\overrightarrow{RS}$ and $\overrightarrow{RU}$ such that $m\angle TRS = m\angle TRU$. What do we call a ray such as $\overrightarrow{RT}$?
9. An angle is its own supplement. What kind of an angle must it be?

The next 3 problems refer to the following figure.



10. Name a right angle.
11. Name a pair of complementary angles.
12. Name four obtuse angles.
13. Two lines intersect at W to form 4 angles. The degree measure of one of those angles is 115°. What is the measure of its nonadjacent angle?

14. Refer to the description in problem 20. What is the measure of its adjacent angle?

15. Line *l* is parallel to line *m*. Line *n* is also parallel to line *m*. What relationship if any exists between lines *l* and *n*?

16. $m\angle DEF = 47°$. What is the degree measure of its complement?

17. Two lines intersect at right angles. What is their relationship to each other?

18. What is the name of two angles that share the same vertex and are separated by a ray with the same vertex?

Answers

1. **They're on the same line.** (Lines , p. 37)

2. **1** (Postulate 3) (Postulates and Theorems, p. 39)

3. **14** (Segment Addition and Midpoint, p. 42)

4. **33.5** (Segment Addition and Midpoint, p. 42)

5. **$\angle LKJ$** (Forming and Naming Angles, p. 45)

6. **They have different endpoints and are completely different rays.** (Rays, p. 43)

7. **Vertex** (Forming and Naming Angles, p. 45)

8. **Angle bisector** **(**Angle Bisector, p. 47)

9. **A right angle** (or a 90° angle) (Supplementary Angles, p. 47)

10. **$\angle PRN$ or $\angle KRN$** (Right Angles, p. 47)

11. **$\angle ORN$ and $\angle ORP$, $\angle KRL$ and $\angle LRN$, and others** (Complementary Angles, p. 51)

12. **Any four of: $\angle QRN$, $\angle MRQ$, $\angle PRM$, $\angle PRL$, $\angle ORL$, $\angle ORK$, lower $\angle KRQ$** (Obtuse Angles, p. 48)

13. **115°** (Vertical Angles, p. 50)

14. **65°** (Supplementary Angles , p. 51)

15. **They're parallel.** (Parallel Lines and Segments, p. 54)

16. **43°** (Complementary Angles, p. 51)

17. **They're perpendicular.** (Perpendicular Lines and Segments, p. 54)

18. **Adjacent angles** (Adjacent Angles, p. 50)