1

Introduction to Sonar

SONAR (SOund NAvigation and Ranging) systems have many similarities to radar and electro-optical systems. The operation of sonar is based on the propagation of waves between a target and a receiver. The two most common types of sonar systems are passive and active. In a passive sonar system, energy originates at a target and propagates to a receiver, analogous to passive infrared detection. In an active sonar system, waves propagate from a transmitter to a target and back to a receiver, analogous to pulse-echo radar. In addition to these two types, there is also daylight or ambient sonar, where the environment is the source of the sound, which bounces off or is blocked by the target, and the effects of which are observed by the receiver. This latter type of sonar is analogous to human sight.

Sonar differs fundamentally from radar and electro-optical systems because the energy observed by sonar is transferred by mechanical vibrations propagating in water, solids, gases, or plasma, as opposed to electromagnetic waves. Today, sonar refers not only to systems that detect and/or transmit sound, but to the science of sound technology as well.

In military applications, sonar systems are used for detection, classification, localization, and tracking of submarines, mines, or surface contacts, as well as for communication, navigation, and identification of obstructions or hazards (e.g., polar ice). In commercial applications, sonar is used in fish finders, medical imaging, material inspection, and seismic exploration.

Figures 1.1, 1.2, and 1.3 illustrate the basic passive, active, and daylight/ambient sonar systems.

1.1 Acoustic Waves

The term "acoustic" refers to sound waves in any medium. Acoustic waves come in two types: longitudinal or compression and transverse or shear. In fluids, only longitudinal or compression waves are supported because fluids lack shear strength. The easiest way to visualize these two types of waves is to consider a Slinky (see Figure 1.4). If the end or middle portion of a Slinky is moved side to side or up and down, a transverse or shear wave will move along it. This method displaces the material of the Slinky in a direction perpendicular to the direction of travel. As the material is moved off the axis, the spring force exerts a restoring force that pulls it back on axis. If several of the Slinky coils are compressed or stretched, then releasing them

Underwater Acoustics: Analysis, Design and Performance of Sonar Richard P. Hodges

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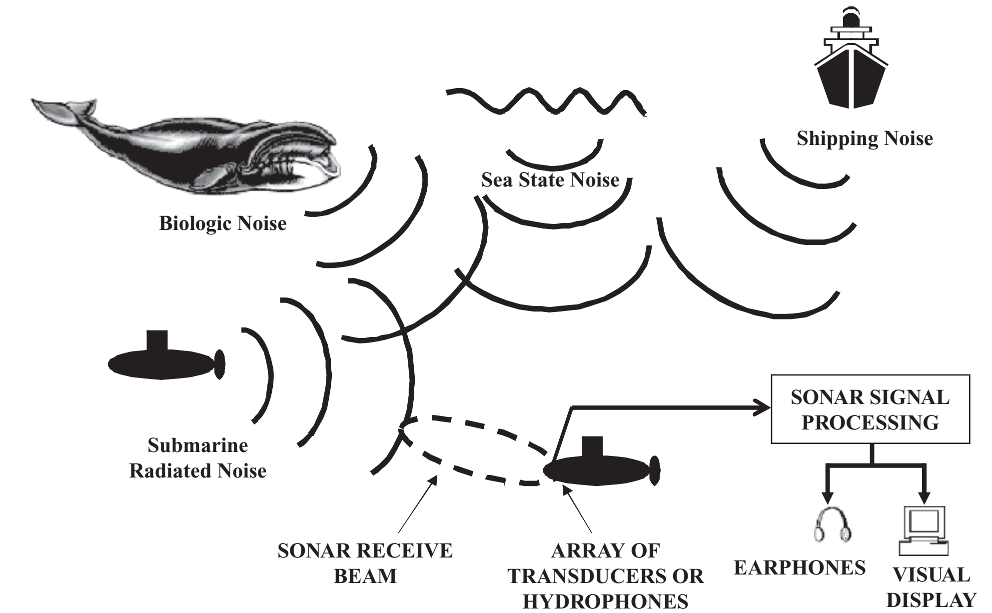

Figure 1.1 Passive sonar system

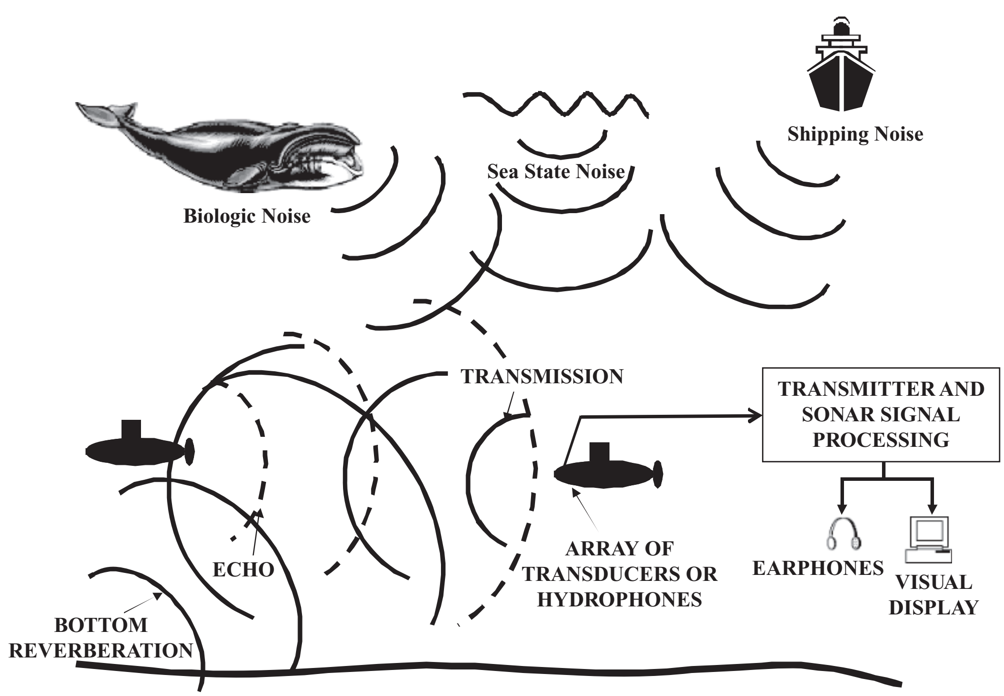

Figure 1.2 Active sonar system

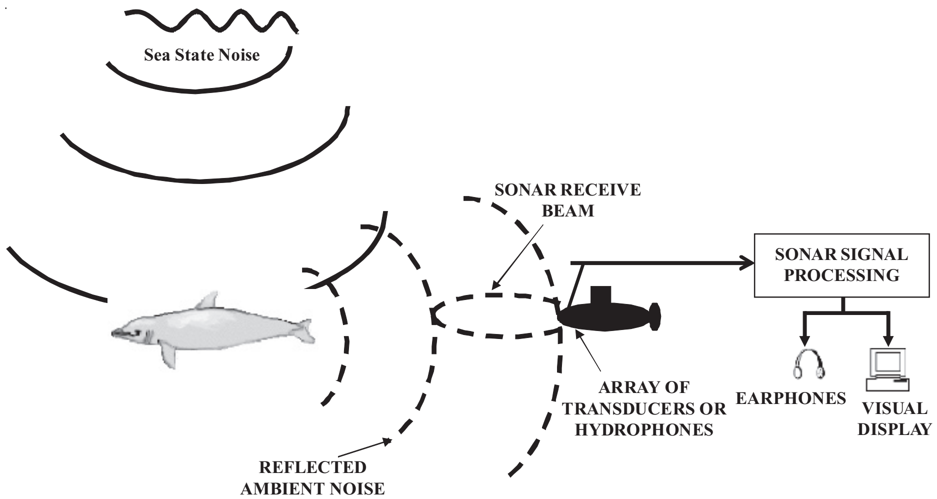

Figure 1.3 Daylight/ambient sonar system

will propagate a longitudinal or compression wave along the Slinky. This method displaces the material of the Slinky along the direction of travel. Again, the restoring force will tend to push the material back into place. In this book, we will deal with transverse or shear waves only occasionally, so unless specifically stated, longitudinal or compression waves are assumed.

1.1.1 Compressions and Rarefactions

Longitudinal waves are composed of compressions, where the parts of the medium (coils of the Slinky) are closer together than normal, and rarefactions, where the parts of the medium are farther apart than normal.

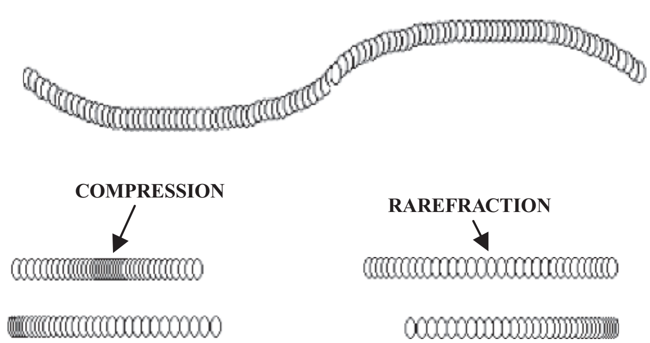

Figure 1.4 Transverse (top) and longitudinal (bottom) waves on a Slinky

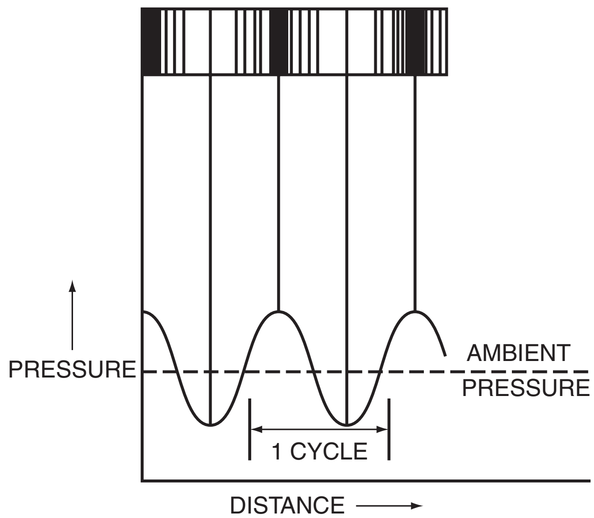

Figure 1.5 Pressure wave

The fundamental parameter of an acoustic wave is pressure. When water or air molecules are pushed or pulled apart, they exert a restoring force that resists the motion. The force will be felt locally as pressure or force per unit area. The amplitude of the wave will be the peak pressure reached in one cycle. The disturbance of the medium that propagates is the distance between molecules. Figure 1.5 illustrates the fundamentals of a pressure wave.

1.2 Speed of Propagation

For a nondispersive medium, one in which different wavelengths propagate at the same phase velocities (e.g., water), we would expect the same type of relationship between wavelength and frequency as with electromagnetic waves:

$$c = \lambda f \tag{1.1}$$

where

λ = wavelength, the distance between corresponding points (peak to peak or valley to valley) on a wave
f = frequency, the number per unit time the wave performs a cycle

The speed of propagation for sound waves is much slower than electromagnetic radiation, on the order of 1500 m/s in water. The speed of propagation is a function of ambient temperature (T), pressure (p) and salinity (S) of the water [1]. Therefore, we can write:

$$c = F(T, p, S) \tag{1.2}$$

Given the complexity of this function, the following rules of thumb apply:

+1 °C change in temperature	= +4.6 m/s at 0.0 °C increase in speed +2.5 m/s at 21.1 °C increase in speed
+100 m of depth increase	= +1.7 m/s increase in speed
+1 ppt (part per thousand) increase in salinity	= +1.4 m/s increase in speed
+1 °F change in temperature	= +8.4 ft/s at 32 °F increase in speed +4.6 ft/s at 70 °F increase in speed
+100 ft of depth increase	= +1.7 ft/s increase in speed
+1 ppt (part per thousand) increase in salinity	= +4 ft/s increase in speed

As these rules show, the greatest variation in speed occurs with changes in temperature. Fluctuations in temperature, by as much as 30 °C, are possible in submarine operational areas. The change in depth required to change the speed of propagation by the same amount is more than 5000 m or 16 000 ft. (Note that large variations in salinity are limited to regions where fresh and salt water mix, e.g., river outflows or under melting sea ice, which are frequently beyond the regions where antisubmarine warfare (ASW) operations take place.)

A simple empirical equation for the speed of sound in sea water, with reasonable accuracy for the world's oceans, is due to Mackenzie [2]:

$$\begin{aligned} c(T, S, D) = {} & A_1 + A_2 T + A_3 T^2 + A_4 T^3 + A_5(S - 35) \\ & + A_6 D + A_7 D^2 + A_8 T(S - 35) + A_9 T D^3 \end{aligned} \tag{1.3}$$

where

T = temperature in degrees Celsius
S = salinity in parts per thousand
D = depth in meters

The constants used are

$A_1 = 1448.96$	$A_4 = 2.374 \times 10^{-4}$	$A_7 = 1.675 \times 10^{-7}$
$A_2 = 4.591$	$A_5 = 1.340$	$A_8 = -1.025 \times 10^{-2}$
$A_3 = -5.304 \times 10^{-2}$	$A_6 = 1.630 \times 10^{-2}$	$A_9 = -7.139 \times 10^{-13}$

Solving this equation gives 1550.74 m/s for $T = 25$ °C, $S = 35$ ‰, $D = 1000$ m. The standard error for salinities between 25 and 40 ppt is 0.070 m/s. Other, far more complicated, equations for sound speed in sea water are accurate over a wider range of conditions (e.g., Del Grosso [3] and Chen and Millero [4]).

Table 1.1 shows sound speeds for select liquids at 1 atmosphere and 25 °C, unless otherwise noted.

1.3 Acoustic Wave Parameters

The two fundamental parameters of an acoustic wave are frequency and amplitude, a pressure measured in units of force per area. The System International (SI) unit of pressure is the Pascal

Table 1.1 Sound speeds for select liquids

Liquid	Sound velocity (m/s)	Liquid	Sound velocity (m/s)	Liquid	Sound velocity (m/s)
Acetic acid	1584	Chloroform	995	Phenol	1274
Acetone	1174	Ether	985	Toluene	1275
Alcohol, ethyl (ethanol)	1144	Ethylene Glycol	1644	Turpentine	1240
Alcohol, methyl	1205	Glycerol (Glycerine)	1904	Water	1493
Alcohol, propyl	1205	Heptane	1138	Water, 0 °C	1402
Benzene	1298	Hexane	1203	Water, 20 °C	1482
Carbon disulfide	1149	Kerosene	1324	Water, sea	1533
Carbon tetrachloride	926	Mercury	1450	Water, sea, 20 °C	1522
Castor oil	1474	Octane	1171		

(Pa), where 1 Pa = 1 N/m^2. For those readers not comfortable with this unit, Table 1.2 gives the most commonly encountered units of pressure and their equivalents.

As anyone who has flown in an airplane knows, changes of a fraction of an atmosphere can be quite painful. The human ear is capable of hearing sound with pressure changes as small as 2×10^{-10} atmospheres. This value was used in older literature as a reference pressure for measurements, 0.0002 μbar.

In general, longitudinal waves are the most important type of waves in acoustics, particularly when discussing the underwater environment. Consequently, the analysis in this book will be specifically for this type of wave. When a simple harmonic wave propagates, the magnitude of the acoustic disturbance varies sinusoidally in time at every place in the medium. The spatial distribution of the disturbance at any fixed time is also sinusoidal.

The surface joining regions within the medium undergoing the same amount of perturbation during the same compression or rarefaction cycle is known as a wave front. The shape of the wave front enables the classification of acoustic waves to be subdivided further. Waves generated in a homogeneous medium, from a point source that is very small compared to the wavelength, propagate with spherical symmetry. These wave fronts are spherical in shape and are known as spherical waves. If instead the medium is bounded by two parallel planes (as in the case of the sea), waves generated by a point source will eventually spread with circular symmetry only in the horizontal plane. These wave fronts will be cylinders and are known as

Table 1.2 Common units of pressure

Normal atmospheric pressure atmospheres (atm)	= 1.01325 bar
	= 1.01325 $\times 10^5$ Pascal (Pa)
	= 1.01325 $\times$ 10^{10} dynes/m^2
	= 1.01325 $\times$ 10^6 dynes/cm^2
	= 14.6960 pounds/in^2 (psi)
	= 29.9213 inches of Hg
	= 760 mm of Hg (torr)
	= 406.8 inches of water

cylindrical waves. If the source is an infinite plane surface, the resulting wave fronts are also a plane. No spreading occurs and the waves are known as plane waves. Although plane waves cannot be generated in practice, both spherical and cylindrical waves approximate plane waves when they are sufficiently far from their source.

An acoustic pressure wave applies a stress to successive elements of the medium through which it propagates. The resulting particle motion in each element is determined by the mechanical properties of the medium, i.e., its elastic modulus describing the difficulty with which it is compressed and its density (ρ).

In a solid, the elastic modulus is frequently dependent upon the orientation of the medium relative to the acoustic wave. In a completely anisotropic solid, 21 constants are required to specify completely the stress–strain relationship. Being isotropic, fluids and gases require only one elastic constant, compressibility, s (m^2/N), defined as the volumetric strain produced per unit of applied stress:

$$s = \frac{\Delta v / v_0}{p} \tag{1.4}$$

where Δv is the change in the original volume, v_0, caused by the application of a pressure, p. The reciprocal of compressibility is known as the volume elasticity or bulk modulus, κ, and is usually used in acoustic expressions instead of compressibility:

$$\kappa = \frac{1}{s} = \frac{p}{\Delta v / v_0} \tag{1.5}$$

For the following discussion, we will assume that the bulk modulus is constant. This is essentially true for low-amplitude acoustic waves. As an acoustic wave moves though a medium it can be characterized by certain parameters that vary periodically with both time and space.

Particle displacement, ξ, is the amount of displacement of a particle from its mean position within the medium under the action of the acoustic pressure.

Particle velocity, u, is the velocity of a particle in the medium, given by the time derivative of the particle displacement:

$$u = \frac{\mathrm{d}\xi}{\mathrm{d}t} \tag{1.6}$$

Particle acceleration, a, is the time derivative of the particle velocity:

$$a = \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} \tag{1.7}$$

Acoustic or excess pressure, p, is the change in pressure from the mean value. It is the difference between the instantaneous pressure, P, and the ambient pressure, P_0:

$$p = P - P_0 \tag{1.8}$$

Condensation, S, is the fractional change in density resulting from the acoustic pressure:

$$S = \frac{\rho - \rho_0}{\rho_0} \tag{1.9}$$

where ρ and ρ_0 are the instantaneous and mean densities respectively. Since mass must be conserved, this can also be written in terms of volume as

$$S = \frac{\Delta v}{v_0} \tag{1.10}$$

Combining these last two equations gives an equation relating bulk modulus and excess pressure:

$$p = \kappa S \tag{1.11}$$

Propagation speed, c, is the speed with which the acoustic wave passes through a fluid medium. It is determined by the mechanical properties of the medium, by

$$c = \sqrt{\frac{\kappa}{\rho_0}} \tag{1.12}$$

The derivation of this equation assumes that the propagation takes place at a constant temperature, i.e., isothermal. In reality, the temperature rises during compression and drops during rarefaction. In general, the actual temperature gradient formed is small if the pressure changes are small (this would not be true for a shock wave). This, combined with the short duration between compressions and rarefactions is usually insufficient for any significant heat flow to occur. As a result, a better assumption is that the propagation is an adiabatic process. A more general form for this is

$$c = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho}} \tag{1.13}$$

For isothermal and adiabatic conditions, the excess pressure and density are related by:

$$p = A\left(\rho - \rho_0\right) \tag{1.14}$$

and

$$P = A'\left(\rho - \rho_0\right)^{\gamma} \tag{1.15}$$

respectively, where γ is the ratio of specific heats, C_p and C_v, of the medium, measured under constant pressure and constant volume respectively. The isothermal and adiabatic propagation speeds are

$$\begin{aligned} c_{\mathrm{is}}^2 &= A = \frac{p}{(\rho - \rho_0)} \\ c_{\mathrm{ad}}^2 &= A'\gamma\ (\rho - \rho_0)^{\gamma-1} = \gamma \frac{p}{(\rho - \rho_0)} \end{aligned} \tag{1.16}$$

or

$$c_{\mathrm{ad}}^2 = \gamma c_{\mathrm{is}}^2 = \gamma \frac{\kappa}{\rho} \tag{1.17}$$

For sea water at 13 °C, $\gamma = 1.01$, $\kappa = 2.28 \times 10^9$ Pa (N/m^2), $\rho = 1026$ kg/m^3, and the speed of propagation is 1498 m/s. This value assumes one atmosphere (at the surface) and a salinity of 35 ppt. Note that the calculation assuming isothermal speeds would have been 1491 m/s or an error of about 0.5 %.

1.4 Doppler Shift

If a sound source is moving relative to the medium with a component of v_s towards a fixed receiver, the observed frequency must change. At the receiver, the observed frequency (f_r) is given by

$$f_{\mathrm{r}} = f_{\mathrm{s}} \frac{c}{c - v_{\mathrm{s}}} \tag{1.18}$$

where

c = speed of sound in the medium
f_s = source frequency
v_s = source relative speed (positive if toward the receiver)

Similarly, if the receiver is moving with a component, v_r, relative to the source, the frequency is altered:

$$f_{\mathrm{r}} = f_{\mathrm{s}} \frac{c - v_{\mathrm{r}}}{c} \tag{1.19}$$

In general, if both the source and medium are moving, the received frequency is given by

$$f_{\mathrm{r}} = f_{\mathrm{s}} \frac{c - v_{\mathrm{r}}}{c - v_{\mathrm{s}}} \tag{1.20}$$

The frequency change resulting from relative motion of the source and receiver is known as the Doppler shift. The change in frequency is defined as

$$\Delta f = f_{\mathrm{s}} \frac{v_{\mathrm{s}} - v_{\mathrm{r}}}{c - v_{\mathrm{s}}} \tag{1.21}$$

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Frequently, the speeds are very small relative to the speed of sound, therefore

$$\Delta f = f_s \frac{\Delta v}{c} \tag{1.22}$$

where Δv is the combined relative speed component (positive for closing). If frequency is in kHz, Δv is in knots, and Δf is in Hz, then the change in frequency in the ocean is defined as

$$\Delta f(\text{ocean}) \cong 0.35 f_s \Delta v \tag{1.23}$$

The change in frequency in air is about five times larger because of the lower speed of sound:

$$\Delta f(\text{air}) \cong 1.7 f_s \Delta v \tag{1.24}$$

A special case of interest is the Doppler shift as observed by a monostatic active sonar (Figure 1.2), where the signal has twice the shift (out and return):

$$\Delta f(\text{monostatic active sonar in ocean}) \cong 0.7 f_s \Delta v \tag{1.25}$$

1.5 Intensity, SPL, and Decibels

The energy flow in an acoustic wave is similar to that of radar and electro-optics. The power per unit area in an acoustic wave, referred to as intensity, I, varies as the square of the pressure. This relationship is written as

$$I \propto p^2$$

For the purpose of this discussion, we will use a term called sound pressure level, or SPL, which is defined as

$$\text{SPL} = 20 \log \left(\frac{p}{p_0} \right) \tag{1.26}$$

where p_0 is the reference pressure, frequently identified as 1 μPa, 1 μbar, and 0.0002 μbar (see Table 1.2). Here, the units are decibels (dB). Therefore, the exact coefficients of the proportionality are irrelevant, since SPL is the log of a ratio.

It should be noted that a factor of 20 is used because intensity is proportional to the square of the pressure and 10 log of power is the decibels. Decibels express a ratio of powers, which in this case is proportional to the square of pressure. Using the properties of logarithms, the exponent is brought down in front and multiplies the normal factor of 10. For example, 10 log (x^2) = 20 log (x). This is a very important lesson to understand. Too often decibels are manipulated without truly being understood.

If a reference pressure is not stated, then the SPL is **absolutely meaningless**. For example, if an SPL is assumed to have a reference pressure of 1 μPa and it is actually 1 μbar, the numerical value will be off by 120 dB or one has missed the power by a multiplicative factor of 1 trillion. What may have been thought of as 1 milliwatts could actually be 10 megawatts! The problem could be even worse if the focus was on not only a single frequency but power over a band of frequencies or because sometimes measurements are made at a distance (1 yd, 1 m, 100 yd, etc.).

A true description of SPL must include: (1) the reference pressure (or sometimes power density, e.g., W/cm^2), (2) the frequency range over which the power is measured (e.g., 1 Hz or one-third of an octave), and (3) in the case of signals radiated from a source, the reference range being used (e.g., 1 yd). Surprisingly, most people do not know that an octave is a factor of 2 in frequency. In music, middle C (the center of a piano keyboard) is about 261.6 Hz, with each note above it increasing by a twelfth of an octave, a factor of $2^{1/12}$. Seen as a geometric progression, C sharp, which is also D flat, is about 277.2 Hz and C flat is 246.9 Hz, and so forth. Middle A is actually the reference at 440.0 Hz. The third of an octave mentioned above is a factor of $2^{1/3}$ or 1.2599. In other words, the frequency band from 1000 to 1260 Hz is roughly a one-third of the octave band, and would be from 100 to 126 Hz. Another common bandwidth is a tenth decade of $10^{1/10}$, which is a factor of 1.2589 and is so close to the one-third octave that many analysts use the two interchangeably.

Decibels come in two forms: the first form, just discussed, which represents a power density, and the second form, which represents a gain or loss. This latter form is dimensionless. For example, a linear amplifier might provide 20 dB gain; i.e., the signal provided within the linear range of the amplifier will be raised by 20 dB (in power, a factor of 100, or in voltage, a factor of 10). As a result, the value is not dependent on what units are being used for expressing the input level.

Acoustic analysis is done in decibels because it is a convenient way to handle the wide range of values and because the addition of decibels is equivalent to the multiplication of the underlying quantities. For example, the human ear is capable of hearing sound with pressure changes as small as 2×10^{-10} atmospheres. The level at which sound becomes painful is about 120 dB higher. Thus, the human ear has an enormous dynamic range of about 10^{12} in power.

1.6 Combining Acoustic Waves

Consider two acoustic waves arriving at some point in a medium. To compute the combined intensity we must superimpose the signals. At each instant in time, the pressures will add. Therefore, in order to compute the intensity we must sum the pressures and average over time. There are two ways that these signals can combine: coherently and incoherently, the latter sometimes referred to as power addition. In reality, signals always combine coherently in a medium in the sense that the pressure signatures sum. However, averaged over time, the two signals might or might not maintain coherence that is in phase for a sine wave.

Let us start with two sine waves with the same frequency, amplitude and phase, as stated below:

$$S_1 = A \sin(\omega t)$$
$$S_2 = A \sin(\omega t)$$

The average power per unit area, at the reference distance (R) of each individual signal is

$$\text{Power}_1 = \text{Power}_2 = \frac{A^2}{2\rho c R^2} \tag{1.27}$$

$$\text{Power(surface)} = \frac{4\pi R^2 A^2}{2\rho c R^2} = \frac{2\pi A^2}{\rho c} \tag{1.28}$$

When these are added:

$$S_c = S_1 + S_2 = 2A\sin(\omega t) \tag{1.29}$$

the average power per unit area (W) at a distance R is

$$W = \frac{\int_0^{2\pi/\omega} \frac{[2A\sin(\omega t)]^2}{\rho c R^2}\,\mathrm{d}t}{\int_0^{2\pi/\omega} 1\,\mathrm{d}t} = \frac{2A^2}{\rho c R^2} \tag{1.30}$$

Note that this is four times the power per unit area of each signal individually. If we were to assume that this happens over the surface of a sphere of radius R then we get

$$W(\text{over sphere?}) = 4\pi R^2 \frac{2A^2}{\rho c R^2} = \frac{8\pi A^2}{\rho c} \tag{1.31}$$

Clearly this cannot be the case because energy must be conserved, so what went wrong?

Consider two point sources, one radiating S_1 above and the other S_2, separated by a distance d. The pressure arriving from each (P_1 and P_2) at a point (x, y, z) is given by

$$\begin{aligned} P_1 &= \frac{A\sin(\omega t + \Delta_1)}{\sqrt{x^2 + y^2 + (z - d/2)^2}} \\ \Delta_1 &= \frac{\sqrt{x^2 + y^2 + (z - d/2)^2}}{c} \\ P_2 &= \frac{A\sin(\omega t + \Delta_2)}{\sqrt{x^2 + y^2 + (z + d/2)^2}} \\ \Delta_2 &= \frac{\sqrt{x^2 + y^2 + (z + d/2)^2}}{c} \end{aligned} \tag{1.32}$$

Assuming the distances to the spherical surface from each source is very large compared to d, the distances can be replaced by a single value R. Summing the pressure at that point yields

$$P_1 + P_2 = \frac{A}{R}\{\sin[\omega(t + \Delta_1)] + \sin[\omega(t + \Delta_2)]\} \tag{1.33}$$

The average power of the sum is then given by

$$\text{Power(average)} = \frac{\int_0^{2\pi/\omega} \frac{A^2}{\rho c R^2} \{\sin[\omega(t+\Delta_1)] + \sin[\omega(t+\Delta_2)]\}^2 \, \mathrm{d}t}{\int_0^{2\pi/\omega} 1 \, \mathrm{d}t} \tag{1.34}$$

Converting to polar coordinates:

$$\begin{aligned} x &= R\sin(\theta)\sin(\phi) \\ y &= R\cos(\theta)\sin(\phi) \\ z &= R\cos(\phi) \end{aligned}$$

gives

$$\begin{aligned} \Delta_1 c &= \sqrt{R^2[\sin(\phi)]^2 + R^2[\cos(\phi)]^2 - Rd\cos(\phi) + d^2/4} \\ \Delta_2 c &= \sqrt{R^2[\sin(\phi)]^2 + R^2[\cos(\phi)]^2 + Rd\cos(\phi) + d^2/4} \end{aligned} \tag{1.35}$$

Since $R \gg d$:

$$\begin{aligned} \Delta_1 &\cong \frac{R}{c}\left[1 - \frac{d\cos(\phi)}{2R}\right] \\ \Delta_2 &\cong \frac{R}{c}\left[1 + \frac{d\cos(\phi)}{2R}\right] \end{aligned} \tag{1.36}$$

Substituting and simplifying yields

$$\begin{aligned} \text{Power (average)} &= \frac{\int_0^{2\pi/\omega} \frac{A^2}{\rho c R^2}\left[4\sin(\omega t)^2 \cos\left(\frac{\omega \mathrm{d}\cos(\phi)}{2c}\right)^2\right] \mathrm{d}t}{\int_0^{2\pi/\omega} 1 \, \mathrm{d}t} \\ &= \frac{2A^2\cos\left(\frac{\omega \mathrm{d}\cos(\phi)}{2c}\right)^2}{\rho c R^2} \end{aligned} \tag{1.37}$$

Integrating over the whole surface yields

$$\text{Power(surface)} = \int_0^{\pi}\int_0^{2\pi} \frac{2A^2\left[\cos\left(\frac{\omega \mathrm{d}\cos(\phi)}{2c}\right)\right]^2}{\rho c R^2} R^2 \sin(\phi)\, \mathrm{d}\theta\, \mathrm{d}\phi \tag{1.38}$$

$$\text{Power(surface)} = \frac{4\pi A^2}{\rho c}\left[1 + \frac{\sin\left(\frac{2\pi \mathrm{d}}{\lambda}\right)}{\left(\frac{2\pi}{\lambda}\right)}\right] \tag{1.39}$$

Since $\omega = 2\pi f$ and $\lambda = c/f$.

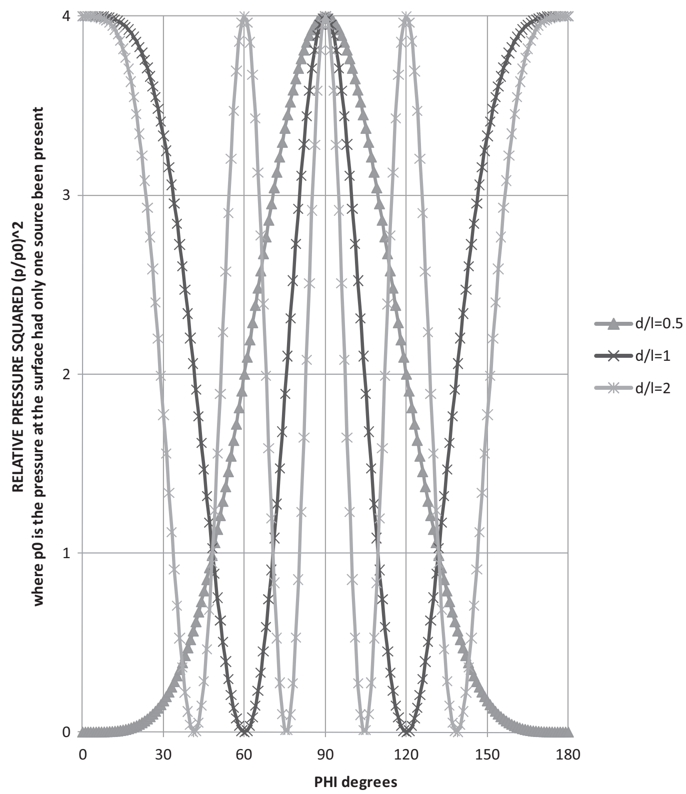

Figure 1.6 Power versus conical angle for two separated equal intensity in-phase sources

The first term is the sum of the powers of the two sources individually, as expected. The resolution to the possible violation of the conservation of energy is simply when two sources combine; the power per unit area will vary from zero (out of phase) to four times the individual power density (in phase).

The second term is the near field effect. If the two sources are very close together, more power is required to make each source put out the desired level because of the pressure field generated by the other source. This term disappears if the separation, d, is an integer multiple of half the wavelength. This term becomes very small if $d \gg \lambda$, such as when the interaction between the sources becomes very small. Figure 1.6 shows the power per unit area for a north–south oriented dipole located at the center of a sphere as a function of the angle from the pole.

1.7 Comparative Parameter for Sound in Water and Air

Table 1.3 shows select sound levels of interest for water. Table 1.4 shows similar levels and corresponding parameters of air.

Table 1.3 Select sound pressure levels and associated parameters for water (impedance of sea water = $\rho c = 1.5 \times 10^6$ kg/m^2 s)

Sound level	Intensity I(W/m^2)	Pressure rms p(N/m^2)	Particle velocity u(m/s)	Particle displacement u/ω (m) at 3 kHz
Sea state 3 (50 dB re: 1 μPa)	6.7×10^{-14}	3.2×10^{-4}	2.1×10^{-10}	1.1×10^{-14}
Active source (220 dB re: 1 μPa)	6.7×10^{3}	1.0×10^{5}	6.7×10^{-2}	3.5×10^{-6}

Note. The pressure for active source is 1 atmosphere.

Table 1.4 Select sound pressure levels and associated parameters for air (impedance of sea water = $\rho c = 415$ kg/m^2 s)

Sound level	Intensity I(W/m^2)	Pressure rms p(N/m^2)	Particle velocity u(m/sec)	Particle displacement u/ω (m) at 440 Hz
Threshold of human hearing (26 dB re: 1 μPa)	9.6×10^{-13}	2.0×10^{-5}	4.8×10^{-5}	1.7×10^{-11}
Normal conversation (106 dB re: 1 μPa)	9.6×10^{-7}	2.0×10^{-2}	4.8×10^{-5}	1.7×10^{-8}
Rock band / threshold of pain (146 dB re: 1 μPa)	1	2.0×10^{1}	4.8×10^{-2}	1.7×10^{-3}

Note. Human hearing works over a vast power range, 10^{12}. The particle displacement at the threshold of human hearing at middle C is about 6 % of the diameter of a hydrogen molecule, so the ear is able to detect tiny movements.

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P1: OTA/XYZ P2: ABC
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