

# Index

- A posteriori learning
  - algorithms, 129, 288
  - definitions, 287
  - error adaptation, 87, 129
  - geometric interpretation, 288
- Activation functions
  - adaptive amplitude, 120
  - complex nonlinearity, 9, 16, 29, 214
    - singularities, 9, 54
  - elementary transcendental functions (ETF), 51, 52, 82, 89, 98, 194, 259
  - fully complex, 9, 51, 54, 100, 102, 197, 259, 260, 267
  - real valued activation functions
    - hyperbolic tangent, 83, 257
    - logistic sigmoid, 102, 257
  - split complex, 9, 49, 52, 54, 102, 214, 260
- Adaptive learning, 47, 55, 266
- Adaptive systems, 13, 266, 274
- Arithmetic operations, 246
- Asymptotic stability, 77
  - attractive* fixed point, 301
  - definition, 292
  - global, 292, 294
  - uniform, 292
- Attractors
  - lorenz, 178
- Augmented statistics, *see* Complex covariance matrix, 161
- Autonomous systems, 295, 296
  - definition, 291
- Autoregressive (AR) models
  - circularly symmetric, 168
  - complex linear AR(4), 165
  - nonlinear AR (NAR), 33
  - widely linear, 174
  - Yule Walker solution, 170
- Autoregressive moving average (ARMA)
  - models, 33, 34, 91, 92, 234
  - nonlinear AR moving average (NARMA), 33
  - widely linear (WL-ARMA), 191
- Averaging, 88, 216
  - wind signal, 233, 243
- Backpropagation
  - fully complex algorithm, 9
- Batch learning, 178, 266
- Bias/variance dilemma, 266
- Bilinear transformation, *see* Möbius transformation, 140
- Bivariate, *see* Complex signals, 233
- Cauchy–Riemann equations, 14, 44, 55, 56, 81, 94, 98, 283
  - Cauchy integral, 251
- Channel equalisation, 9, 27, 71
- Circularity, *see* Complex circularity, 153
- Clifford algebra, 8, 53, 255
- Collaborative filtering, 217
  - hybrid filtering, 208, 210
  - online test for complex circularity, 220
- Complex circularity, 29, 153, 183
  - examples, 165
  - properties, 158
  - random variables, 153
  - second order, 164

- Complex covariance matrix, 90, 151, 154, 171, 199, 202, 249
  - augmented complex statistics, 59, 151, 161, 191, 203
  - pseudocovariance, 90, 151, 153, 161, 171, 173, 233
- Complex least mean square (CLMS) algorithm, 9, 69, 73, 102, 231
  - augmented CLMS (ACLMS), 175, 218
    - weight updates, 187
  - data-reusing form, 129
  - weight updates, 66, 75, 278
  - widely linear complex filter, 187
- Complex matrix differentiation, 249
- Complex multilayer perceptron (CMLP), 9
- Complex nonlinear gradient descent (CNGD) algorithm, 80, 87, 212
  - data-reusing form, 131
  - gradient adaptive stepsize algorithms, 107
  - normalised CNGD, 87, 110, 122, 212
- Complex nonlinearity, *see* Activation functions, 190
- Complex numbers
  - sign* function, 246
  - basic arithmetic, 246
  - complex conjugate, 57, 194, 247
  - complex mean, 246
  - complex random variables, 152
  - higher dimension algebras, 254
  - history, 2
  - matrix representation, 138
  - ordering of numbers, 152, 245
- Complex random variables, 63, 152, 201
- Complex signals, 159
  - 50 Hz wind data, 113
  - bivariate, 10, 233, 240
    - normal distribution, 155
    - wind data, 243
  - cross-multicorrelations, 160
  - dual univariate, 10
  - IPIX radar, 113
  - multicorrelations, 159
  - second order structure, 161
- Complex white noise, 76, 152, 172
  - doubly white, 34, 102, 113
- Constructive learning, 266
- Continuous complex functions, 55, 248
- Contraction mapping theorem, 299
- Data-reusing, 129
  - contractive, 129
- Delay vector variance (DVV), 237
  - complex case, 240
- Deterministic vs stochastic (DVS) plots, 234, 237
- Discrete cosine transform (DCT), 221
- Dual channel
  - adaptive filters, 183, 185
  - DCRLMS algorithm, 185
- Electroencephalogram (EEG), 26
- Embedding dimension, 237, 240
- Empirical mode decomposition (EMD)
  - algorithm, 221
  - as a fixed point operation, 222
  - bivariate EMD (BEMD), 228
  - complex EMD, 227
  - rotation invariant EMD (RIEMD), 227
  - sifting algorithm, 222
- Equilibrium point, 292
- Error criterion
  - deterministic, 72, 170
  - stochastic, 170
- Error function, 265
- Exponential stable, 292
- Extended Kalman filter (EKF), 9
  - augmented complex EKF algorithm (ACEKF), 200
- Feedforward network
  - definition, 273
  - multilayer, 36
- Finite impulse response (FIR) filter, 33, 70
  - learning algorithm, 183, 279
  - nonlinear filter, 33, 107
- Fixed point
  - Brower's theorem, 299
  - iteration, 75, 133, 223
- Forgetting factor, 277
- Fourier transform, 160, 162, 163, 236
  - DFT, 27
  - FFT, 221
  - inverse, 20, 227, 236
  - spectrum, 236
- Fractals and Mandelbrot set, 308
  - complex iterated maps, 28
  - theory of fractals, 145
- Frequency domain, 161
- Frobenius matrix, 295

- Fully complex, *see* Activation functions, 260
- Function definitions  
 bounded, 9, 47, 49, 81, 84, 259, 262  
 conformal, 142  
 differentiable, 9, 29, 47, 84, 194, 262  
 holomorphic, 44, 94, 142, 194, 248  
 meromorphic, 129
- Gaussian complex RVs, 158  
 circular complex noise, 168  
 complex circular, 153  
 complex model, 10, 151
- Generalised normalised gradient descent (GNGD) algorithm, 209, 279, 281  
 complex GNGD (CGNGD), 110  
 nonlinear filter case (CFANNGD), 111
- Gradient adaptive stepsize algorithms  
*see* Complex nonlinear gradient descent (CNGD), 107
- Hilbert transform, 15
- Hybrid filters, *see* Collaborative filtering, 207
- Improper random vectors, 10, 29, 151, 159, 162, 169, 249  
 examples, 168  
 noncircular, 164
- Infinite impulse response (IIR) filter, 34, 91
- Intrinsic mode functions (IMFs), 221
- Kolmogorov function, 269
- Kolmogorov's theorem, 38, 270
- Kolmogorov–Smirnov test, 242
- Kolmogorov–Sprecher's theorem, 270
- Least mean square (LMS) algorithm  
 dual channel real LMS (DCRLMS), 185  
 dual univariate LMS (DULMS), 188  
 gradient adaptive step size (GASS), 279  
 hybrid filters, 208  
 normalised LMS (NLMS), 107, 279
- Linear prediction, 27, 120
- Linear regression, 152, 169  
 widely linear regression, 175
- Liouville's theorem, 9, 48, 208, 251, 259  
 singularities, 208, 259
- Lipschitz function, 270, 291  
 Lipschitz constant, 300  
 Lipschitz continuity, 224, 299  
 Lipschitz continuous mapping, 299
- Lorenz equation, *see* attractors, 178
- Lyapunov's second method, 291
- Möbius transformation, 18, 140  
 all pass systems, 146  
 properties, 140
- Minimum mean square error (MMSE), 72, 73, 173  
 convergence, 77
- Modular group, 144, 145  
 nesting, 145  
 CPRNN architecture, 275
- Monotonic function, 222, 270
- Neural networks  
 homomorphic, 268  
 hypercomplex, 10, 53  
 multivalued neurons (MVN), 27  
 ontogenic, 266, 273  
 terminology, 273
- Neuron  
 clamping functions, 273
- Noise cancellation, 26, 70
- Null hypothesis, 235
- Pattern learning, 266
- Phase space, 237
- Polar coordinates, 16, 153, 247  
 wind representation, 22
- Power spectrum, 20, 80, 163, 172, 188
- Prediction gain, 102, 113, 124, 178, 188
- Probability density functions, 10, 151, 152, 155, 158, 245, 246
- Proper random vectors, 162, 169  
 circular, 164  
 second order circular, 151
- Properties of functions in  $\mathbb{C}$   
 differentiable, 56, 62  
 holomorphic, 57
- Pseudocovariance, *see* Complex covariance matrix, 246
- Quaternions, 7  
 algebra, 255  
 conjugate, 138  
 matrix representation, 137
- MLPs, 10
- Real time recurrent learning (RTRL)  
 adaptive amplitude CRTRL (AACRTRL), 122

- Real time recurrent learning (RTRL)  
*(Continued)*  
 augmented complex valued RTRL  
 (ACRTRL) algorithm, 197  
 complex RTRL (CRTRL) algorithm, 99, 275  
 data-reusing CRTRL, 134
- Recurrent neural networks (RNNs)  
 complex valued pipelined (CPRNN), 275
- Recursive algorithm, 91, 192, 267
- Regularisation factor, 124, 281
- Signal modality characterisation, 207, 211,  
 233  
 signal nonlinearity, 207, 216  
 statistical testing, 235
- Signal nonlinearity, *see* Signal modality  
 characterisation, 207
- Singularities, *see* Liouville's theorem,  
 Activation functions, 259
- Spectral matrix, 163, 164  
 components, 15  
 covariance, 172  
 pseudocovariance, 172, 188
- Split complex nonlinearity, *see* Activation  
 functions, 260
- State space representation, 37, 39, 297  
 Ikeda map, 240
- Stochastic gradient learning, 10, 122, 171, 184,  
 210
- Stochastic matrix, 296
- Stochastic models, 34
- Supervised learning, 220, 267, 273
- Surrogate dataset, 20, 236  
 bivariate iAAFT, 239  
 complex iAAFT, 240  
 iterative Amplitude Adjusted Fourier  
 Transform (iAAFFT), 236
- Unscented Kalman filter (UKF)  
 augmented complex UKF algorithm  
 (ACUKF), 200
- Unsupervised learning, 273
- Vanishing gradient, 275
- Vector and matrix  
 differentiation, 263  
 norm, 293
- Volterra system, 38, 207  
 filters, 208
- Weierstrass theorem, 270
- Weighted sum, 15, 147, 277
- Wide sense stationary (WSS), 162, 168,  
 172
- Widely linear, 171  
 autoregressive model, 174, 191  
 ACLMS algorithm, 220  
 adaptive filters, 10, 185, 187, 194  
 benefits, 175  
 estimator, 174  
 models, 139, 169, 171, 218, 233
- Wiener filter, 71, 152, 171
- Wind data analysis, 228
- Wold decomposition, 234