

Preface

Copula functions represent a methodology which has recently become the most significant new tool to handle in a flexible way the comovement between markets, risk factors and other relevant variables studied in finance. While the tool is borrowed from the theory of statistics, it has been gathering more and more popularity both among academics and practitioners in the field of finance principally because of the huge increase of volatility and erratic behavior of financial markets. These new developments have caused standard tools of financial mathematics, such as the Black and Scholes formula, to become suddenly obsolete. The reason has to be traced back to the overwhelming evidence of non-normality of the probability distribution of financial assets returns, which has become popular well beyond the academia and in the dealing rooms. Maybe for this reason, and these new environments, non-normality has been described using curious terms such as the “smile effect”, which traders now commonly use to define strategies, and the “fat-tails” problem, which is the major topic of debate among risk managers and regulators. The result is that nowadays no one would dare to address any financial or statistical problem connected to financial markets without taking care of the issue of departures from normality.

For one-dimensional problems many effective answers have been given, both in the field of pricing and risk measurement, even though no model has emerged as the heir of the traditional standard models of the Gaussian world.

On top of that, people in the field have now begun to realize that abandoning the normality assumption for multidimensional problems was a much more involved issue. The multidimensional extension of the techniques devised at the univariate level has also grown all the more as a necessity in the market practice. On the one hand, the massive use of derivatives in asset management, in particular from hedge funds, has made the non-normality of returns an investment tool, rather than a mere statistical problem: using non-linear derivatives any hedge fund can design an appropriate probability distribution for any market. As a counterpart, it has the problem of determining the joint probability distribution of those exposures to such markets and risk factors. On the other hand, the need to reach effective diversification has led to new investment products, bound to exploit the credit risk features of the assets. It is particularly for the evaluation of these new products, such as securitized assets (asset-backed securities, such as CDO and the like) and basket credit derivatives (n th to default options) that the need to account for comovement among non-normally distributed variables has become an unavoidable task.

Copula functions have been first applied to the solution of these problems, and have been later applied to the multidimensional non-normality problem throughout all the fields

in mathematical finance. In fact, the use of copula functions enables the task of specifying the marginal distributions to be decoupled from the dependence structure of variables. This allows us to exploit univariate techniques at the first step, and is directly linked to non-parametric dependence measures at the second step. This avoids the flaws of linear correlation that have, by now, become well known.

This book is an introduction to the use of copula functions from the viewpoint of mathematical finance applications. Our method intends to explain copulas by means of applications to major topics such as asset pricing, risk management and credit risk analysis. Our target is to enable the readers to devise their own applications, following the strategies illustrated throughout the book. In the text we concentrate all the information concerning mathematics, statistics and finance that one needs to build an application to a financial problem. Examples of applications include the pricing of multivariate derivatives and exotic contracts (basket, rainbow, barrier options and so on), as well as risk-management applications. Beyond that, references to financial topics and market data are pervasively present throughout the book, to make the mathematical and statistical concepts, and particularly the estimation issues, easier for the reader to grasp.

The audience target of our work consists of academics and practitioners who are eager to master and construct copula applications to financial problems. For this applied focus, this book is, to the best of our knowledge, the first initiative in the market. Of course, the novelty of the topic and the growing number of research papers on the subject presented at finance conferences all over the world allows us to predict that our book will not remain the only one for too long, and that, on the contrary, this topic will be one of the major issues to be studied in the mathematical finance field in the near future.

Outline of the book

Chapter 1 reviews the state of the art in asset pricing and risk management, going over the major frontier issues and providing justifications for introducing copula functions.

Chapter 2 introduces the reader to the bivariate copula case. It presents the mathematical and probabilistic background on which the applications are built and gives some first examples in finance.

Chapter 3 discusses the flaws of linear correlation and highlights how copula functions, along with non-parametric association measures, may provide a much more flexible way to represent market comovements.

Chapter 4 extends the technical tools to a multivariate setting. Readers who are not already familiar with copulas are advised to skip this chapter at first reading (or to read it at their own risk!).

Chapter 5 explains the statistical inference for copulas. It covers both methodological aspects and applications from market data, such as calibration of actual risk factors comovements and VaR measurement. Here the readers can find details on the classical estimation methods as well as on most recent approaches, such as the conditional copula.

Chapter 6 is devoted to an exhaustive account of simulation algorithms for a large class of multivariate copulas. It is enhanced by financial examples.

Chapter 7 presents credit risk applications, besides giving a brief introduction to credit derivative markets and instruments. It applies copulas to the pricing of complex credit structures such as basket default swaps and CDOs. It is shown how to calibrate the pricing

model to market data. Its sensitivity with respect to the copula choice is accounted for in concrete examples.

Chapter 8 covers option pricing applications. Starting from the bivariate pricing kernel, copulas are used to evaluate counterparty risk in derivative transactions and bivariate rainbow options, such as options to exchange. We also show how the barrier option pricing problem can be cast in a bivariate setting and can be represented in terms of copulas. Finally, the estimation and simulation techniques presented in Chapters 5 and 6 are put at work to solve the evaluation problem of a multivariate basket option.

