1

Basic Concepts

1.1 History

Although this book will not follow a strictly historical development, to 'set the scene' this first chapter will start with a brief review of the most important discoveries that led to the separation of nuclear physics from atomic physics as a subject in its own right and later work that in its turn led to the emergence of particle physics from nuclear physics.[1]

1.1.1 The Origins of Nuclear Physics

Nuclear physics as a subject distinct from atomic physics could be said to date from 1896, the year that Becquerel observed that photographic plates were being fogged by an unknown radiation emanating from uranium ores. He had accidentally discovered *radioactivity*: the fact that some nuclei are unstable and spontaneously decay. The name was coined by Marie Curie two years later to distinguish this phenomenon from induced forms of radiation. In the years that followed, radioactivity was extensively investigated, notably by the husband and wife team of Pierre and Marie Curie, and by Rutherford and his collaborators,[2] and it was established that there were two distinct types of radiation involved, named by Rutherford α and β rays. We know now that α rays are bound states of two protons and two neutrons (we will see later that they are the nuclei of helium atoms) and β rays are electrons. In 1900 a third type of decay was discovered by Villard that involved the emission of photons, the quanta of electromagnetic radiation, referred to in this context as γ rays. These historical names are still commonly used.

[1] An interesting account of the early period, with descriptions of the personalities involved, is given in Segrè (1980). An overview of the later period is given in Chapter 1 of Griffiths (1987).

[2] The 1903 Nobel Prize in Physics was awarded jointly to Henri Becquerel for his discovery and to Pierre and Marie Curie for their subsequent research into radioactivity. Ernest Rutherford had to wait until 1908, when he was awarded the Nobel Prize in Chemistry for his 'investigations into the disintegration of the elements and the chemistry of radioactive substances'.

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At about the same time as Becquerel's discovery, J.J. Thomson was extending the work of Perrin and others on the radiation that had been observed to occur when an electric field was established between electrodes in an evacuated glass tube and in 1897 he was the first to definitively establish the nature of these 'cathode rays'. We now know the emanation consists of free *electrons*, (the name 'electron' had been coined in 1894 by Stoney) denoted e^- (the superscript denotes the electric charge) and Thomson measured their mass and charge.[3] The view of the atom at that time was that it consisted of two components, with positive and negative electric charges, the latter now being the electrons. Thomson suggested a model where the electrons were embedded and free to move in a region of positive charge filling the entire volume of the atom – the so-called 'plum pudding model'.

This model could account for the stability of atoms, but could not account for the discrete wavelengths observed in the spectra of light emitted from excited atoms. Neither could it explain the results of a classic series of experiments started in 1911 by Rutherford and continued by his collaborators, Geiger and Marsden. These consisted of scattering α particles by very thin gold foils. In the Thomson model, most of the α particles would pass through the foil, with only a few suffering deflections through small angles. Rutherford suggested they look for large-angle scattering and indeed they found that some particles were scattered through very large angles, even greater than 90 degrees. Rutherford showed that this behaviour was not due to multiple small-angle deflections, but could only be the result of the α particles encountering a very small positively charged central *nucleus.* (The reason for these two different behaviours is discussed in Appendix C.)

To explain the results of these experiments, a 'planetary' model was formulated where the atom was likened to a planetary system, with the electrons (the 'planets') occupying discrete orbits about a central positively charged nucleus (the 'Sun'). Because photons of a definite energy would be emitted when electrons moved from one orbit to another, this model could explain the discrete nature of the observed electromagnetic spectra when excited atoms decayed. In the simplest case of hydrogen, the nucleus is a single *proton* (p) with electric charge $+e$, where e is the magnitude of the charge on the electron,[4] orbited by a single electron. Heavier atoms were considered to have nuclei consisting of several protons. This view persisted for a long time and was supported by the fact that the masses of many naturally occurring elements are integer multiples of a unit that is about 1 % smaller than the mass of the hydrogen atom. Examples are carbon and nitrogen, with masses of 12.0 and 14.0 in these units. But it could not explain why not all atoms obeyed this rule. For example, chlorine has a mass of 35.5 in these units. However, about the same time, the concept of *isotopism* (a name coined by Soddy) was conceived. *Isotopes* are atoms whose nuclei have different masses, but the same charge. Naturally occurring elements were postulated to consist of a mixture of different isotopes, giving rise to the observed masses.[5]

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[3] J.J. Thomson received the 1906 Nobel Prize in Physics for his discovery. A year earlier, Philipp von Lenard had received the Physics Prize for his work on cathode rays.

[4] Why the charge on the proton should have exactly the same magnitude as that on the electron is a puzzle of very long-standing, the solution to which is suggested by some as yet unproven, but widely believed, theories of particle physics that will be briefly discussed in Section 9.5.1.

[5] Frederick Soddy was awarded the 1921 Nobel Prize in Chemistry for his work on isotopes.

The explanation of isotopes had to wait twenty years until a classic discovery by Chadwick in 1932. His work followed earlier experiments by Irène Curie (the daughter of Pierre and Marie Curie) and her husband Frédéric Joliot.[6] They had observed that neutral radiation was emitted when α particles bombarded beryllium and later work had studied the energy of protons emitted when paraffin was exposed to this neutral radiation. Chadwick refined and extended these experiments and demonstrated that they implied the existence of an electrically neutral particle of approximately the same mass as the proton. He had discovered the *neutron* (n) and in so doing had produced almost the final ingredient for understanding nuclei.[7]

There remained the problem of reconciling the planetary model with the observation of stable atoms. In classical physics, the electrons in the planetary model would be constantly accelerating and would therefore lose energy by radiation, leading to the collapse of the atom. This problem was solved by Bohr in 1913. He applied the newly emerging quantum theory and the result was the now well-known Bohr model of the atom. Refined modern versions of this model, including relativistic effects described by the Dirac equation (the relativistic analogue of the Schrödinger equation that applies to electrons), are capable of explaining the phenomena of atomic physics. Later workers, including Heisenberg, another of the founders of quantum theory, applied quantum mechanics to the nucleus, now viewed as a collection of neutrons and protons, collectively called *nucleons*. In this case however, the force binding the nucleus is not the electromagnetic force that holds electrons in their orbits, but is a short-range[8] force whose magnitude is independent of the type of nucleon, proton or neutron (i.e. charge-independent). This binding interaction is called the *strong nuclear force*.

These ideas still form the essential framework of our understanding of the nucleus today, where nuclei are bound states of nucleons held together by a strong charge-independent short-range force. Nevertheless, there is still no single theory that is capable of explaining all the data of nuclear physics and we shall see that different models are used to interpret different classes of phenomena.

1.1.2 The Emergence of Particle Physics: the Standard Model and Hadrons

By the early 1930s, the nineteenth-century view of atoms as indivisible *elementary particles* had been replaced and a larger group of physically smaller entities now enjoyed this status: electrons, protons and neutrons. To these we must add two electrically neutral particles: the *photon* (γ) and the *neutrino* (ν). The photon had been postulated by Planck in 1900 to explain black-body radiation, where the classical description of electromagnetic radiation led to results incompatible with experiments.[9] The neutrino was postulated by Pauli in 1930[10] to explain the apparent nonconservation of energy observed in the decay products

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[6] Irène Curie and Frédéric Joliot received the 1935 Nobel Prize in Chemistry for 'synthesizing new radioactive elements'.

[7] James Chadwick received the 1935 Nobel Prize in Physics for his discovery of the neutron.

[8] The concept of range will be discussed in more detail in Section 1.5.1, but for the present it may be taken as the effective distance beyond which the force is insignificant.

[9] X-rays had already been observed by Röntgen in 1895 (for which he received the first Nobel Prize in Physics in 1901) and γ-rays were seen by Villard in 1900, but it was Max Planck who first made the startling suggestion that electromagnetic energy was quantized. For this he was awarded the 1918 Nobel Prize in Physics. Many years later, he said that his hypothesis was an 'act of desperation' as he had exhausted all other possibilities.

[10] The name was later given by Fermi and means 'little neutron'.

of some unstable nuclei where β rays are emitted, the so-called β *decays*. Prior to Pauli's suggestion, β decay had been viewed as a parent nucleus decaying to a daughter nucleus and an electron. As this would be a two-body decay, it would imply that the electron would have a unique momentum, whereas experiments showed that the electron actually had a momentum *spectrum*. Pauli's hypothesis of a third particle (the neutrino) in the final state solved this problem, as well as a problem with angular momentum conservation, which was apparently also violated if the decay was two-body. The β-decay data implied that the neutrino mass was very small and was compatible with the neutrino being massless.[11] It took more than 25 years before Pauli's hypothesis was confirmed by Reines and Cowan in a classic experiment in 1956 that detected free neutrinos from β decay.[12]

The 1950s also saw technological developments that enabled high-energy beams of particles to be produced in laboratories. As a consequence, a wide range of controlled scattering experiments could be performed and the greater use of computers meant that sophisticated analysis techniques could be developed to handle the huge quantities of data that were being produced. By the 1960s this had resulted in the discovery of a very large number of unstable particles with very short lifetimes and there was an urgent need for a theory that could make sense of all these states. This emerged in the mid 1960s in the form of the so-called *quark model*, first suggested by Gell-Mann, and independently and simultaneously by Zweig, who postulated that the new particles were bound states of three families of more fundamental physical particles.

Gell-Mann called these particles *quarks* (q).[13] Because no free quarks were detected experimentally, there was initially considerable scepticism for this view. We now know that there is a fundamental reason why quarks cannot be observed as free particles (it is discussed in Section 5.1), but at the time most physicists looked upon quarks as a convenient mathematical description, rather than physical particles.[14] However, evidence for the existence of quarks as real particles came in the 1960s from a series of experiments analogous to those of Rutherford and his co-workers, where high-energy beams of electrons and neutrinos were scattered from nucleons. (These experiments are discussed in Section 5.8.) Analysis of the angular distributions of the scattered particles showed that the nucleons were themselves bound states of three point-like charged entities, with properties consistent with those hypothesized in the quark model. One of these properties was unusual: quarks have fractional electric charges, in practice $-\frac{1}{3}e$ and $+\frac{2}{3}e$. This is essentially the picture today, where elementary particles are now considered to be a small number of physical entities, including quarks, the electron, neutrinos, the photon and a few others we shall meet, but no longer nucleons.

The best theory of elementary particles we have at present is called, rather prosaically, the *standard model*. This aims to explain all the phenomena of particle physics, except those due to gravity, in terms of the properties and interactions of a small number of

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[11] However, in Section 3.1.4 we will discuss evidence that shows the neutrino has a nonzero mass, albeit very small.

[12] A description of this experiment is given in Chapter 12 of Trigg (1975). Frederick Reines shared the 1995 Nobel Prize in Physics for his work in neutrino physics and particularly for the detection of the neutrino.

[13] Murray Gell-Mann received the 1969 Nobel Prize in Physics for 'contributions and discoveries concerning the classification of elementary particles and their interactions'. For the origin of the word 'quark', he cited the now famous quotation 'Three quarks for Muster Mark' from James Joyce's book *Finnegans Wake.* George Zweig had suggested the name 'aces'.

[14] This was history repeating itself. In the early days of the atomic model many very distinguished scientists were reluctant to accept that atoms existed, because they could not be 'seen' in a conventional sense.

*elementary (*or *fundamental) particles,* which are now defined as being point-like, without internal structure or excited states. Particle physics thus differs from nuclear physics in having a single theory to interpret its data.

An elementary particle is characterized by, amongst other things, its mass, its electric charge and its *spin*. The latter is a permanent angular momentum possessed by all particles in quantum theory, even when they are at rest. Spin has no classical analogue and is not to be confused with the use of the same word in classical physics, where it usually refers to the (orbital) angular momentum of extended objects. The maximum value of the spin angular momentum about any axis is $s\hbar$ ($\hbar \equiv h/2\pi$), where h is Planck's constant and s is the *spin quantum number*, or *spin* for short. It has a fixed value for particles of any given type (for example $s = \frac{1}{2}$ for electrons) and general quantum mechanical principles restrict the possible values of s to be $0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$. Particles with half-integer spin are called *fermions* and those with integer spin are called *bosons*. There are three families of elementary particles in the standard model: two spin-$\frac{1}{2}$ families of fermions called *leptons* and *quarks*; and one family of spin-1 bosons. In addition, at least one other spin-0 particle, called the *Higgs boson*, is postulated to explain the origin of mass within the theory.[15]

The most familiar elementary particle is the electron, which we know is bound in atoms by the *electromagnetic interaction*, one of the four forces of nature.[16] One test of the elementarity of the electron is the size of its magnetic moment. A charged particle with spin necessarily has an intrinsic magnetic moment $\boldsymbol{\mu}$. It can be shown from the Dirac equation that a point-like spin-$\frac{1}{2}$ particle of charge q and mass m has a magnetic moment $\boldsymbol{\mu} = (q/m)\,\mathbf{S}$, where $\mathbf{S}$ is its spin vector. Magnetic moment is a vector, and the value μ tabulated is the z component when the z component of spin has is maximum value, i.e. $\mu = q\hbar/2m$. The magnetic moment of the electron obeys this relation to one part in 10^4.[17]

The electron is a member of the family of leptons. Another is the neutrino, which was mentioned earlier as a decay product in β decays. Strictly this particle should be called the *electron neutrino,* written ν_e, because it is always produced in association with an electron. (The reason for this is discussed in Section 3.1.1.) The force responsible for beta decay is an example of a second fundamental force, the *weak interaction*. Finally, there is the third force, the (fundamental) *strong interaction*, which, for example, binds quarks in nucleons. The strong nuclear force mentioned in Section 1.1.1 is not the same as this fundamental strong interaction, but is a consequence of it. The relation between the two will be discussed in more detail in Section 7.1.

The standard model also specifies the origin of these three forces. In classical physics the electromagnetic interaction is propagated by electromagnetic waves, which are continuously emitted and absorbed. While this is an adequate description at long distances, at short distances the quantum nature of the interaction must be taken into account. In quantum theory, the interaction is transmitted discontinuously by the exchange of photons, which are members of the family of fundamental spin-1 bosons of the standard model. Photons

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[15] In the theory without the Higgs boson, all elementary particles are predicted to have zero mass, in obvious contradiction with experiment. A solution to this problem involving the Higgs boson is briefly discussed in Section 9.3.1, and Section D.2.

[16] Although an understanding of all four forces will ultimately be essential, gravity is so weak that it can be neglected in nuclear and particle physics at presently accessible energies. Because of this, we will often refer in practice to the *three* forces of nature.

[17] Polykarp Kusch shared the 1955 Nobel Prize in Physics for the first precise determination of the magnetic moment of the electron.

are referred to as the *gauge bosons*, or 'force carriers', of the electromagnetic interaction. The use of the word 'gauge' originates from the fact that the electromagnetic interaction possesses a fundamental symmetry called *gauge invariance*. For example, Maxwell's equations of classical electromagnetism are invariant under a specific phase transformation of the electromagnetic fields – the gauge transformation. This property is common to all the three interactions of nature we will be discussing and has profound consequences, but we will not need its details in this book.[18] The weak and strong interactions are also mediated by the exchange of spin-1 gauge bosons. For the weak interaction these are the W^+, W^- and Z^0 bosons (again the superscripts denote the electric charges) with masses about 80–90 times the mass of the proton. For the strong interaction, the force carriers are called *gluons*. There are eight gluons, all of which have zero mass and are electrically neutral.[19]

In addition to the elementary particles of the standard model, there are other important particles we will be studying. These are the *hadrons*, the bound states of quarks. Nucleons are examples of hadrons,[20] but there are several hundred more, not including nuclei, most of which are unstable and decay by one of the three interactions. It was the abundance of these states that drove the search for a simplifying theory that would give an explanation for their existence and led to the quark model in the 1960s. The most common unstable example of a hadron is the *pion*, which exists in three electrical charge states, written (π^+, π^0, π^-). Hadrons are important because free quarks are unobservable in nature and so to deduce their properties we are forced to study hadrons. An analogy would be if we had to deduce the properties of nucleons by exclusively studying the properties of nuclei.

Since nucleons are bound states of quarks and nuclei are bound states of nucleons, the properties of nuclei should in principle be deducible from the properties of quarks and their interactions, i.e. from the standard model. In practice, however, this is beyond present calculational techniques and sometimes nuclear and particle physics are treated as two almost separate subjects. However, there are many connections between them and in introductory treatments it is still useful to present both subjects together.

The remaining sections of this chapter are devoted to introducing some of the basic theoretical tools needed to describe the phenomena of both nuclear and particle physics, starting with a key concept: antiparticles.

1.2 Relativity and Antiparticles

Elementary particle physics is also called high-energy physics. One reason for this is that if we wish to produce new particles in a collision between two other particles, then because of the relativistic mass-energy relation $E = mc^2$, energies are needed at least as great as the rest masses of the particles produced. The second reason is that to explore the structure of a particle requires a probe whose wavelength λ is smaller than the structure to be explored.

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[18] A brief description of gauge invariance and some of its consequences is given in Appendix D.

[19] Note that the word 'electrical' has been used when talking about charge. This is because the weak and strong interactions also have associated 'charges' which determine the strengths of the interactions, just as the electric charge determines the strength of the electromagnetic interaction. This is discussed in more detail in later chapters.

[20] The magnetic moments of the proton and neutron do not obey the prediction of the Dirac equation and this is evidence that nucleons have structure and are not elementary. The proton magnetic moment was first measured by Otto Stern using a molecular beam method that he developed and for this he received the 1943 Nobel Prize in Physics.

By the de Broglie relation $\lambda = h/p$, this implies that the momentum p of the probing particle, and hence its energy, must be large. For example, to explore the internal structure of the proton using electrons requires wavelengths that are much smaller than the classical radius of the proton, which is roughly 10^{-15} m. This in turn requires electron energies that are greater than 10^3 times the rest energy of the electron, implying electron velocities very close to the speed of light. Hence any explanation of the phenomena of elementary particle physics must take account of the requirements of the theory of special relativity, in addition to those of quantum theory. There are very few places in particle physics where a nonrelativistic treatment is adequate, whereas the need for a relativistic treatment is less in nuclear physics.

Constructing a quantum theory that is consistent with special relativity leads to the conclusion that for every particle of nature, there must exist an associated particle, called an *antiparticle*, with the same mass as the corresponding particle. This important theoretical prediction was first made by Dirac and follows from the solutions of the equation he postulated to describe relativistic electrons.[21] The *Dirac equation* for a particle of mass m and momentum $\mathbf{p}$ moving in free space is of the form[22]

$$i\hbar \frac{\partial \mathbf{\Psi}(\mathbf{r}, t)}{\partial t} = H(\mathbf{r}, \hat{\mathbf{p}})\mathbf{\Psi}(\mathbf{r}, t), \tag{1.1}$$

where $\hat{\mathbf{p}} = -i\hbar\boldsymbol{\nabla}$ is the usual quantum mechanical momentum operator and the Hamiltonian was postulated by Dirac to be

$$H = c\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta mc^2. \tag{1.2}$$

The coefficients $\boldsymbol{\alpha}$ and β are determined by the requirement that the solutions of (1.1) are also solutions of the free-particle *Klein-Gordon equation*[23]

$$-\hbar^2 \frac{\partial^2 \mathbf{\Psi}(\mathbf{r}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \mathbf{\Psi}(\mathbf{r}, t) + m^2 c^4 \mathbf{\Psi}(\mathbf{r}, t). \tag{1.3}$$

This leads to the conclusion that $\boldsymbol{\alpha}$ and β cannot be simple numbers; their simplest forms are 4×4 matrices. Thus the solutions of the Dirac equation are four-component wavefunctions (called *spinors*) with the form[24]

$$\mathbf{\Psi}(\mathbf{r}, t) = \begin{pmatrix} \psi_1(\mathbf{r}, t) \\ \psi_2(\mathbf{r}, t) \\ \psi_3(\mathbf{r}, t) \\ \psi_4(\mathbf{r}, t) \end{pmatrix}. \tag{1.4}$$

The interpretation of (1.4) is that the four components describe the two spin states of a negatively charged electron with positive energy and the two spin states of a corresponding particle having the same mass, but with negative energy. Two spin states arise because in quantum mechanics the projection in any direction of the spin vector of a spin-$\frac{1}{2}$ particle

[21] Paul Dirac shared the 1933 Nobel Prize in Physics with Erwin Schrödinger. The somewhat cryptic citation stated 'for the discovery of new productive forms of atomic theory'.
[22] We use the notation $\mathbf{r} = (x_1, x_2, x_3) = (x, y, z)$.
[23] This is a relativistic equation, which follows from using the usual quantum mechanical operator substitutions, $\hat{\mathbf{p}} = -i\hbar\nabla$ and $E = i\hbar\, \partial/\partial t$ in the relativistic mass-energy relation $E^2 = p^2c^2 + m^2c^4$.
[24] The details may be found in many quantum mechanics books, e.g. pp. 475–477 of Schiff (1968).

can only result in one of the two values $\pm\frac{1}{2}$, referred to as 'spin up' and 'spin down', respectively. The two energy solutions arise from the two solutions of the relativistic mass-energy relation $E = \pm\sqrt{p^2c^2 + m^2c^4}$. The negative-energy states can be shown to behave in all respects as *positively* charged electrons (called *positrons*), but with *positive* energy. The positron is referred to as the antiparticle of the electron. The discovery of the positron by Anderson in 1933, with all the predicted properties, was a spectacular verification of the Dirac prediction.

Although Dirac originally made his prediction for electrons, the result is general and is true whether the particle is an elementary particle or a hadron. If we denote a particle by P, then the antiparticle is in general written with a bar over it, i.e. $\bar{P}$. For example, the antiparticle of the proton p is the antiproton $\bar{p}$,[25] with negative electric charge; and associated with every quark, q, is an antiquark, $\bar{q}$. However, for some very common particles the bar is usually omitted. Thus, for example, in the case of the positron e^+, the superscript denoting the charge makes explicit the fact that the antiparticle has the opposite electric charge to that of its associated particle. Electric charge is just one example of a *quantum number* (spin is another) that characterizes a particle, whether it is elementary or composite (i.e. a hadron).

Many quantum numbers differ in sign for particle and antiparticle, and electric charge is an example of this. We will meet others later. When brought together, particle-antiparticle pairs, each of mass m, can annihilate, releasing their combined rest energy $2mc^2$ as photons or other particles. Finally, we note that there is symmetry between particles and antiparticles, and it is a convention to call the electron the particle and the positron its antiparticle. This reflects the fact that the normal matter contains electrons rather than positrons.

1.3 Space-Time Symmetries and Conservation Laws

Symmetries and the invariance properties of the underlying interactions play an important role in physics. Some lead to conservation laws that are universal. Familiar examples are translational invariance, leading to the conservation of linear momentum; and rotational invariance, leading to conservation of angular momentum. The latter plays an important role in nuclear and particle physics as it leads to a scheme for the classification of states based, among other quantum numbers, on their spins. This is similar to the scheme used to classify states in atomic physics.[26] Another very important invariance that we have briefly mentioned is gauge invariance. This fundamental property of all three interactions restricts their forms in a profound way that initially is contradicted by experiment. This is the prediction of zero masses for all elementary particles, mentioned earlier. There are theoretical solutions to this problem whose experimental verification (the discovery of the Higgs boson), or otherwise, is the most eagerly awaited result in particle physics today.[27]

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[25] Carl Anderson shared the 1936 Nobel Prize in Physics for the discovery of the positron. The 1959 Prize was awarded to Emilio Segrè and Owen Chamberlain for their discovery of the antiproton.
[26] These points are explored in more detail in, for example, Chapter 5 of Martin and Shaw (2008).
[27] Experimental searches for the Higgs boson are discussed in Section 9.3.2, and a very brief explanation of the so-called 'Higgs mechanism', that generates particle masses, is given in Section D.2.

In nuclear and particle physics we need to consider additional symmetries of the Hamiltonian and the conservation laws that follow and in the remainder of this section we discuss three space-time symmetries that we will need later – *parity*, *charge conjugation* and *time-reversal*.

1.3.1 Parity

Parity was first introduced in the context of atomic physics by Wigner in 1927.[28] It refers to the behaviour of a state under a spatial reflection, i.e. $\mathbf{r} \to -\mathbf{r}$. If we consider a single-particle state, represented for simplicity by a nonrelativistic wavefunction $\psi(\mathbf{r}, t)$, then under the parity operator $\hat{P}$,

$$\hat{P}\psi(\mathbf{r}, t) \equiv P\psi(-\mathbf{r}, t). \tag{1.5}$$

Applying the operator again, gives

$$\hat{P}^2\psi(\mathbf{r}, t) = P\hat{P}\psi(-\mathbf{r}, t) = P^2\psi(\mathbf{r}, t), \tag{1.6}$$

implying $P = \pm 1$. If the particle is an eigenfunction of linear momentum $\mathbf{p}$, i.e.

$$\psi(\mathbf{r}, t) \equiv \psi_p(\mathbf{r}, t) = \exp[i(\mathbf{p} \cdot \mathbf{r} - Et)/\hbar], \tag{1.7}$$

then

$$\hat{P}\psi_{\mathbf{p}}(\mathbf{r}, t) = P\psi_{\mathbf{p}}(-\mathbf{r}, t) = P\psi_{-\mathbf{p}}(\mathbf{r}, t) \tag{1.8}$$

and so a particle at rest, with $\mathbf{p} = \mathbf{0}$, is an eigenstate of parity. The eigenvalue $P = \pm 1$ is called the *intrinsic parity*, or just the *parity*, of the state. By considering a multiparticle state with a wavefunction that is the product of single-particle wavefunctions, it is clear that parity is a multiplicative quantum number.

The strong and electromagnetic interactions, but not the weak interactions, are invariant under parity, i.e. the Hamiltonian of the system, and hence the equation of motion, remains unchanged under a parity transformation on the position vectors of all particles in the system. Parity is therefore conserved, by which we mean that the total parity quantum number remains unchanged in the interaction. Compelling evidence for parity conservation in the strong and electromagnetic interactions comes from the suppression of transitions between nuclear states that would violate parity conservation. Such decays are not absolutely forbidden, because the Hamiltonian responsible for the transition will always have a small admixture due to the weak interactions between nucleons. However, the observed rates are extremely small compared to analogous decays that do not violate parity, and are entirely consistent with the transitions being due to this very small weak interaction component. The evidence for nonconservation of parity in the weak interaction will be discussed in detail in Section 6.2.

In addition to intrinsic parity, there is a contribution to the total parity if the particle has an orbital angular momentum l. In this case its wave function is a product of a radial part R_{nl} and an angular part $Y_l^m(\theta, \phi)$:

$$\psi_{lmn}(\mathbf{r}) = R_{nl}Y_l^m(\theta, \phi), \tag{1.9}$$

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[28] Eugene Wigner shared the 1963 Nobel Prize in Physics, principally for his work on symmetries.

where n and m are the principal and magnetic quantum numbers and $Y_l^m(\theta, \phi)$ is a spherical harmonic. It is straightforward to show from the relations between Cartesian (x, y, z) and spherical polar co-ordinates (r, θ, ϕ), i.e.

$$x = r \sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi, \quad z = r \cos\theta, \tag{1.10}$$

that the parity transformation $\mathbf{r} \to -\mathbf{r}$ implies

$$r \to r, \quad \theta \to \pi - \theta, \quad \phi \to \pi + \phi, \tag{1.11}$$

and from this it can be shown that

$$Y_l^m(\theta, \phi) \to Y_l^m(\pi - \theta, \pi + \phi) = (-)^l Y_l^m(\theta, \phi). \tag{1.12}$$

Equation (1.12) may easily be verified directly for specific cases; for example, for the first three spherical harmonics,

$$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}, \quad Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta, \quad Y_1^{\pm 1} = \left(\frac{3}{8\pi}\right)^{1/2} \sin\theta \, e^{\pm i\phi}. \tag{1.13}$$

Hence

$$\hat{P}\psi_{lmn}(\mathbf{r}) = P\psi_{lmn}(-\mathbf{r}) = P(-)^l \psi_{lmn}(\mathbf{r}), \tag{1.14}$$

i.e. $\psi_{lmn}(\mathbf{r})$ is an eigenstate of parity with eigenvalue $P(-1)^l$.

An analysis of the Dirac equation (1.1) for relativistic electrons, shows that it is invariant under a parity transformation only if $P(e^+e^-) = -1$. This is a general result for all fermion-antifermion pairs, so it is a convention to assign $P = +1$ to all leptons and $P = -1$ to their antiparticles. We will see in Chapter 3 that in strong and electromagnetic interactions quarks can only be created as part of a quark-antiquark pair, so the intrinsic parity of a single quark cannot be measured. For this reason, it is also a convention to assign $P = +1$ to quarks. Since quarks are fermions, it follows from the Dirac result that $P = -1$ for antiquarks. The intrinsic parities of hadrons then follow from their structure in terms of quarks and the orbital angular momentum between the constituent quarks, using (1.14). This will be explored in Chapter 3 as part of the discussion of the quark model.

1.3.2 Charge Conjugation

Charge conjugation is the operation of changing a particle into its antiparticle. Like parity, it gives rise to a multiplicative quantum number that is conserved in strong and electromagnetic interactions, but violated in the weak interaction. In strong interactions this can be tested experimentally, by for example measuring the rates of production of positive and negative mesons in $p\bar{p}$ annihilations, and is found to hold.

In discussing charge conjugation, we will need to distinguish between states such as the photon γ and the neutral pion π^0 that do not have distinct antiparticles and those such as the π^+ and the neutron, which do. Particles in the former class we will collectively denote by a, and those of the latter type will be denoted by b. It is also convenient at this point to extend our notation for states. Thus we will represent a state of type a having a wavefunction ψ_a by $|a, \psi_a\rangle$ and similarly for a state of type b.[29] Then under the charge conjugation operator $\hat{C}$,

$$\hat{C}|a, \psi_a\rangle = C_a|a, \psi_a\rangle, \tag{1.15a}$$

and

$$\hat{C}|b, \psi_b\rangle = |\bar{b}, \psi_{\bar{b}}\rangle, \tag{1.15b}$$

where C_a is a phase factor analogous to the phase factor in (1.5).[30] Applying the operator twice, in the same way as for parity, leads to $C_a = \pm 1$. From (1.15a), we see that states of type a are eigenstates of $\hat{C}$ with eigenvalues ± 1, called their *C parities*. As an example, consider the π^0. This decays via the electromagnetic interaction to two photons: $\pi^0 \to \gamma\gamma$. The C parity of the photon follows directly from the invariance of Maxwell's equations under charge conjugation and is $C_\gamma = -1$[31] and hence $C_{\pi^0} = 1$. It follows that the decay $\pi^0 \to \gamma\gamma\gamma$ is forbidden by C invariance. The experimental limit for the ratio of rates $\pi^0 \to 3\gamma/\pi^0 \to 2\gamma$ is less than 3×10^{-8}, which is strong evidence for C invariance in electromagnetic interactions. The evidence for the violation of C invariance in the weak interaction is discussed in detail in Chapter 6.

States with distinct antiparticles can only form eigenstates of $\hat{C}$ as linear combinations. As an example of the latter, consider a $\pi^+\pi^-$ pair with orbital angular momentum L between them. In this case

$$\hat{C}|\pi^+\pi^-; L\rangle = (-1)^L|\pi^+\pi^-; L\rangle, \tag{1.16}$$

because interchanging the pions reverses their relative positions in the spatial wavefunction. The same factor occurs for spin-$\frac{1}{2}$ fermion pairs $f\bar{f}$, but in addition there are two other factors. The first is $(-1)^{S+1}$, where S is the total spin of the pair. This follows directly from the structure of the spin wavefunctions:

$$\left.\begin{array}{ll} \uparrow_1\uparrow_2 & S_z = 1 \\ \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 + \downarrow_1\uparrow_2) & S_z = 0 \\ \downarrow_1\downarrow_2 & S_z = -1 \end{array}\right\} \quad S = 1 \tag{1.17a}$$

and

$$\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2) \quad S_z = 0 \quad S = 0 \tag{1.17b}$$

where $\uparrow_i$ ($\downarrow_i$) represents particle i having spin 'up' ('down') in the z direction. A second factor (-1) arises whenever fermions and antifermions are interchanged. This has its origins in quantum field theory.[32] Combining these factors, finally we have

$$\hat{C}|f\,\bar{f}; J, L, S\rangle = (-1)^{L+S}|f\,\bar{f}; J, L, S\rangle, \tag{1.18}$$

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[29] This is part of the so-called 'Dirac notation' in quantum mechanics. However, we will only need the notation and not the associated mathematics.

[30] A phase factor could have been inserted in (1.15b), but it is straightforward to show that the relative phase of the two states b and $\bar{b}$ cannot be measured and so a phase introduced in this way would have no physical consequences. (See Problem 1.4.)

[31] A proof of this is given in Section 5.4.1 of Martin and Shaw (2008). An alternative argument is that electromagnetic fields are produced by moving electric charges, which change sign under charge conjugation, and hence $C_\gamma = -1$.

[32] See, for example, pp. 249–250 of Gottfried and Weisskopf (1986).

for fermion-antifermion pairs having total, orbital and spin angular momentum quantum numbers J, L and S, respectively.

1.3.3 Time Reversal

Time-reversal invariance is defined as invariance under the transformation

$$t \to t' = -t, \tag{1.19}$$

leaving all position vectors unchanged. Like parity and charge conjugation invariance, it is a symmetry of the strong and electromagnetic interactions, but is violated by the weak interactions. However, unlike parity and charge conjugation, there is no associated quantum number that is conserved when weak interactions are neglected. To understand this we consider the transformation of a single-particle wavefunction, which must satisfy

$$|\psi(\mathbf{r}, t)|^2 \xrightarrow{T} |\psi'(\mathbf{r}, t)|^2 = |\psi(\mathbf{r}, -t)|^2 \tag{1.20}$$

if the system is T invariant, so that the probability of finding the particle at position $\mathbf{r}$ at time $-t$ becomes the probability of finding it at position $\mathbf{r}$ at time t in the transformed system. In addition, since in classical mechanics linear and angular momentum change sign under (1.19), we would expect the same result

$$\mathbf{p} \xrightarrow{T} \mathbf{p}' = -\mathbf{p}; \qquad \mathbf{J} \xrightarrow{T} \mathbf{J}' = -\mathbf{J} \tag{1.21}$$

to hold in quantum mechanics by the correspondence principle. Hence a free-particle wavefunction

$$\psi_{\mathbf{p}}(\mathbf{r}, t) = \exp[i(\mathbf{p} \cdot \mathbf{r} - Et)/\hbar],$$

corresponding to momentum $\mathbf{p}$ and energy $E = p^2/2m$, must transform into a wavefunction corresponding to momentum $-\mathbf{p}$ and energy E, i.e.

$$\psi_{\mathbf{p}}(\mathbf{r}, t) \xrightarrow{T} \psi'_{\mathbf{p}}(\mathbf{r}, t) = \psi_{-\mathbf{p}}(\mathbf{r}, t) = \exp[-i(\mathbf{p} \cdot \mathbf{r} + Et)/\hbar]. \tag{1.22}$$

A suitable transformation that satisfies both (1.20) and (1.22) is

$$\psi(\mathbf{r}, t) \xrightarrow{T} \psi'(\mathbf{r}, t) = \psi^*(\mathbf{r}, -t) \equiv \hat{T}\, \psi(\mathbf{r}, t), \tag{1.23}$$

where we have introduced the time reversal operator $\hat{T}$ by analogy with the parity operator $\hat{P}$ introduced in Equation (1.5). However, quantum mechanical operators $\hat{O}$ that correspond to physical observables must be both linear

$$\hat{O}(\alpha_1\psi_1 + \alpha_2\psi_2) = \alpha_1(\hat{O}\psi_1) + \alpha_2(\hat{O}\psi_2), \tag{1.24a}$$

(to ensure that the superposition principle holds), and Hermitian

$$\int \mathrm{d}x (\hat{O}\psi_1)^*\psi_2 = \int \mathrm{d}x\ \psi_1^*(\hat{O}\psi_2), \tag{1.24b}$$

(to ensure that the the eigenvalues of $\hat{O}$, i.e. the observed values, are real), where $\psi_{1,2}$ are arbitrary wavefunctions and $\alpha_{1,2}$ are arbitrary complex numbers. In contrast, the definition (1.23) implies

$$\hat{T}(\alpha_1\psi_1 + \alpha_2\psi_2) = \alpha_1^*(\hat{T}\psi_1) + \alpha_2^*(\hat{T}\psi_2) \neq \alpha_1(\hat{T}\psi_1) + \alpha_2(\hat{T}\psi_2)$$

for complex α_1 and α_2, and one easily verifies that (1.24b) is also not satisfied by $\hat{T}$. Thus the time reversal operator does not correspond to a physical observable, and there is no observable analogous to parity that is conserved as a consequence of T invariance.

Although T invariance does not give rise to a conservation law, it does lead to a relation between any reaction and the 'time-reversed' process related to it by (1.19). Thus reactions like

$$a(\mathbf{p}_a, m_a) + b(\mathbf{p}_b, m_b) \to c(\mathbf{p}_c, m_c) + d(\mathbf{p}_d, m_d) \tag{1.25a}$$

and their time-reversed counterparts

$$c(-\mathbf{p}_c, -m_c) + d(-\mathbf{p}_d, -m_d) \to a(-\mathbf{p}_a, -m_a) + b(-\mathbf{p}_b, -m_b), \tag{1.25b}$$

in which the initial and final states are interchanged and the particle momenta ($\mathbf{p}_a$ etc) and z components of their spins (m_a etc) are reversed in accordance with (1.21), are related. In particular, if weak interactions are neglected, the rates for reactions (1.25a) and (1.25b) must be equal.

A more useful relation between reaction rates can be obtained if we combine time reversal with parity invariance. Under the parity transformation (1.5), momenta $\mathbf{p}$ change sign while orbital angular momenta $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ do not. If we assume the same behaviour holds for spin angular momenta, then

$$\mathbf{p} \xrightarrow{P} \mathbf{p}' = -\mathbf{p}; \qquad \mathbf{J} \xrightarrow{P} \mathbf{J}' = \mathbf{J} \tag{1.26}$$

under parity. The parity-transformed reaction corresponding to (1.25b) is

$$c(\mathbf{p}_c, -m_c) + d(\mathbf{p}_d, -m_d) \to a(\mathbf{p}_a, -m_a) + b(\mathbf{p}_b, -m_b) \tag{1.25c}$$

so that if both P and T invariance holds, all three reactions (1.25a, b, c) must have the same rate. If we average over all spin projections

$$m_i = -S_i, -S_i + 1, \ldots S_i \quad (i = a, b, c, d),$$

where S_i is the spin of particle i, then reactions (1.25a) and (1.25c) differ only by the interchange of initial and final states. Consequently, the rates for the reactions

$$i \equiv a(\mathbf{p}_a) + b(\mathbf{p}_b) \leftrightarrow c(\mathbf{p}_c) + d(\mathbf{p}_d) \equiv f \tag{1.27}$$

should be equal, provided that we average over all possible spin states. This relation is called the *principle of detailed balance*, and has been accurately confirmed experimentally in a variety of strong and electromagnetic reactions.

Finally, although the weak interaction is not invariant under the above transformations, there is a general result, called the *CPT theorem*, which states that under very general conditions *any* relativistic field theory is invariant under the combined operation of CPT, taken in any order. Among other things, CPT invariance predicts that the masses and lifetimes of a particle and its antiparticle must be exactly equal. This prediction is accurately verified by experimental measurements on a number of particles, including e^+e^- pairs.

1.4 Interactions and Feynman Diagrams

We now turn to a discussion of particle interactions and how they can be described by the very useful pictorial methods of Feynman diagrams.

1.4.1 Interactions

Interactions involving elementary particles and/or hadrons are conveniently summarized by ‘equations’ in analogy to chemical reactions, in which the different particles are represented by symbols, which usually, but not always, have a superscript to denote their electric charge. In the interaction

$$\nu_e + n \to e^- + p \tag{1.28}$$

for example, an electron neutrino ν_e collides with a neutron n to produce an electron e^- and a proton p; while the equation

$$e^- + p \to e^- + p \tag{1.29}$$

represents an electron and proton interacting to give the same particles in the final state, but in general travelling in different directions. In such equations, conserved quantum numbers must have the same total values in initial and final states.

Particles may be transferred from initial to final states and *vice versa*, when they become antiparticles. Thus starting from the process

$$\pi^- + p \to \pi^- + p, \tag{1.30a}$$

and taking the proton from the initial state to an antiproton in the final state and the negatively charged pion in the final state to a positively charged pion in the initial state, we obtain

$$\pi^+ + \pi^- \to p + \bar{p}. \tag{1.31}$$

It follows that if (1.30a) does not violate any relevant quantum numbers, then neither does reaction (1.31) and so is also in principle an allowed reaction. The qualification is needed because although (1.31) does not violate any quantum numbers, energy conservation leads to a minimum total energy $E_{min} = (m_p + m_{\bar{p}})c^2$ below which it cannot proceed.

The interactions (1.29) and (1.30a), in which the particles remain unchanged, are examples of *elastic scattering*, in contrast to reactions (1.28) and (1.31), where the final-state particles differ from those in the initial state. Collisions between a given pair of initial particles do not always lead to the same final state, but can lead to different final states with different probabilities. For example, the collision of a negatively charged pion and a proton can give rise to elastic scattering (1.30a) and a variety of other reactions, such as

$$\pi^- + p \to n + \pi^0 \quad \text{and} \quad \pi^- + p \to p + \pi^- + \pi^- + \pi^+, \tag{1.30b}$$

depending on the initial energy. In particle physics it is common to refer (rather imprecisely) to such interactions as ‘inelastic’ scattering.

Similar considerations apply to nuclear physics, but the term *inelastic scattering* is reserved for the case where the final state is an excited state of the parent nucleus A, that subsequently decays, for example via photon emission, i.e.

$$a + A \to a + A^*; \qquad A^* \to A + \gamma, \tag{1.32}$$

where a is a projectile and A^* is an excited state of A. A useful shorthand notation used in nuclear physics for the general reaction $a + A \to b + B$ is $A(a, b)B$. It is usual in nuclear physics to further subdivide types of interactions according to the underlying mechanism that produced them. We will return to this in Section 2.9, as part of a more general discussion of nuclear reactions.

Finally, many particles are unstable and spontaneously decay to other, lighter (i.e. having less mass) particles. An example of this is the free neutron (i.e. one not bound in a nucleus), which decays by the β-decay

$$n \to p + e^- + \bar{\nu}_e, \tag{1.33}$$

with a mean lifetime of about 900 seconds.[33] The same notation can also be used in nuclear physics. For example, many nuclei decay via the β-decay mechanism. Thus, denoting a nucleus with Z protons and N neutrons as (Z, N), we have

$$(Z, N) \to (Z - 1, N) + e^+ + \nu_e. \tag{1.34}$$

This is also a weak interaction. This reaction is effectively the decay of a proton bound in a nucleus. Although a *free* proton cannot decay by the beta decay $p \to n + e^+ + \nu_e$ because it violates energy conservation (the final-state particles have greater total mass than the proton), a proton bound in a nucleus can decay because of its binding energy. The explanation for this is given in Chapter 2.

1.4.2 Feynman Diagrams

The forces producing all the above interactions are due to the exchange of particles and a convenient way of illustrating this is to use *Feynman diagrams*. There are mathematical rules and techniques associated with these that enable them to be used to calculate the quantum mechanical probabilities for given reactions to occur, but in this book Feynman diagrams will only be used as a convenient very useful pictorial description of reaction mechanisms.

We first illustrate them at the level of elementary particles for the case of electromagnetic interactions, which arise from the emission and/or absorption of photons. For example, the dominant interaction between two electrons is due to the exchange of a single photon, which is emitted by one electron and absorbed by the other. This mechanism, which gives rise to the familiar Coulomb interaction at large distances, is illustrated in the Feynman diagram Figure 1.1a.

In such diagrams, we will use the convention that particles in the initial state are shown on the left and particles in the final state are shown on the right. (Some authors take time to run along the y axis.) Spin-$\frac{1}{2}$ fermions (such as the electron) are drawn as solid lines and photons are drawn as wiggly lines. Arrowheads pointing to the right indicate that the solid lines represent electrons. In the case of photon exchange between two positrons, which is

[33] The reason that this decay involves an antineutrino rather than a neutrino will become clear in Chapter 3.

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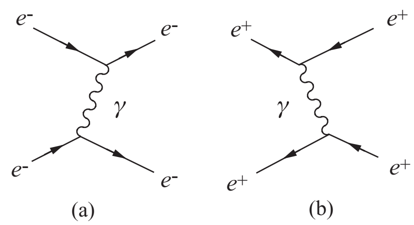

Figure 1.1 Single-photon exchange in (a) $e^- + e^- \to e^- + e^-$ and (b) $e^+ + e^+ \to e^+ + e^+$. Time as usual runs from left to right.

shown in Figure 1.1b, the arrowheads on the antiparticle (positron) lines are conventionally shown as pointing to the left. In interpreting these diagrams, it is important to remember that the direction of the arrows on fermion lines does *not* indicate the particle's direction of motion, but merely whether the fermions are particles or antiparticles; and that particles in the initial state are always to the left and particles in the final state are always to the right.

A feature of the above diagrams is that they are constructed from combinations of simple three-line vertices. This is characteristic of electromagnetic processes. Each vertex has a line corresponding to a single photon being emitted or absorbed, while one fermion line has the arrow pointing towards the vertex and the other away from the vertex, guaranteeing charge conservation at the vertex, which is one of the rules of Feynman diagrams.[34] For example, a vertex like Figure 1.2 would correspond to a process in which an electron emitted a photon and turned into a positron. This would violate charge conservation and is therefore forbidden.

Feynman diagrams can also be used to describe the fundamental weak and strong interactions. This is illustrated by Figure 1.3a, which shows contributions to the elastic weak scattering reaction $e^- + \nu_e \to e^- + \nu_e$ due to the exchange of a Z^0 and by Figure 1.3b that shows the exchange of a gluon g (represented by a coiled line) between two quarks, which is a strong interaction.

Feynman diagrams that involve hadrons can also be drawn. As illustrations, Figure 1.4a shows the decay of a neutron via an intermediate W boson; and Figure 1.4b denotes the exchange of a charged pion (shown as a dashed line) between a proton and a neutron. We shall see later that the latter mechanism is a major contribution to the strong nuclear force between a proton and a neutron.

We turn now to consider in more detail the relation between exchanged particles and forces.

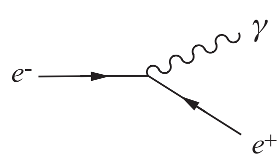

Figure 1.2 The forbidden vertex $e^- \to e^+ + \gamma$.

[34] Compare Kirchhoff's laws in electromagnetism.

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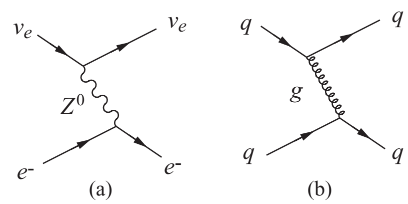

Figure 1.3 (a) Contributions of Z^0 exchange to the elastic weak scattering reaction $e^- + \nu_e \to e^- + \nu_e$; (b) gluon exchange contribution to the strong interaction $q + q \to q + q$.

1.5 Particle Exchange: Forces and Potentials

This section starts with a discussion of the important relationship between forces and particle exchanges and then relates this to potentials. Although the idea of a potential has its greatest use in nonrelativistic physics, nevertheless it is useful to illustrate concepts and is used in later sections as an intermediate step in relating theoretical Feynman diagrams to measurable quantities. The results can be extended to more general situations.

1.5.1 Range of Forces

At each vertex of a Feynman diagram, charge is conserved by construction. We will see later that, depending on the nature of the interaction (strong, weak or electromagnetic), other quantum numbers are also conserved. However, it is easy to show that energy and momentum *cannot* be conserved simultaneously.

Consider the general case of a reaction $A + B \to A + B$ mediated by the exchange of a particle X, as shown in Figure 1.5. In the rest frame of the incident particle A, the lower vertex represents the *virtual* process ('virtual' because X does not appear as a real particle in the final state),

$$A(M_A c^2, \mathbf{0}) \to A(E_A, \mathbf{p}_A c) + X(E_X, -\mathbf{p}_A c), \tag{1.35}$$

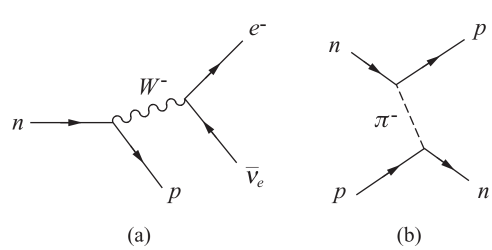

Figure 1.4 (a) The decay of a neutron via an intermediate W boson; and (b) single-pion exchange in the reaction $p + n \to n + p$.

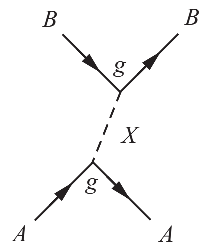

Figure 1.5 Exchange of a particle X in the reaction $A + B \to A + B$.

where E_A is the *total* energy of the final particle A and $\mathbf{p}_A$ is its 3-momentum.[35] Thus, if we denote by P_A the 4-momentum for particle A,

$$P_A = (E_A/c, \mathbf{p}_A) \tag{1.36}$$

and

$$P_A^2 = E_A^2/c^2 - \mathbf{p}_A^2 = M_A^2 c^2. \tag{1.37}$$

Applying this to the diagram and imposing momentum conservation, gives

$$E_A = \left(p^2c^2 + M_A^2 c^4\right)^{1/2} \quad \text{and} \quad E_X = \left(p^2c^2 + M_X^2 c^4\right)^{1/2}, \tag{1.38}$$

where $p = |\mathbf{p}_A|$. The energy difference between the final and initial states is given by

$$\begin{aligned} \Delta E = E_X + E_A - M_A c^2 &\to 2pc, \quad p \to \infty \\ &\to M_X c^2, \; p \to 0 \end{aligned} \tag{1.39}$$

and thus $\Delta E \geq M_X c^2$ for all p, i.e. energy is not conserved. However, by the Heisenberg uncertainty principle, such an energy violation is allowed, but only for a time $\tau \leq \hbar/\Delta E$, so we immediately obtain

$$r \leq R \equiv \hbar/M_X c \tag{1.40}$$

as the maximum distance over which X can propagate before being absorbed by particle B. This maximum distance is called the *range* of the interaction and this was the sense of the word used in Section 1.1.1.

The electromagnetic interaction has an infinite range, because the exchanged particle is a massless photon. In contrast, the weak interaction is associated with the exchange of very heavy particles – the W and Z bosons. These lead to ranges that from (1.40) are of order $R_{W,Z} \approx 2 \times 10^{-18}$ m. The fundamental strong interaction has infinite range because, like the photon, gluons have zero mass. On the other hand, the strong nuclear force, as exemplified by Figure 1.4b, has a much shorter range of approximately $(1-2) \times 10^{-15}$ m. We will comment briefly on the relation between these two different manifestations of the strong interaction in Section 7.1.

[35] A résumé of relativistic kinematics is given in Appendix B.

1.5.2 The Yukawa Potential

In the limit that M_A becomes large, we can regard B as being scattered by a static potential of which A is the source. This potential will in general be spin dependent, but its main features can be obtained by neglecting spin and considering X to be a spin-0 boson, in which case it will obey the Klein-Gordon equation,

$$-\hbar^2 \frac{\partial^2 \phi(\mathbf{r}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \phi(\mathbf{r}, t) + M_X^2 c^4 \phi(\mathbf{r}, t). \tag{1.41}$$

The static solution of this equation satisfies

$$\nabla^2 \phi(\mathbf{r}) = \frac{M_X^2 c^2}{\hbar^2} \phi(\mathbf{r}), \tag{1.42}$$

where $\phi(\mathbf{r})$ is interpreted as a static potential. For $M_X = 0$ this equation is the same as that obeyed by the electrostatic potential, and for a point charge $-e$ interacting with a point charge $+e$ at the origin, the appropriate solution is the Coulomb potential

$$V(r) = -e\phi(r) = -\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r}, \tag{1.43}$$

where $r = |\mathbf{r}|$ and ε_0 is the dielectric constant. The corresponding solution in the case where $M_X^2 \neq 0$ is easily verified by substitution to be

$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-r/R}}{r}, \tag{1.44}$$

where R is the range defined earlier and g, the so-called *coupling constant,* is a parameter associated with each vertex of a Feynman diagram and represents the basic strength of the interaction.[36] For simplicity, we have assumed equal strengths for the coupling of particle X to the particles A and B.

The form of $V(r)$ in (1.44) is called a *Yukawa potential*, after the physicist who in 1935 first introduced the idea of forces due to the exchange of massive particles.[37] As $M_X \to 0$, $R \to \infty$ and the Coulomb potential is recovered from the Yukawa potential, while for very large masses the interaction is approximately point-like (zero range). It is conventional to introduce a dimensionless parameter α_X by

$$\alpha_X = \frac{g^2}{4\pi\hbar c}, \tag{1.45}$$

that characterizes the strength of the interaction at short distances $r \leq R$. For the electromagnetic interaction this is the *fine structure constant*

$$\alpha \equiv e^2/4\pi\varepsilon_0\hbar c \approx 1/137 \tag{1.46}$$

that governs the splittings of atom energy levels.[38]

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[36] Although we call g a (point) coupling *constant*, in general it will have a dependence on the momentum carried by the exchanged particle. We ignore this in what follows.

[37] For this insight, Hideki Yukawa received the 1949 Nobel Prize in Physics.

[38] Like g, the coupling α_X will in general have a dependence on the momentum carried by particle X. In the case of the electromagnetic interaction, this dependence is relatively weak.

The forces between hadrons are also generated by the exchange of particles. Thus, in addition to the electromagnetic interaction between charged hadrons, all hadrons, whether charged or neutral, experience a strong *short-range* interaction, which in the case of two nucleons, for example, has a range of about 10^{-15} m, corresponding to the exchange of a particle with an effective mass of about $\frac{1}{7}$th the mass of the proton. The dominant contribution to this force is the exchange of a single pion, as shown in Figure 1.4b. This nuclear strong interaction is a complicated effect that has its origins in the fundamental strong interactions between the quark distributions within the two hadrons. Two neutral atoms also experience an electromagnetic interaction (the van der Waals force), which has its origins in the fundamental Coulomb forces, but is of much shorter range. Although an analogous mechanism is not in fact responsible for the nuclear strong interaction, it is a useful reminder that the force between two *distributions* of particles can be much more complicated than the forces between the individual components. We will return to this point when we discuss the nature of the nuclear potential in more detail in Section 7.1.

1.6 Observable Quantities: Cross-sections and Decay Rates

We have mentioned earlier that Feynman diagrams can be translated into probabilities for a process by a set of mathematical rules (the *Feynman Rules*) that can be derived from the quantum theory of the underlying interaction. In the case of the electromagnetic interaction, the theory is called Quantum Electrodynamics (QED) and is spectacularly successful in explaining experimental results.[39] We will not pursue this in detail in this book, but rather will show in principle their relation to *observables*, i.e. things that can be measured, concentrating on the cases of two-body scattering reactions and decays of unstable states.

1.6.1 Amplitudes

The intermediate step is the *amplitude* $\mathcal{M}$, the modulus squared of which is directly related to the probability of the process occurring. To get some qualitative idea of the structure of $\mathcal{M}$, we will use nonrelativistic quantum mechanics and assume that the coupling constant g^2 is small compared to $4\pi\hbar c$, so that the interaction is a small perturbation on the free particle solution, which will be taken as plane waves.

In lowest-order perturbation theory, the probability amplitude for a particle with initial momentum $\mathbf{q}_i$ to be scattered to a final state with momentum $\mathbf{q}_f$ by a potential $V(\mathbf{r})$ is proportional to[40]

$$\mathcal{M}(\mathbf{q}) = \int \mathrm{d}^3\mathbf{r}\, V(\mathbf{r}) \exp(i\mathbf{q}\cdot\mathbf{r}/\hbar), \qquad (1.47)$$

[39] Richard Feynman, Sin-Itiro Tomonoga and Julian Schwinger shared the 1965 Nobel Prize in Physics for their work on formulating quantum electrodynamics. The Feynman rules are discussed in an accessible way in Griffiths (1987).

[40] This is called the Born approximation. For a discussion, see, for example, Section 10.2.2 of Mandl (1992), or pp. 397–399 of Gasiorowicz (1974).

where $\mathbf{q} \equiv \mathbf{q}_i - \mathbf{q}_f$ is the momentum transfer. The integration may be done using polar co-ordinates. Taking $\mathbf{q}$ in the z direction, gives

$$\mathbf{q} \cdot \mathbf{r} = |\mathbf{q}|\, r \cos\theta \tag{1.48}$$

and

$$\mathrm{d}^3\mathbf{r} = r^2 \sin\theta \,\mathrm{d}\theta\, \mathrm{d}r\, \mathrm{d}\phi, \tag{1.49}$$

where $r \equiv |\mathbf{r}|$. For the Yukawa potential, the integral (1.47) gives

$$\mathcal{M}(\mathbf{q}^2) = \frac{-g^2\hbar^2}{|\mathbf{q}|^2 + M_X^2 c^2}. \tag{1.50}$$

In deriving (1.50) for the scattering amplitude, we have used potential theory, treating the particle A as a static source. The particle B then scatters through some angle without loss of energy, so that $|\mathbf{q}_i| = |\mathbf{q}_f|$ and the initial and final energies of particle B are equal, $E_i = E_f$. While this is a good approximation at low energies, at higher energies the recoil energy of the target particle cannot be neglected, so that the initial and final energies of B are no longer equal. A full relativistic calculation taking account of this is beyond the scope of this book, but in lowest-order perturbation theory the result is

$$\mathcal{M}(q^2) = \frac{g^2\hbar^2}{q^2 - M_X^2 c^2}, \tag{1.51}$$

where

$$q^2 \equiv (E_f - E_i)^2/c^2 - (\mathbf{q}_f - \mathbf{q}_i)^2 \tag{1.52}$$

is the squared four-momentum transfer. The denominator in (1.51) is called the *propagator*. In the low-energy limit, $E_i = E_f$ and (1.51) reduces to (1.50). However, in contrast to (1.50), which was derived in the rest frame of particle A, the form (1.51) is explicitly Lorentz invariant and holds in all inertial frames of reference. It is thus also called the *invariant amplitude*.[41]

In the zero-range approximation, (1.51) reduces to a constant. To see this, we note that this approximation is valid when the range $R = \hbar/M_X c$ is very small compared with the de Broglie wavelengths of all the particles involved. In particular, this implies $q^2 \ll M_X^2 c^2$ and neglecting q^2 in (1.51) gives

$$\mathcal{M}(q^2) = -G, \tag{1.53}$$

where the constant G is given by

$$\frac{G}{(\hbar c)^3} = \frac{1}{\hbar c}\left(\frac{g}{M_X c^2}\right)^2 \equiv \frac{4\pi\alpha_X}{(M_X c^2)^2} \tag{1.54}$$

and the right-hand side has the dimensions of inverse energy squared. Thus we see that in the zero-range approximation, the resulting point interaction between A and B is characterized by a single dimensioned coupling constant G and not g and M_X separately. As we shall see

[41] Relativistic kinematics will be used in Chapter 5 when we discuss the scattering of high-energy leptons from nucleons.

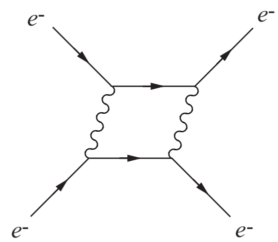

Figure 1.6 Two-photon exchange in the reaction $e^- + e^- \to e^- + e^-$.

later, this approximation is extremely useful in weak interactions, where the corresponding *Fermi coupling constant*, measured for example in nuclear β decay, is given by

$$\frac{G_F}{(\hbar c)^3} = 1.166 \times 10^{-5}\ \text{GeV}^{-2}. \tag{1.55}$$

The amplitude (1.50) corresponds to the exchange of a single particle, as shown for example in Figure 1.5. It is also possible to draw more complicated Feynman diagrams that correspond to the exchange of more than one particle. An example of such a diagram for elastic e^-e^- scattering, where two photons are exchanged, is shown in Figure 1.6. Multiparticle exchange corresponds to higher orders in perturbation theory and higher powers of g^2.

The number of vertices in any diagram is called the *order n*, and when the amplitude associated with any given Feynman diagram is calculated, it always contains a factor of $(\sqrt{\alpha})^n$. Since the probability is proportional to the square of the modulus of the amplitude, the former will contain a factor α^n. The probability associated with the single-photon exchange diagrams of Figure 1.1 thus contain a factor of α^2 and the contribution from two-photon exchange is of order α^4. As $\alpha \sim 1/137$, the latter is usually very small compared to the contribution from single-photon exchange. This is a general feature of electromagnetic interactions: because the fine structure constant is very small, in most cases only the lowest-order diagrams that contribute to a given process need be taken into account, and more complicated higher-order diagrams with more vertices can to a good approximation be ignored in many applications.

1.6.2 Cross-sections

The next step is to relate the amplitude to measurables. For scattering reactions, the appropriate observable is the *cross-section*. In a typical scattering experiment, a beam of particles is allowed to hit a target and the rates of production of various particles in the final state are counted.[42] It is clear that the rates will be proportional to: (a) the number N of particles in the target illuminated by the beam; and (b) the rate per unit area at which beam particles cross a small surface placed in the beam at rest with respect to the target

[42] The practical aspects of experiments are discussed in Chapter 4.

and perpendicular to the beam direction. The latter is called the *flux* and is given by

$$J = n_b \upsilon_i, \tag{1.56}$$

where n_b is the number density of particles in the beam and υ_i is their velocity[43] in the rest frame of the target. Hence the rate W_r at which a specific reaction r occurs in a particular experiment can be written in the form

$$W_r = JN\sigma_r, \tag{1.57a}$$

where σ_r, the constant of proportionality, is called the *cross-section* for reaction r. If the beam has a cross-sectional area S, its intensity is $I = JS$ and so an alternative expression for the rate is

$$W_r = N\sigma_r I/S = I\sigma_r n_t t, \tag{1.57b}$$

where n_t is the number of target particles per unit volume and t is the thickness of the target. If the target consists of an isotopic species of atomic mass M_A in atomic mass units (these are defined in Section 1.7 below), then $n_t = \rho N_A/M_A$, where ρ is the density of the target and N_A is Avogadro's constant. Thus, (1.57b) may be written

$$W_r = I\sigma_r(\rho t)N_A/M_A, \tag{1.57c}$$

where (ρt) is a measure of the amount of material in the target, expressed in units of mass per unit area. The form (1.57c) is particularly useful for the case of thin targets commonly used in experiments (such as those of Rutherford and his collaborators) to reduce the probability of multiple scattering. In the above, the product JN is called the *luminosity* L, i.e.

$$L \equiv JN \tag{1.58}$$

and contains all the dependencies on the densities and geometries of the beam and target. The cross-section is independent of these factors.

It can be seen from the above equations that the cross-section has the dimensions of an area; the rate per target particle $J\sigma_r$ at which the reaction occurs is equal to the rate at which beam particles would hit a surface of area σ_r, placed in the beam at rest with respect to the target and perpendicular to the beam direction. Since the area of such a surface is unchanged by a Lorentz transformation in the beam direction, the cross-section is the same in all inertial frames of reference; i.e. it is a Lorentz invariant.

The quantity σ_r is better named the *partial cross-section*, because it is the cross-section for a particular reaction r. The *total cross-section* σ_{tot} is defined by

$$\sigma_{tot} \equiv \sum_r \sigma_r, \tag{1.59}$$

where the summation is over all allowed reactions. Another useful quantity is the *differential cross-section*, $\mathrm{d}\sigma_r(\theta,\phi)/\mathrm{d}\Omega$, for a particular reaction r, which is defined by

$$\mathrm{d}W_r \equiv JN\frac{\mathrm{d}\sigma_r(\theta,\phi)}{\mathrm{d}\Omega}\mathrm{d}\Omega, \tag{1.60}$$

[43] Strictly, their *speed*, but we will conform to common usage and call υ_i the velocity.

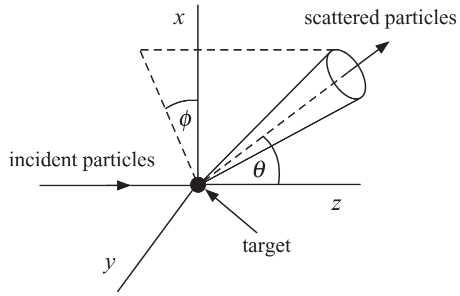

Figure 1.7 Geometry of the differential cross-section. A beam of particles is incident along the z axis and collides with a stationary target at the origin. The differential cross-section is proportional to the rate for particles to be scattered into a small solid angle $\mathrm{d}\Omega$ in the direction (θ, ϕ).

where $\mathrm{d}W_r$ is the measured rate for the particles to be emitted into an element of solid angle $\mathrm{d}\Omega = \mathrm{d}\cos\theta\,\mathrm{d}\phi$ in the direction (θ, ϕ), as shown in Figure 1.7. The partial cross-section σ_r is obtained by integrating the differential cross-section over all angles, i.e.,

$$\sigma_r = \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}\cos\theta\, \frac{\mathrm{d}\sigma_r(\theta, \phi)}{\mathrm{d}\Omega}. \tag{1.61}$$

The final step is to write these formulas in terms of the scattering amplitude $\mathcal{M}(\mathbf{q}^2)$ appropriate for describing the scattering of a nonrelativistic spinless particle from a potential. To do this it is convenient to consider a single beam particle interacting with a single target particle and to confine the whole system in an arbitrary volume V (which cancels in the final result). The incident flux is then given by

$$J = n_b \upsilon_i = \upsilon_i / V \tag{1.62}$$

and since the number of target particles is $N = 1$, the differential rate is

$$\mathrm{d}W_r = \frac{\upsilon_i}{V} \frac{d\sigma_r(\theta, \phi)}{\mathrm{d}\Omega} \mathrm{d}\Omega. \tag{1.63}$$

In quantum mechanics, provided the interaction is not too strong, the transition rate for any process is given in perturbation theory by the Born approximation[44]

$$\mathrm{d}W_r = \frac{2\pi}{\hbar} \left| \int \mathrm{d}^3\mathbf{r}\, \psi_r^* V(\mathbf{r}) \psi_i \right|^2 \rho(E_f). \tag{1.64}$$

The term $\rho(E_f)$ is the *density-of-states factor* (see below) and we take the initial and final state wavefunctions to be plane waves:

$$\psi_i = \frac{1}{\sqrt{V}} \exp(i\mathbf{q}_i \cdot \mathbf{r}/\hbar), \qquad \psi_f = \frac{1}{\sqrt{V}} \exp(i\mathbf{q}_f \cdot \mathbf{r}/\hbar), \tag{1.65}$$

[44] This equation is a form of the *Second Golden Rule* in quantum mechanics. It is discussed in Section A.3.

where the final momentum $\mathbf{q}_f$ lies within a small solid angle $\mathrm{d}\Omega$ located in the direction (θ, ϕ). (See Figure 1.7.) Then, by direct integration,

$$\mathrm{d}W_r = \frac{2\pi}{\hbar V^2}|\mathcal{M}(\mathbf{q}^2)|^2 \rho(E_f), \tag{1.66}$$

where $\mathcal{M}(\mathbf{q}^2)$ is the scattering amplitude defined in (1.50).

The density of states $\rho(E_f)$ that appears in (1.64) is the number of possible final states with energy lying between E_f and $E_f + \mathrm{d}E_f$ and is given by[45]

$$\rho(E_f) = \frac{V}{(2\pi\hbar)^3} q_f^2 \frac{\mathrm{d}q_f}{\mathrm{d}E_f} \mathrm{d}\Omega, \tag{1.67}$$

where, nonrelativistically,

$$\mathrm{d}q_f/\mathrm{d}E_f = 1/\upsilon_f. \tag{1.68}$$

If we use (1.66), (1.67) and (1.68) in (1.63), we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4\pi^2\hbar^4} \frac{q_f^2}{\upsilon_i \upsilon_f} |\mathcal{M}(\mathbf{q}^2)|^2. \tag{1.69}$$

Although this result has been derived in the laboratory system, because we have taken a massive target it is also valid in the centre-of-mass system.

The only place where nonrelativistic kinematics have been explicitly used in obtaining (1.69) is in the derivation of the density-of-states factor, so to have a formula that is also true for the general two-body relativistic scattering process $a + b \to c + d$, we have to re-examine the derivative (1.68) using relativistic kinematics. In this case we can use

$$E_f = E_c + E_d = \left(q_f^2 c^2 + m_c^2 c^4\right)^{1/2} + \left(q_f^2 c^2 + m_d^2 c^4\right)^{1/2} \tag{1.70}$$

to give

$$\frac{\mathrm{d}E_f}{\mathrm{d}q_f} = q_f c^2 \left(\frac{1}{E_c} + \frac{1}{E_d}\right), \tag{1.71}$$

which, using the relativistic relation $\mathbf{v} = \mathbf{p}c^2/E$ (see Equation (B.11) of Appendix B) and noting that in the centre-of-mass system $\mathbf{p}_c = -\mathbf{p}_d$, yields

$$\frac{\mathrm{d}q_f}{\mathrm{d}E_f} = \frac{1}{\upsilon_f}, \tag{1.72}$$

where υ_f is the modulus of the relative velocity of particles c and d. Thus the general interpretation of (1.69) is that $q_f = |\mathbf{q}_c| = |\mathbf{q}_d|$ is the centre-of-mass momentum of the final-state particles and $\upsilon_{i,f}$ are the relative velocities in the centre-of-mass of particles a and b, and c and d, respectively.

All the above is for spinless particles, so finally we have to generalize (1.69) to include the effects of spin. Suppose the initial-state particles a and b, have spins S_a and S_b and

[45] The derivation is given in detail in Section A.2.

the final-state particles c and d have spins S_c and S_d. The total numbers of spin substates available to the initial and final states are g_i and g_f, respectively, given by

$$g_i = (2S_a + 1)(2S_b + 1) \quad \text{and} \quad g_f = (2S_c + 1)(2S_d + 1). \tag{1.73}$$

If the initial particles are unpolarized (which is the most common case in practice), then we must average over all possible initial spin configurations (because each is equally likely) and sum over the final configurations. Thus, (1.69) becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{g_f}{4\pi^2\hbar^4}\frac{q_f^2}{\upsilon_i \upsilon_f}|\mathcal{M}_{fi}|^2, \tag{1.74}$$

where

$$|\mathcal{M}_{fi}|^2 \equiv \overline{|\mathcal{M}(\mathbf{q}^2)|^2} \tag{1.75}$$

and the bar over the amplitude denotes a spin-average of the squared matrix element.

1.6.3 Unstable States

In the case of an unstable state, the observable of interest is its *lifetime at rest* τ, or equivalently its *natural decay width*, given by $\Gamma = \hbar/\tau$, which is a measure of the rate of the decay reaction. In general, an initial unstable state will decay to several final states and in this case we define Γ_f as the *partial width* for a specific final state f and

$$\Gamma = \sum_f \Gamma_f \tag{1.76}$$

as the *total decay width*, while

$$B_f \equiv \Gamma_f / \Gamma \tag{1.77}$$

is defined as the *branching ratio* for decay to the state f.

The energy distribution of an isolated unstable state to a final state f has the *Breit-Wigner* form

$$N_f(W) \propto \frac{\Gamma_f}{(W - M)^2 c^4 + \Gamma^2/4}, \tag{1.78}$$

where M is the mass of the decaying state and W is the invariant mass of the decay products.[46] The Breit-Wigner formula is shown in Figure 1.8 and is the same formula that describes the widths of atomic and nuclear spectral lines. (The overall factor depends on the spins of the particles involved.) It is a symmetrical bell-shaped curve with a maximum at $W = M$ and a full width Γ at half the maximum height of the curve. It is proportional to the number of events with invariant mass W.

If an unstable state is produced in a scattering reaction, then the cross-section for that reaction will show an enhancement described by the same Breit-Wigner formula. In this case we say we have produced a *resonance state*. In the vicinity of a resonance of mass M,

[46] This form arises from a state that decays exponentially with time, although a proof of this is quite lengthy. See, for example, Appendix B of Martin and Shaw (2008).

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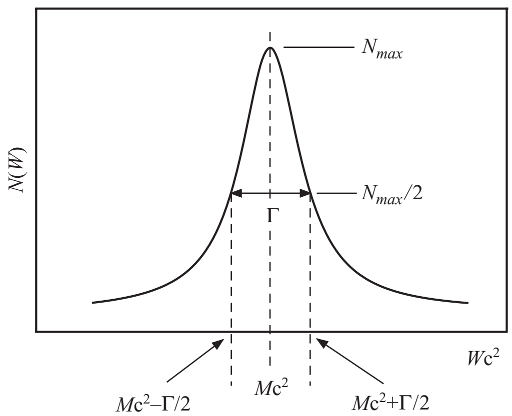

Figure 1.8 The Breit-Wigner formula (1.78).

and width Γ, the cross-section for the reaction $i \to f$ has the form

$$\sigma_{fi} \propto \frac{\Gamma_i \Gamma_f}{(E - Mc^2)^2 + \Gamma^2/4}, \tag{1.79}$$

where E is the total energy of the system. Again, the form of the overall constant will depend on the spins of the particles involved. Thus, for example, if the resonance particle has spin j and the spins of the initial particles are S_1 and S_2, then

$$\sigma_{fi} = \frac{\pi \hbar^2}{q_i^2} \frac{2j + 1}{(2S_1 + 1)(2S_2 + 1)} \frac{\Gamma_i \Gamma_f}{(E - Mc^2)^2 + \Gamma^2/4}. \tag{1.80}$$

In practice there will also be kinematical and angular momentum effects that will distort this formula from its perfectly symmetric shape.

An example of resonance formation in $\pi^- p$ interactions is given in Figure 1.9, which shows the $\pi^- p$ total cross-section in the centre-of-mass energy range 1.2–2.4 GeV. (The units used in the plots will become clear after the next section.) Two enhancements can be seen that are of the approximate Breit-Wigner resonance form and there are two other maxima at higher energies. In principle, the mass and width of a resonance may be obtained by using a Breit-Wigner formula and varying M and Γ to fit the cross-section in the region of the enhancement. In practice more sophisticated methods are used simultaneously that fit a wide range of data, including differential cross-sections, and also take account of nonresonant contributions to the scattering. The widths obtained from such analyses are of the order of 100 MeV, with corresponding interaction times of order 10^{-23} s, which is consistent with the time taken for a relativistic pion to transit the dimension of a proton. Resonances are also a prominent feature of interactions in nuclear physics and we will return to this in Section 2.9 when we discuss nuclear reaction mechanisms.

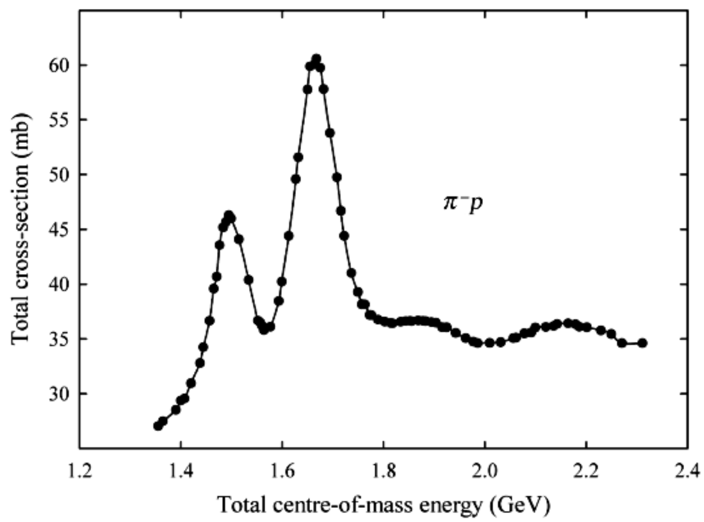

Figure 1.9 Total cross-sections for $\pi^- p$ interactions. (Data from Carter *et al*. (1968)).

1.7 Units: Length, Mass and Energy

Most branches of science introduce special units that are convenient for their own purposes. Nuclear and particle physics are no exceptions. Distances tend to be measured in femtometres or, equivalently *fermis*, with 1 fm $\equiv 10^{-15}$ m. In these units, the radius of the proton is about 0.8 fm. The range of the strong nuclear force between protons and neutrons is of order 1–2 fm, while the range of the weak force is of order 10^{-3} fm. For comparison, the radii of atoms are of order 10^5 fm. A common unit for area is the *barn* defined by 1 b $= 10^{-28}$ m^2. For example, the total cross-section for pp scattering (a strong interaction) is a few tens of millibarns (mb) (compare also the $\pi^- p$ total cross-section in Figure 1.9), whereas the same quantity for νp scattering (a weak interaction) is a few tens of femtobarns (fb), depending on the energies involved. Nuclear cross-sections are very much larger and increase approximately like $A^{2/3}$, where A is the total number of nucleons in the nucleus.

Energies are invariably specified in terms of the electron volt, eV, defined as the energy required to raise the electric potential of an electron or proton by one volt. In S.I. units, 1 eV $= 1.6 \times 10^{-19}$ joules. The units 1 keV $= 10^3$ eV, 1 MeV $= 10^6$ eV, 1 GeV $= 10^9$ eV and 1 TeV $= 10^{12}$ eV are also in general use. In terms of these units, atomic ionization energies are typically a few eV, the energies needed to bind nucleons in heavy nuclei are typically 7–8 MeV per particle, and the highest particle energies produced in the laboratory are of order of a few TeV for protons. Momenta are specified in eV/c, MeV/c etc.

In order to create a new particle of mass M, an energy at least as great as its rest energy Mc^2 must be supplied. The rest energies of the electron and proton are 0.51 MeV and 0.94 GeV respectively, whereas the W and Z^0 bosons have rest energies of 80 GeV and 91 GeV, respectively. Correspondingly their masses are conveniently measured in MeV/c^2

or GeV/c^2, so that, for example,

$$\begin{aligned} M_e &= 0.51\,\text{MeV/c}^2, \quad M_p = 0.94\,\text{GeV/c}^2, \\ M_W &= 80.4\,\text{GeV/c}^2, \quad M_Z = 91.2\,\text{GeV/c}^2. \end{aligned} \tag{1.81}$$

In S.I. units, $1\,\text{MeV/c}^2 = 1.78 \times 10^{-30}$ kg. In nuclear physics it is also common to express masses in *atomic mass units* (u), defined as $\frac{1}{12}$ the mass of the commonest isotope of carbon: $1\,\text{u} = 1.661 \times 10^{-27}\,\text{kg} = 931.5\,\text{MeV/c}^2$.

Although practical calculations are expressed in the above units, it is usual in particle physics to make theoretical calculations in units chosen such that $\hbar \equiv h/2\pi = 1$ and $c = 1$ (called *natural units*) and many books do this. However, as this book is about both nuclear and particle physics, practical units will be used, the sole exception being in Appendix D. A table giving numerical values of fundamental and derived constants, together with some useful conversion factors is given in Section E.1.

Problems

1.1 'Derive' the Klein-Gordon equation using the information in Footnote 23 and verify that the Yukawa potential (1.44) is a static solution of the equation.

1.2 Verify that the spherical harmonic $Y_1^1 = \sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$ is an eigenfunction of parity with eigenvalue $P = -1$.

1.3 A proton and antiproton at rest in an S-state annihilate to produce $\pi^0\pi^0$ pairs. Show that this reaction cannot be a strong interaction.

1.4 Suppose that an intrinsic C-parity factor is introduced into (1.15b), which then becomes

$$\hat{C}|b, \psi_b\rangle = C_b|\bar{b}, \psi_{\bar{b}}\rangle.$$

Show that the eigenvalue corresponding to any eigenstate of $\hat{C}$ is independent of C_b, so that C_b cannot be measured.

1.5 In classical physics, in the absence of explicit electric charges, the electromagnetic field may be described by an electric field vector, $\mathbf{E}(\mathbf{r}, t)$ or a vector potential $\mathbf{A}(\mathbf{r}, t)$. These are related by $\mathbf{E} = -\partial\mathbf{A}/\partial t$. If the electromagnetic interaction is invariant under charge conjugation, deduce the C parity of the photon.

1.6 Show that a collection of i particles with electric charges q_i and position vectors $\mathbf{r}_i$ will have a zero electric dipole moment if time-reversal invariance holds.

1.7 Use the principle of detailed balance applied to the reactions $pp \rightleftarrows \pi^+ d$ to deduce that the spin of the π^+ may be found from the expression

$$S_\pi = \frac{1}{2}\left[\frac{4R}{3}\left(\frac{p_p}{p_\pi}\right)^2 - 1\right],$$

where $p_{p,\pi}$ are the magnitudes of the proton and pion momenta and

$$R = \frac{\mathrm{d}\sigma(pp \to \pi^+ d)/\mathrm{d}\Omega}{\mathrm{d}\sigma(\pi^+ d \to pp)/\mathrm{d}\Omega},$$

where the differential cross-sections are at the same total centre-of-mass energy and both beams and projectiles are unpolarized.

1.8 Consider the reaction $\pi^- d \to nn$, where d is a spin–1 S-wave bound state of a proton and a neutron called the deuteron and the initial pion is at rest. Deduce the intrinsic parity of the negative pion.

1.9 Write down equations in symbol form that describe the following interactions:

(a) elastic scattering of an electron antineutrino and a positron;

(b) inelastic production of a pair of neutral pions in proton-proton interactions;

(c) the annihilation of an antiproton with a neutron to produce three pions.

1.10 Draw a lowest-order Feynman diagram for the following processes: (a) $\nu_e \nu_\mu$ elastic scattering and (b) $e^+e^- \to e^+e^-$; and (c) a fourth-order diagram for the reaction $\gamma + \gamma \to e^+ + e^-$.

1.11 Calculate the energy-momentum transfer between two particles equivalent to a distance of approach of (a) 1 fm and (b) 10^{-3} fm. Assuming that the intrinsic strengths of the fundamental weak and electromagnetic interactions are approximately equal, compare the relative sizes of the invariant (scattering) amplitudes for weak and electromagnetic processes at these two energy-momentum transfers.

1.12 Verify by explicit integration that

$$\mathcal{M}(q^2) = -g^2\hbar^2(q^2 + m^2c^2)^{-1}$$

is the amplitude corresponding to the Yukawa potential (1.44).

1.13 Two beams of particles, consisting of n bunches with N_i $(i = 1, 2)$ particles in each, traverse circular paths and collide 'head-on'. Show that in this case the general expression for the luminosity (1.58) reduces to $L = nN_1N_2 f/A$, where A is the cross-sectional area of the beam and f is the frequency, i.e. $f = 1/T$, where T is the time taken for the particles to make one traversal of the ring.

1.14 A thin ('density' 1 mg cm^{-2}) target of ^{24}Mg ($M_A = 24.3$ atomic mass units) is bombarded with a 10 nA beam of alpha particles. A detector subtending a solid angle of 2×10^{-3} sr, records 20 protons per second. If the scattering is isotropic, determine the cross-section for the ^{24}Mg(α, p) reaction.

1.15 The cross-section for photon scattering from free electrons when $E_\gamma \ll m_e c^2$ is given *in natural units* by

$$\sigma = \frac{8\pi\alpha^2}{3m_e^2}.$$

What is the value of σ in mb?