

Chapter 8

POWERS OF TEN AND SCIENTIFIC NOTATION

Chapter Check-In

- Operations with powers of ten
- Changing to scientific notation
- Operations with scientific notation

Powers of Ten

Since our number system is based on **powers** of ten, you should understand the notation and how to work with these powers, as follows:

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1,000$$

and so on.

$$10^{-1} = \frac{1}{10} = .1$$

$$10^{-2} = \frac{1}{100} = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = .01$$

$$10^{-3} = \frac{1}{1000} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000} = .001$$

and so on.

Multiplying powers of ten

To multiply powers of ten, add the exponents.

Example 1: Multiply the following and leave the answers in powers of ten.

- (a) 100×10
 (b) $1,000 \times 100$
 (c) $.01 \times .001$
 (d) $10,000 \times .01$
 (e) $.0001 \times 1,000$
- (a) $100 \times 10 = 10^2 \times 10^1 = 10^{(2+1)} = 10^3$
 (b) $1,000 \times 100 = 10^3 \times 10^2 = 10^{(3+2)} = 10^5$
 (c) $.01 \times .001 = 10^{-2} \times 10^{-3} = 10^{[-2+(-3)]} = 10^{-5}$
 (d) $10,000 \times .01 = 10^4 \times 10^{-2} = 10^{[4+(-2)]} = 10^2$
 (e) $.0001 \times 1,000 = 10^{-4} \times 10^3 = 10^{(-4+3)} = 10^{-1}$

Dividing powers of ten

To divide powers of 10, subtract the exponents; that is, subtract the exponent of the second number (the *divisor*).

Example 2: Divide the following and leave the answers in powers of ten.

- (a) $1,000 \div 100$
 (b) $100 \div 10,000$
 (c) $1 \div .01$
 (d) $.001 \div .01$
 (e) $10,000 \div .1$
- (a) $1,000 \div 100 = 10^3 \div 10^2 = 10^{(3-2)} = 10^1$ or 10
 (b) $100 \div 10,000 = 10^2 \div 10^4 = 10^{(2-4)} = 10^{-2}$
 (c) $1 \div .01 = 10^0 \div 10^{-2} = 10^{[0-(-2)]} = 10^{(0+2)}$ or 10^2
 (d) $.001 \div .01 = 10^{-3} \div 10^{-2} = 10^{[-3-(-2)]} = 10^{(-3+2)} = 10^{-1}$
 (e) $10,000 \div .1 = 10^4 \div 10^{-1} = 10^{[4-(-1)]} = 10^{(4+1)} = 10^5$

Scientific Notation

Very large or very small numbers are sometimes written in **scientific notation**. A number written in scientific notation is a number between 1 and 10 multiplied by a power of 10.

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Example 3: Express the following in scientific notation.

- (a) 3,400,000
- (b) .0000008
- (c) .0047
- (d) 27,410

(a) 3,400,000 written in scientific notation is 3.4×10^6 . Simply place the decimal point to get a number between 1 and 10 and then count the digits to the right of the decimal to get the power of 10.

3.400000. moved 6 digits to the left.

(b) .0000008 written in scientific notation is 8×10^{-7} . Simply place the decimal point to get a number between 1 and 10 and then count the digits from the original decimal point to the new one.

.0000008. moved 7 digits to the right.

Notice that whole numbers have positive exponents and fractions have negative exponents.

(c) .004.7 = 4.7×10^{-3}

(d) 2.7430. = 2.743×10^0

Multiplication in scientific notation

To multiply numbers in scientific notation, simply multiply the numbers that are between 1 and 10 together to get the first number and add the powers of ten to get the second number.

Example 4: Multiply the following and express the answers in scientific notation.

(a) $(2 \times 10^2)(3 \times 10^4)$

(b) $(6 \times 10^5)(5 \times 10^7)$

(c) $(4 \times 10^{-4})(2 \times 10^5)$

(d) $(5 \times 10^4)(9 \times 10^2)$

(e) $(2 \times 10^2)(4 \times 10^4)(5 \times 10^6)$

$$\begin{aligned} \text{(a)} \quad (2 \times 10^2)(3 \times 10^4) &= (2 \times 10^2)(3 \times 10^4) \\ &= 6 \times 10^6 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (6 \times 10^5)(5 \times 10^7) &= (6 \times 10^5)(5 \times 10^7) \\ &= 30 \times 10^{12} \end{aligned}$$

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This answer must be changed to scientific notation (first number from 1 to 9).

$$\begin{aligned} 30 \times 10^{12} &= 3.0 \times 10^1 \times 10^{12} \\ &= 3.0 \times 10^{13} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad (4 \times 10^{-4})(2 \times 10^5) &= (4 \times 10^{-4})(2 \times 10^5) \\ &= 8 \times 10^1 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad (5 \times 10^4)(9 \times 10^2) &= (5 \times 10^4)(9 \times 10^2) \\ &= 45 \times 10^6 \\ &= 4.5 \times 10^1 \times 10^6 \\ &= 4.5 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad (2 \times 10^2)(4 \times 10^4)(5 \times 10^6) &= (2 \times 10^2)(4 \times 10^4)(5 \times 10^6) \\ &= 40 \times 10^{12} \\ &= 4.0 \times 10^1 \times 10^{12} \\ &= 4.0 \times 10^{13} \end{aligned}$$

Division in scientific notation

To divide numbers in scientific notation, simply divide the numbers that are between 1 and 10 to get the first number and subtract the powers of ten to get the second number.

Example 5: Divide the following and express the answers in scientific notation.

$$\text{(a)} \quad (8 \times 10^5) \div (2 \times 10^2)$$

$$\text{(d)} \quad (2 \times 10^4) \div (5 \times 10^2)$$

$$\begin{aligned} \text{(b)} \quad & \frac{7 \times 10^{11}}{4 \times 10^5} \end{aligned}$$

$$\text{(e)} \quad (8.4 \times 10^5) \div (2.1 \times 10^{-4})$$

$$\text{(c)} \quad (6 \times 10^7) \div (3 \times 10^9)$$

$$\begin{aligned} \text{(a)} \quad (8 \times 10^5) \div (2 \times 10^2) &= (8 \times 10^5) \div (2 \times 10^2) \\ &= 4 \times 10^3 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{7 \times 10^{11}}{4 \times 10^5} &= (7 \div 4)(10^9 \div 10^3) \\ &= 1.75 \times 10^6 \end{aligned}$$

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$$(c) \quad (6 \times 10^7) \div (3 \times 10^9) = \overbrace{(6 \times 10^7)}^{+} \div \overbrace{(3 \times 10^9)}^{-}$$

$$= 2 \times 10^{-2}$$

$$(d) \quad (2 \times 10^4) \div (5 \times 10^2) = \overbrace{(2 \times 10^4)}^{+} \div \overbrace{(5 \times 10^2)}^{-}$$

$$= .4 \times 10^2$$

This answer must be changed to scientific notation.

$$.4 \times 10^2 = 4 \times 10^{-1} \times 10^2$$

$$= 4 \times 10^1$$

$$(e) \quad (8.4 \times 10^5) \div (2.1 \times 10^{-4}) = \overbrace{(8.4 \times 10^5)}^{+} \div \overbrace{(2.1 \times 10^{-4})}^{-}$$

$$= 4 \times 10^{5-(-4)}$$

$$= 4 \times 10^9$$

Chapter Checkout

Q&A

1. Multiply $1,000 \times 100$ and leave the answer in powers of ten.
2. Multiply $.1 \times .001$ and leave the answer in powers of ten.
3. Divide $100 \div 1,000$ and leave the answer in powers of ten.
4. Express 27,000 in scientific notation.
5. Express .0005 in scientific notation.
6. $(3 \times 10^4)(2 \times 10^3) = \underline{\hspace{2cm}}$. (in scientific notation)
7. $(4 \times 10^{-7})(5 \times 10^2) = \underline{\hspace{2cm}}$. (in scientific notation)
8. $(9 \times 10^4) \div (3 \times 10^{-2}) = \underline{\hspace{2cm}}$. (in scientific notation)

Answers: 1. 10^5 2. 10^{-4} 3. 10^{-1} 4. 2.7×10^4 5. 5×10^{-4} 6. 6×10^7
7. 2×10^{-4} 8. 3×10^6