

Contents at a Glance

<i>Introduction</i>	1
<i>Part I: Set It Up, Solve It, Graph It</i>	7
Chapter 1: Pre-Pre-Calculus	9
Chapter 2: Dealing with Real Numbers	21
Chapter 3: The Foundation of Pre-Calc: Functions	33
Chapter 4: Finding and Using Roots to Graph Polynomial Functions	67
Chapter 5: Powering Up with Exponential and Logarithmic Functions	97
<i>Part II: The Essentials of Trigonometry</i>	117
Chapter 6: Angling In on the Unit Circle	119
Chapter 7: Graphing and Transforming Trig Functions	147
Chapter 8: Using Trig Identities: The Basics	177
Chapter 9: Pre-Calc, Here I Come! Advanced Identities Lead the Way	195
Chapter 10: Solving Oblique Triangles with the Laws of Sines and Cosines	217
<i>Part III: Analytic Geometry and System Solving</i>	235
Chapter 11: A New Plane of Thinking: Complex Numbers and Polar Coordinates	237
Chapter 12: Cutting It Up with Conics	253
Chapter 13: Solving Systems and Mingling with Matrices	283
Chapter 14: Sequences, Series, and Expanding Binomials	315
Chapter 15: Looking Forward to Calculus	337
<i>Part IV: The Part of Tens</i>	349
Chapter 16: Ten Habits That Help You Attack Calculus	351
Chapter 17: Ten Habits to Break before Calculus	359
<i>Index</i>	365

Table of Contents

.....

***Introduction* 1**

About This Book	1
Conventions Used in This Book	2
Foolish Assumptions	3
How This Book Is Organized	3
Part I: Set It Up, Solve It, Graph It	3
Part II: The Essentials of Trigonometry	3
Part III: Analytic Geometry and System Solving	4
Part IV: The Part of Tens	4
Icons Used in This Book	4
Where to Go from Here	5

***Part I: Set It Up, Solve It, Graph It* 7**

Chapter 1: Pre-Pre-Calculus 9

Pre-Calculus: An Overview	9
All the Number Basics (No, Not How to Count Them!)	11
The multitude of number types: Terms to know	11
The fundamental operations you can perform on numbers	12
The properties of numbers: Truths to remember	13
Putting Mathematical Statements in Visual Form: Fun with Graphs	14
Digesting basic terms and concepts	15
Graphing equalities versus inequalities	16
Gathering information from graphs	16
Getting a Grip on a Graphing Calculator	18

Chapter 2: Dealing with Real Numbers 21

Solving Inequalities	21
A brief how-to inequality recap	22
Solving equations and inequalities when absolute value is involved	22
Expressing solutions for inequalities with interval notation	24
Variations on Dividing and Multiplying: Working with Radicals and Exponents	26
Defining and relating radicals and exponents	26
Rewriting radicals as exponents (or, creating rational exponents)	27
Getting a radical out of a denominator: Rationalizing	28

Chapter 3: The Foundation of Pre-Calc: Functions	33
Qualities of Even and Odd Functions and Their Graphs	34
Dealing with Parent Functions (The Most Common) and Their Graphs	34
Quadratic functions	34
Square root functions	35
Absolute value functions	36
Cubic functions	37
Cube root functions	37
Transforming the Parent Graphs	38
Vertical transformations	39
Horizontal transformations	40
Translations	41
Reflections	43
Combining various transformations (a transformation in itself!)	44
Transforming functions point by point	46
Graphing Functions that Have More than One Rule: Piece-Wise Functions	47
Calculating Outputs for Rational Functions	49
Step 1: Search for vertical asymptotes	50
Step 2: Look for horizontal asymptotes	51
Step 3: Seek out oblique asymptotes	51
Step 4: Locate the x - and y -intercepts	52
Putting the Output to Work: Graphing Rational Functions	52
The denominator has the greater degree	53
The numerator and denominator have equal degrees	55
The numerator has the greater degree	57
No Scalpel Necessary: Operating on Functions	58
Adding and subtracting	58
Multiplying and dividing	59
Breaking down a composition of functions	60
Adjusting the domain and range of combined functions (if applicable)	60
Flip-Flopping with Inverse Functions	63
Graphing an inverse	63
Inverting a function to find its inverse	64
Verifying an inverse	65
Chapter 4: Finding and Using Roots to Graph Polynomial Functions	67
The Function of Degrees and Roots	68
Factoring a Polynomial Expression	69
Always the first step: Look for a GCF	70
Wrap it up: The FOIL method for trinomials	71
Recognizing and factoring special types of polynomials	73
Grouping to factor four or more terms	77

Finding the Roots of a Factored Equation	78
Cracking a Quadratic Equation When It Won't Factor	78
Using the quadratic formula	79
Completing the square	79
Solving Unfactorable Polynomials with a Degree Higher than Two	80
Counting a polynomial's total number of roots	81
Tallying the real roots: Descartes' Rule of Signs	81
Accounting for imaginary roots: The Fundamental Theorem of Algebra	82
Guessing and checking the real roots	84
Put It in Reverse: Using Solutions to Find Factors	91
Graphing Polynomials	91
When all the roots are real numbers	92
When some (or all) of the roots are imaginary numbers: Combining all techniques	95

Chapter 5: Powering Up with Exponential and Logarithmic Functions 97

Exploring Exponential Functions	98
Searching the ins and outs of an exponential function	98
Graphing and transforming an exponential function	100
Logarithms: Investigating the Inverse of Exponential Functions	102
Getting a better handle on logarithms	103
Managing the properties and identities of logs	104
Changing a log's base (when the log isn't natural or common)	105
Calculating a number when you know its log: Inverse logs	106
Graphing logs	106
Solving Equations with Exponents and Logs	109
Stepping through the process of exponential equation solving	110
Taking steps to solve logarithm equations	112
Surviving Exponential Word Problems	113

Part II: The Essentials of Trigonometry 117

Chapter 6: Angling In on the Unit Circle 119

Introducing Radians: The Basic Pre-Calc Measurement	119
Trig Ratios: Taking Right Triangles a Step Further	120
Making a sine	121
Looking for a cosine	122
Going on a tangent	123
Discovering the flip side: Reciprocal trig functions	124
Working in reverse: Inverse trig functions	125
Understanding How Trig Ratios Work on the Coordinate Plane	126

Getting a Good Grasp on the Unit Circle	128
Familiarizing yourself with the most common angles	128
Drawing uncommon angles	130
Digesting Special Triangle Ratios	131
The 45er: 45°-45°-90° triangles	131
The old 30-60: 30°-60°-90° triangles	132
The Fusion of Triangles and the Unit Circle:	
Working Together for Good	134
Placing the major angles correctly, sans protractor	134
Retrieving trig-function values on the unit circle	136
Finding the reference angle to solve for angles	
on the unit circle	140
Not Just a Job for Noah: Making and Measuring Arcs	145
Chapter 7: Graphing and Transforming Trig Functions	147
Drafting the Sine and Cosine Parent Graphs	148
The sine graph	148
The cosine graph	150
Graphing Tangent and Cotangent	152
Tangent	152
Cotangent	154
Putting Secant and Cosecant in Picture	156
Secant	156
Cosecant	158
Transforming Trig Graphs	159
Screwing with sine and cosine graphs	160
Tweaking tangent and cotangent graphs	170
Transforming the graphs of secant and cosecant	173
Chapter 8: Using Trig Identities: The Basics	177
Keeping the End in Mind: A Quick Primer on Identities	178
Lining Up the Means to the End: Basic Trig Identities	178
Reciprocal identities	179
Pythagorean identities	181
Even-odd identities	183
Co-function identities	185
Periodicity identities	187
Tackling Difficult Trig Proofs: Some Techniques to Know	189
Dealing with dreaded denominators	189
Going solo on each side	193
Chapter 9: Pre-Calc, Here I Come! Advanced Identities	
Lead the Way	195
Finding Trig Functions of Sums and Differences	196
Searching out the sine of $(a \pm b)$	197
Calculating the cosine of $(a \pm b)$	200
Taming the tangent of $(a \pm b)$	202

Doubling an Angle's Trig Value
 without Knowing the Angle205
 Finding the sine of a doubled angle205
 Calculating cosines for two207
 Squaring your cares away208
 Having twice the fun with tangents208
 Taking Trig Functions of Common Angles Divided in Two210
 A Glimpse of Calculus: Traveling from Products to Sums and Back211
 Expressing products as sums (or differences)212
 Transporting from sums (or differences) to products213
 Eliminating Exponents on Trig Functions
 with Power-Reducing Formulas214

**Chapter 10: Solving Oblique Triangles with
 the Laws of Sines and Cosines217**

Solving a Triangle with the Law of Sines218
 When you know two angle measures219
 When you know two consecutive side lengths (SSA)221
 Conquering a Triangle with the Law of Cosines228
 SSS: Finding angles using only sides228
 SAS: Tagging the angle in the middle (and the two sides)230
 Filling in the Triangle by Calculating Area232
 Finding area with two sides and an included angle
 (for SAS scenarios)232
 Heron's Formula (for SSS scenarios)233

Part III: Analytic Geometry and System Solving235

**Chapter 11: A New Plane of Thinking:
 Complex Numbers and Polar Coordinates237**

Understanding Real versus Imaginary
 (According to Mathematicians)238
 Combining Real and Imaginary: The Complex Number System239
 Grasping the usefulness of complex numbers239
 Performing operations with complex numbers240
 Graphing Complex Numbers242
 Plotting around a Pole: Polar Coordinates243
 Wrapping your brain around the polar coordinate plane243
 Graphing polar coordinates with negative values246
 Changing to and from polar coordinates247
 Picturing polar equations250

Chapter 12: Cutting It Up with Conics	253
Cone to Cone: Identifying the Four Conic Sections	254
In picture (graph form)	254
In print (equation form)	256
Going Round and Round with Circles	257
Graphing a circle	257
Riding the Ups and Downs with Parabolas	259
Labeling the parts	260
Understanding the characteristics of a standard parabola	261
Plotting the variations: Parabolas all over the plane (not at the origin)	261
Finding the vertex, axis of symmetry, focus, and directrix	263
Identifying the min and max on vertical parabolas	266
The Fat and the Skinny on the Ellipse (A Fancy Word for Oval)	268
Labeling ellipses and expressing them with algebra	269
Identifying the parts of the oval: Vertices, co-vertices, axes, and foci	270
Pair Two Parabolas and What Do You Get? Hyperbolas	273
Visualizing the two types of hyperbolas and their bits and pieces	273
Graphing a hyperbola from an equation	275
Finding the equation of asymptotes	277
Expressing Conics Outside the Realm of Cartesian Coordinates	278
Graphing conic sections in parametric form	278
The equations of conic sections on the polar coordinate plane	280
Chapter 13: Solving Systems and Mingling with Matrices	283
A Primer on Your System-Solving Options	284
Finding Solutions of Two-Equation Systems Algebraically	285
Solving linear systems	285
Working nonlinear systems	288
Solving Systems with More than Two Equations	291
Decomposing Partial Fractions	293
Surveying Systems of Inequalities	295
Introducing Matrices: The Basics	296
Applying basic operations to matrices	297
Multiplying matrices by each other	298
Simplifying Matrices to Ease the Solving Process	301
Writing a system in matrix form	301
Reduced row echelon form	302
Augmented form	305

Conquering Matrices	305
Using Gaussian elimination to solve systems	306
Multiplying a matrix by its inverse	309
Using determinants: Cramer's rule	311
Chapter 14: Sequences, Series, and Expanding Binomials	315
Speaking Sequentially: Grasping the General Method	316
Calculating a sequence's terms by using	
the sequence expression	316
Working in reverse: Forming an expression from terms	317
Recursive sequences: One type of general sequence	318
Covering the Distance between Terms: Arithmetic Sequences	319
Using consecutive terms to find another	
in an arithmetic sequence	319
Using any two terms	320
Sharing Ratios with Consecutive Paired Terms:	
Geometric Sequences	321
Identifying a term when you know consecutive terms	322
Going out of order: Finding a term when	
the terms are nonconsecutive	323
Creating a Series: Summing Terms of a Sequence	324
Reviewing general summation notation	324
Summing an arithmetic sequence	325
Seeing how a geometric sequence adds up	326
Expanding with the Binomial Theorem	329
Breaking down the binomial theorem	330
Starting at the beginning: Binomial coefficients	330
Expanding by using the binomial theorem	332
Chapter 15: Looking Forward to Calculus	337
The Differences between Pre-Calc and Calc	338
Understanding and Communicating about Limits	339
Finding the Limit of a Function	339
Graphically	340
Analytically	341
Algebraically	342
Operating on Limits: The Limit Laws	345
Exploring Continuity in Functions	346
Determining whether a function is continuous	347
Dealing with discontinuity	347

Part IV: The Part of Tens	349
Chapter 16: Ten Habits That Help You Attack Calculus	351
Figure Out What the Problem Is Asking	351
Draw Pictures (And Plenty of 'Em)	352
Plan Your Attack	352
Write Down Any Formulas	354
Show Each Step of Your Work	354
Know When to “Quit”	354
Check Your Answers	355
Practice Plenty of Problems	356
Make Sure You Understand the Concepts	356
Pepper Your Teacher with Questions	357
Chapter 17: Ten Habits to Break before Calculus	359
Operating Out of Order	359
Squaring without FOILing	360
Splitting Up Denominators	360
Combining the Wrong Terms	360
Forgetting the Reciprocal	360
Losing Track of Minus Signs	361
Oversimplifying Radicals	361
Erring in Exponential Dealings	362
Canceling Out too Quickly	362
Distributing Improperly	363
Index	365