CONTENTS

Preface xiii

1 Introduction to Optimization 1

1.1 Requirements for the Application of Optimization Methods / 2
- 1.1.1 Defining the System Boundaries / 2
- 1.1.2 Performance Criterion / 3
- 1.1.3 Independent Variables / 4
- 1.1.4 System Model / 5

1.2 Applications of Optimization in Engineering / 6
- 1.2.1 Design Applications / 8
- 1.2.2 Operations and Planning Applications / 15
- 1.2.3 Analysis and Data Reduction Applications / 20
- 1.2.4 Classical Mechanics Applications / 26
- 1.2.5 Taguchi System of Quality Engineering / 27

1.3 Structure of Optimization Problems / 28

1.4 Scope of This Book / 29

References / 30

2 Functions of a Single Variable 32

2.1 Properties of Single-Variable Functions / 32

2.2 Optimality Criteria / 35

2.3 Region Elimination Methods / 45
- 2.3.1 Bounding Phase / 46
- 2.3.2 Interval Refinement Phase / 48
- 2.3.3 Comparison of Region Elimination Methods / 53

2.4 Polynomial Approximation or Point Estimation Methods / 55
- 2.4.1 Quadratic Estimation Methods / 56

COPYRIGHTED MATERIAL

2.4.2 Successive Quadratic Estimation Method / 58

2.5 Methods Requiring Derivatives / 61
2.5.1 Newton–Raphson Method / 61
2.5.2 Bisection Method / 63
2.5.3 Secant Method / 64
2.5.4 Cubic Search Method / 65

2.6 Comparison of Methods / 69

2.7 Summary / 70

References / 71

Problems / 71

3 Functions of Several Variables **78**

3.1 Optimality Criteria / 80

3.2 Direct-Search Methods / 84
3.2.1 The S^2 (Simplex Search) Method / 86
3.2.2 Hooke–Jeeves Pattern Search Method / 92
3.2.3 Powell's Conjugate Direction Method / 97

3.3 Gradient-Based Methods / 108
3.3.1 Cauchy's Method / 109
3.3.2 Newton's Method / 111
3.3.3 Modified Newton's Method / 115
3.3.4 Marquardt's Method / 116
3.3.5 Conjugate Gradient Methods / 117
3.3.6 Quasi-Newton Methods / 123
3.3.7 Trust Regions / 127
3.3.8 Gradient-Based Algorithm / 128
3.3.9 Numerical Gradient Approximations / 129

3.4 Comparison of Methods and Numerical Results / 130

3.5 Summary / 137

References / 137

Problems / 141

4 Linear Programming **149**

4.1 Formulation of Linear Programming Models / 149

4.2 Graphical Solution of Linear Programs in Two Variables / 154

4.3 Linear Program in Standard Form / 158
- 4.3.1 Handling Inequalities / 159
- 4.3.2 Handling Unrestricted Variables / 159

4.4 Principles of the Simplex Method / 161
- 4.4.1 Minimization Problems / 172
- 4.4.2 Unbounded Optimum / 173
- 4.4.3 Degeneracy and Cycling / 174
- 4.4.4 Use of Artificial Variables / 174
- 4.4.5 Two-Phase Simplex Method / 176

4.5 Computer Solution of Linear Programs / 177
- 4.5.1 Computer Codes / 177
- 4.5.2 Computational Efficiency of the Simplex Method / 179

4.6 Sensitivity Analysis in Linear Programming / 180

4.7 Applications / 183

4.8 Additional Topics in Linear Programming / 183
- 4.8.1 Duality Theory / 184
- 4.8.2 Dual Simplex Method / 188
- 4.8.3 Interior Point Methods / 189
- 4.8.4 Integer Programming / 205
- 4.8.5 Goal Programming / 205

4.9 Summary / 206

References / 206

Problems / 207

5 Constrained Optimality Criteria 218

5.1 Equality-Constrained Problems / 218

5.2 Lagrange Multipliers / 219

5.3 Economic Interpretation of Lagrange Multipliers / 224

5.4 Kuhn–Tucker Conditions / 225
- 5.4.1 Kuhn–Tucker Conditions or Kuhn–Tucker Problem / 226
- 5.4.2 Interpretation of Kuhn–Tucker Conditions / 228

5.5 Kuhn–Tucker Theorems / 229

5.6 Saddlepoint Conditions / 235
5.7 Second-Order Optimality Conditions / 238
5.8 Generalized Lagrange Multiplier Method / 245
5.9 Generalization of Convex Functions / 249
5.10 Summary / 254
References / 254
Problems / 255

6 Transformation Methods **260**

6.1 Penalty Concept / 261
6.1.1 Various Penalty Terms / 262
6.1.2 Choice of Penalty Parameter R / 277
6.2 Algorithms, Codes, and Other Contributions / 279
6.3 Method of Multipliers / 282
6.3.1 Penalty Function / 283
6.3.2 Multiplier Update Rule / 283
6.3.3 Penalty Function Topology / 284
6.3.4 Termination of the Method / 285
6.3.5 MOM Characteristics / 286
6.3.6 Choice of R-Problem Scale / 289
6.3.7 Variable Bounds / 289
6.3.8 Other MOM-Type Codes / 293
6.4 Summary / 293
References / 294
Problems / 298

7 Constrained Direct Search **305**

7.1 Problem Preparation / 306
7.1.1 Treatment of Equality Constraints / 306
7.1.2 Generation of Feasible Starting Points / 309
7.2 Adaptations of Unconstrained Search Methods / 309
7.2.1 Difficulties in Accommodating Constraints / 310
7.2.2 Complex Method / 312
7.2.3 Discussion / 320

7.3 Random-Search Methods / 322
7.3.1 Direct Sampling Procedures / 322
7.3.2 Combined Heuristic Procedures / 326
7.3.3 Discussion / 329
7.4 Summary / 330
References / 330
Problems / 332

8 Linearization Methods for Constrained Problems 336

8.1 Direct Use of Successive Linear Programs / 337
8.1.1 Linearly Constrained Case / 337
8.1.2 General Nonlinear Programming Case / 346
8.1.3 Discussion and Applications / 355
8.2 Separable Programming / 359
8.2.1 Single-Variable Functions / 359
8.2.2 Multivariable Separable Functions / 362
8.2.3 Linear Programming Solutions of Separable Problems / 364
8.2.4 Discussion and Applications / 368
8.3 Summary / 372
References / 373
Problems / 374

9 Direction Generation Methods Based on Linearization 378

9.1 Method of Feasible Directions / 378
9.1.1 Basic Algorithm / 380
9.1.2 Active Constraint Sets and Jamming / 383
9.1.3 Discussion / 387
9.2 Simplex Extensions for Linearly Constrained Problems / 388
9.2.1 Convex Simplex Method / 389
9.2.2 Reduced Gradient Method / 399
9.2.3 Convergence Acceleration / 403
9.3 Generalized Reduced Gradient Method / 406
9.3.1 Implicit Variable Elimination / 406
9.3.2 Basic GRG Algorithm / 410

9.3.3 Extensions of Basic Method / 419
9.3.4 Computational Considerations / 427

9.4 Design Application / 432
9.4.1 Problem Statement / 433
9.4.2 General Formulation / 434
9.4.3 Model Reduction and Solution / 437

9.5 Summary / 441

References / 441

Problems / 443

10 Quadratic Approximation Methods for Constrained Problems 450

10.1 Direct Quadratic Approximation / 451

10.2 Quadratic Approximation of the Lagrangian Function / 456

10.3 Variable Metric Methods for Constrained Optimization / 464

10.4 Discussion / 470
10.4.1 Problem Scaling / 470
10.4.2 Constraint Inconsistency / 470
10.4.3 Modification of $\mathbf{H}^{(t)}$ / 471
10.4.4 Comparison of GRG with CVM / 471

10.5 Summary / 475

References / 476

Problems / 477

11 Structured Problems and Algorithms 481

11.1 Integer Programming / 481
11.1.1 Formulation of Integer Programming Models / 482
11.1.2 Solution of Integer Programming Problems / 484
11.1.3 Guidelines on Problem Formulation and Solution / 492

11.2 Quadratic Programming / 494
11.2.1 Applications of Quadratic Programming / 494
11.2.2 Kuhn–Tucker Conditions / 498

11.3 Complementary Pivot Problems / 499

11.4 Goal Programming / 507

11.5 Summary / 518

References / 518

Problems / 521

12 Comparison of Constrained Optimization Methods 530

12.1 Software Availability / 530

12.2 A Comparison Philosophy / 531

12.3 Brief History of Classical Comparative Experiments / 533
12.3.1 Preliminary and Final Results / 535

12.4 Summary / 539

References / 539

13 Strategies for Optimization Studies 542

13.1 Model Formulation / 543
13.1.1 Levels of Modeling / 544
13.1.2 Types of Models / 548

13.2 Problem Implementation / 552
13.2.1 Model Assembly / 553
13.2.2 Preparation for Solution / 554
13.2.3 Execution Strategies / 580

13.3 Solution Evaluation / 588
13.3.1 Solution Validation / 589
13.3.2 Sensitivity Analysis / 590

13.4 Summary / 594

References / 594

Problems / 597

14 Engineering Case Studies 603

14.1 Optimal Location of Coal-Blending Plants by Mixed-Integer Programming / 603
14.1.1 Problem Description / 604
14.1.2 Model Formulation / 604
14.1.3 Results / 609

14.2 Optimization of an Ethylene Glycol–Ethylene Oxide Process / 610
14.2.1 Problem Description / 610

14.2.2 Model Formulation / 612
14.2.3 Problem Preparation / 618
14.2.4 Discussion of Optimization Runs / 618

14.3 Optimal Design of a Compressed Air Energy Storage System / 621
14.3.1 Problem Description / 621
14.3.2 Model Formulation / 622
14.3.3 Numerical Results / 627
14.3.4 Discussion / 629

14.4 Summary / 630

References / 631

Appendix A Review of Linear Algebra **633**

A.1 Set Theory / 633

A.2 Vectors / 633

A.3 Matrices / 634
A.3.1 Matrix Operations / 635
A.3.2 Determinant of a Square Matrix / 637
A.3.3 Inverse of a Matrix / 637
A.3.4 Condition of a Matrix / 639
A.3.5 Sparse Matrix / 639

A.4 Quadratic Forms / 640
A.4.1 Principal Minor / 641
A.4.2 Completing the Square / 642

A.5 Convex Sets / 646

Appendix B Convex and Concave Functions **648**

Appendix C Gauss–Jordan Elimination Scheme **651**

Author Index 653

Subject Index 659