

On the (Non)Acceptance of Innovations

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1.1 INTRODUCTION

Since its birth as an independent branch of social sciences, finance has witnessed three major revolutions:

- mean–variance, 1952–56
- continuous-time models, 1969–73
- risk measures, 1997–

Markowitz (1952, 1959) proposed to measure risk associated with the return of each investment by means of the deviation from the mean of the return distribution i.e., the variance, and, in the case of a combination (portfolio) of assets, to gauge the risk level via the covariance between all pairs of investments, i.e.:

$$\text{Cov}[X, Y] = E[X, Y] - E[X]E[Y],$$

where X and Y are random returns. The main innovation introduced by Markowitz is to measure the risk of a portfolio via the **joint** (multivariate) **distribution** of returns of all assets. Multivariate distributions are characterized by the statistical (**marginal**) properties of all component random variables and by their **dependence** structure. Markowitz described the former by the first two moments of the univariate distributions – the asset returns – and the latter via the linear (Pearson) correlation coefficient between **each pair** of random returns, i.e.:

$$\rho(X, Y) = \text{Cov}[X, Y] / (\sigma_X^2 \sigma_Y^2)^{1/2}$$

where σ_X and σ_Y denote the standard deviations of the univariate random variables X and Y , respectively. Note that a measure of dispersion can be adopted as a measure of risk only if the relevant distribution is symmetric.

The correlation coefficient, while allowing us to fully describe a multivariate distribution by taking into account the dependence structure among all pairs of components only, is strictly related to the slope parameter of a linear regression of the random variable Y on the random variable X , and it measures only the co-dependence between the linear components of X and Y . Indeed:

$$\rho(X, Y)^2 = \{\sigma_Y^2 - \min E[(Y - (aX + b))^2]\} / \sigma_Y^2,$$

that is the relative variation of σ_Y^2 by linear regression on X .

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It can be proved that for all vectors z and random vectors X , the variance of the linear combination $z^T X$, satisfies the relationship:

$$\sigma^2(z^T X) = z^T \text{Cov}(X) z$$

which is essential in Markowitz portfolio theory. The linear correlation co-dependence measure is indeed very intuitive and appealing in its simplicity, but has a limited range of applicability (Alexander, 2002).

We must recall that the Markowitz model goes hand in hand with appropriate utility functions, which allows a subjective preference ordering of assets and their combinations. In the case of non-normal, albeit symmetric, distributions the utility functions must be quadratic. In practice this limitation restricts the use of this model to portfolios characterized by normal joint return distribution, i.e. to the case in which the returns of all assets as well as their dependence structure is normal.

The second revolution was started by Robert Merton, Fisher Black and Myron Scholes (see Merton, 1989), and it can be labelled as “continuous-time finance”. It made it possible to crack many problems associated with option-pricing and other derivatives. The concept of contingent claim, essential in modern finance, is a byproduct of these theories.

The third revolution is much more recent and started in 1997.¹ Indeed in the last six years, there has been a great momentum in research on this subject, which has touched nine different, but interconnected, problems:

- critique of current risk measures
- definition of risk measure
- construction of (coherent) risk measures
- rationality of insurance premia
- conditional CAPM²
- “good deals”³
- asset pricing in incomplete markets⁴
- generalized hyperbolic Lévy processes⁵
- copulas for the study of co-dependence⁶
- crash-prediction methods⁷

The linear correlation coefficient, while very intuitive and appealing in its simplicity, if used in the case of non-elliptic distributions, may lead to incorrect results.⁸ The concept of “incorrect” must, however, be specified, since it requires an agreement on a “correct” dependence measure. In the absence of such a measure this comparison can be performed only via simulations.⁹

Recently the class of random variables for which linear correlation can be used as an dependence measure has been fully identified (Cambanis, Huang and Simon, 1981). This

¹ The year in which the first results by Artzner, Delbaen, Eber, and Heath, on coherent risk measures were published. In the same year Wang, Young and Panjer published their work on the axiomatic characterization of insurance prices. See also Chapters 10, 11 and 12 in this volume.

² See Franke *et al.* (2000).

³ See Chapter 21 in this volume.

⁴ See Chapter 21 in this volume.

⁵ See Geman (2002).

⁶ See Chapters 14 and 15 in this volume.

⁷ See Sornette *et al.* (2001).

⁸ See, for instance, Embrechts, McNeil and Straumann (1999) who compare bivariate normal distributions and Gumbel distributions with the same linear correlation coefficient.

⁹ This is the technique used by Embrechts, McNeil and Straumann (1999).

is the class of **elliptic** distributions (see Joe, 1997) characterized by the property that their equi-density surfaces are ellipsoids. Thus the Markowitz model is suited only to the case of elliptic distributions, like normal or t -distributions with finite variances. Note that symmetric distributions are not necessarily elliptic. Recent results (Naldi, 2003) show that even in the case of elliptic distributions, a more general measure of co-dependence, the so-called Kendall- τ , is more robust and efficient than the Pearson correlation coefficient. Clearly one loses the simple quadratic structure connected with the mean–variance model and the possibility of using a closed form expression on the efficient frontier (see, for instance, Szegö, 1980), but nowadays in practice one uses straight numerical techniques without using a closed form expression, first.

The analysis via simulation of the co-dependences when the linear correlation coefficient is (incorrectly) used in the case of non-elliptic distributions is not trivial. For instance, it has been shown that in comparing two different distributions (one normal and one Gumbel) with the same linear correlation coefficient, in the vast majority of cases the two results agree. However, most of the points of disagreement¹⁰ lay in the upper right corner of the distribution, corresponding to extreme losses. We can say that if one uses a variance–covariance model for non-elliptic distributions one can severely underestimate extreme events that cause the most severe losses. On the other hand it must be recognized that in the majority of observations in this specific example in which one of the two distributions (Gumbel) is not elliptical, a wrong measure (linear correlation) seems to describe correctly the degree of co-dependence.

In the 1960s the concept of β (volatility) was introduced. This development was motivated by computational reasons. The complexity of the mean–variance approach was considered too high. After almost 40 years, and the gigantic progress in computers, this is no longer the case. The second motivation for the introduction of the β -based portfolio methods was the insufficient data to compute the variance–covariance matrix (the number of data should be at least twice the number of assets). Now bootstrapping techniques allow us to circumvent this problem and β s are almost abandoned in portfolio management in favour of complete variance–covariance models.

The measure of the linear dependence between the return of each security and that of the market, β , led to the development of the main pricing models, CAPM and APT. These models, while extendable to heavy-tailed distributions (see, for instance, Franke, Härde and Stahl, 2000), have been developed in a “normal world” and lead to misleading results when applied to everyday life situations.

It is unfortunate that the precisely formulated Markowitz model has become a “solution in search of a problem” and incorrectly applied to many cases in which risk cannot be described by variance, dependence cannot be measured by linear correlation coefficient, and utility function does not even dream of being quadratic.¹¹

Multivariate normal distribution-based models are very appealing, because the association between any two random variables can be fully described by their marginal distributions and the linear correlation coefficient. It is evident that these models are only a very initial step towards more realistic ones better tuned to grasp real-life situations, i.e. the case in which investment return cumulative distributions of individual assets are skewed, leptokurtic and/or heavy-tailed. The introduction of these models has been hampered by the lack of a suitable theoretical framework.

Probabilistic models for univariate returns have been investigated and extended to the multivariate case under the assumption that all the combined returns and their dependence structure

¹⁰ In the simulations performed by Embrechts, McNeil and Straumann (1999), 0.8% of the cases.

¹¹ I could quote a large number such examples, but ... I have already a sufficient number of enemies!

have the same probabilistic structure. This severe drawback can be overcome with the use of copula functions¹² suitable to the analysis multivariate distributions with almost arbitrary univariate components and dependence structure. Only recently (Embrechts, Küppelberg and Mikosch, 1997) the problem of the study of extreme events, i.e. of the tails of the distribution, has received due attention. Most of the research on new risk measures has been stimulated by “dependent extreme events”.¹³

1.2 THE PATH TOWARDS ACCEPTANCE OF PREVIOUS INNOVATIONS

In the pre-Markowitz era financial risk was considered as a correcting factor of expected return, and risk-adjusted returns were defined on an ad hoc basis. These primitive measures had the advantage of allowing an immediate preferential order of all investments.

One would expect that such important developments would obviously have immediate acceptance and implementation from everybody; but that is not the case. The Markowitz theory was refused. His 1952 paper had difficulties in being accepted by the *Journal of Finance*. Nowadays it is easy to make a show about the shortsightedness of referees and editors in the first part of the last century. Before passing any hurried judgement we must remember that in the early 1950 not only there were no digital computers, but that nobody had any idea of what they could be. How could anybody seriously propose to solve a quadratic programming problem? Even after the birth of digital computers, quadratic programming problems were considered to be so outlandish that Sharpe (1963) became famous also because of his proposal of the “diagonal model”, i.e. a linear programming portfolio selection model. In addition to that, the data collection problem was practically insurmountable¹⁴ in times when data were fed to computers via punched cards. Because of that Markowitz theory had a wider audience in the academic world¹⁵ than in the Street.

Continuous-time finance met the opposite: it was welcomed by the Street, but less by the academia. Indeed by the late-1970s derivatives had become very popular; the problems of their pricing was a critical issue and all the needed computer power was available. On the other hand continuous-time finance required a completely new type of mathematics and for that reason was not popular in the academic world.

Let us analyse the reaction to the New Results that have presented in the last six years, summarily recalled above.

Have they been accepted?

Much to my surprise, again they have been accepted by the Street, rejected by the academic establishment and, so far, disregarded by regulators!

The innovations in risk measures have been accepted, by some important financial institutions.¹⁶

The gut refusals of innovation and in particular of the correct risk measures from the old academic establishment, has expressed itself essentially in the following three forms:

¹² See, for instance, Chapter 15, and also Chapter 14, in this volume.

¹³ Essentially catastrophic events unexpectedly connected. The most typical example has been the increase of spread among sovereigns of countries due to join the Euro in January 1999, following the Russian crisis of August 1998.

¹⁴ To compute correlation coefficients it is advisable to use a number of observations at least double the number of assets.

¹⁵ Graham, Dodd and Cottle (1962) briefly mention the Markowitz model. They reject it essentially as inapplicable from the practical point of view. Sharpe (1970) and Levy and Sarnat (1972) give a complete description of the model. Szegö (1972) presents the model, and provides the first analytical derivation of the mean–variance efficient frontier. I wish to thank Marshall Sarnat for his historic memory!

¹⁶ See Quantitative Credit Research, and in particular O’Kane and Schloegl (2003).

- “I don’t care about this coherent bit; I don’t want any damn mathematician telling me how to measure risk”
- “Who cares about tails; we don’t have sufficient extreme data anyhow”
- “I’m too busy counting out the money to have time to follow esoteric new math” (refusal even to be informed)

1.3 HOW TO ANSWER

We cannot just ignore the negative reactions or attribute them to a perfunctory defence of obsolete techniques. I am therefore planning to carefully analyse these comments, to provide some answers, and to convince the sceptical and the complacent.

I am going to start from refusal to be advised on risk measures. The statement, “I don’t care about this coherent bit; I don’t want any damn mathematician telling me how to measure risk,” follows from a gross misreading of the concept of measure.

A measure of risk is any correspondence, ρ , between a space, X , of random variables (for instance the returns of a given set of investments) and a nonnegative real number, i.e. $\rho : X \rightarrow R$. These correspondences cannot be without restrictions (in this case they would not have any property) that can take the form of binding conditions.¹⁷ What do we ask from risk measures?

- Consistency and repeatability within the same measure
- The possibility of comparing results obtained from different measures

The first property is that the real number that represents the measure of the risk level of a certain event, or the difference between risk levels of different events, does not change if the measurement is repeated again on the same event(s).

The second wish is much more difficult to fulfil: it can be stated with the following question. Is the order of risk levels of different events invariant with respect to the risk measure used?

While the axioms that condition the measures of risk guarantee that each specific measure leads to consistent and repeatable measurements of risk level, there is no guarantee that results obtained via different coherent risk measures are comparable.

Note that the lack of a relationship, and in particular of a linear relationship, between risk levels of the same events measured by different (coherent) risk measures is a price that must be paid for the freedom of adopting the risk measure of your choice. We recall that also in the case of measures of distance (see, for instance, Kolmogorov and Fomin, 1957, pp. 16–23) there does not exist any useful comparison rule,¹⁸ with the exception of the fact that a null distance between two points can occur only in the case in which the two points coincide. This is more or less the situation for risk measures for which, if an event has zero risk in one measure it has zero risk in all measures.

The problem of comparison of risk level obtained by different measures creates some difficulties for regulators. For this reason it is crucial that they adopt an “official” risk meter that is a measure, i.e. that satisfies the required conditions (axioms).¹⁹

¹⁷ Readers must be reminded that in many circumstances it is necessary to impose restrictions in order to obtain meaningful definitions, as in the case of the definition of a distance between two points or of a dynamical system.

¹⁸ The strongest result I am aware of is the following bound on possible differences. For any given distance functions ρ_1, ρ_2 there exist two real numbers $\beta_1 > 0, \beta_2 \geq \beta_1$ such that: $\beta_1 \rho_1(x) \leq \rho_2(x) \leq \beta_2 \rho_1(x)$ for all x . Thus if $x_k \rightarrow 0$ in ρ_1 it will be true for any other ρ .

¹⁹ Many papers in this volume, e.g. Chapters 3, 4 and 10, discuss the consequences of using incorrect risk measures. See also *Journal of Banking and Finance* (2002). From the regulatory point of view we want to recall that the lack of convexity does not allow comparison of results, and the absence of subadditivity makes it possible to cheat.

The class of correspondences that are risk measures (i.e. that satisfy the coherency conditions) is extremely wide and allows us to accommodate all tastes.²⁰ No damn mathematician has ever thought of imposing, or even suggesting, a specific risk measure, but has only pointed out the costs connected with the use of a “bumpy meter”.

The second objection, “Who is afraid of tails?”, is more difficult to handle. It has to do with two problems:

- the economic weight of tail events
- the size of error due to the use of incorrect risk meters

Tail events have, by definition a smaller probability of occurrence than other events. In some cases the number of relevant observations is, however, statistically significant, and the risk measure cannot ignore them. In some other cases, even if extreme events can be characterized by a small number of observations, their occurrence can carry many heavy consequences, and they must be handled in the best possible way. Cutting the tails of the distributions, as done in the case of VaR,²¹ is equivalent to hiding your head in the sand. A stronger argument is contained in section 4 of Chapter 10 and deals with the volatility and efficiency of ES with respect to that of VaR, showing that C-VaR or ES are more stable than VaR for the type of tailed distributions typical of financial problems.

The last, more obtuse, reaction, the refusal to even be informed, can be overcome by running a convincing simulation test to show how much money is lost by using an incorrect risk meter in a portfolio problem. A comparison of the returns of optimal portfolios of randomly chosen sets of different securities obtained by the minimization of a specific risk measure (σ , VaR, C-VaR, ES) for a given expected return could provide the desired answer. The difficulty lies in the non-convexity of VaR which does not allow this simple computation.

1.4 CONCLUSIONS

The reluctance to accept innovations could be overcome by a conclusive argument: some new risk measurement techniques, such as ES, can be implemented much more easily, via linear programming techniques, than the old, possibly incorrect, measures such as variance and VaR. Why pay more to reach possibly worse results?

REFERENCES

- Acerbi, Carlo (2002) Spectral measures of risk: a coherent representation of subjective risk aversion. *Journal of Banking & Finance* **26**, 1505–1518.
- Alexander, Carol (2002) *Market Models*, Wiley, New York.
- Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber and David Heath (1997) Thinking coherently, *Risk*, **10**, 33–49.
- Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber and David Heath (1999) Coherent measures of risk, *Mathematical Finance*, **9**, 203–228.
- Basel Committee on Banking Supervision (1988) *International Convergence of Capital Measurements and Capital Standards*, BIS, Basel, July.

²⁰ Chapter 10 in this volume allows us to fully appreciate this point, in particular the results on the convex combination of risk measures and on the “admissible function” that plays a crucial role in the definition of spectral risk measure. See also Acerbi (2002).

²¹ One recent paper by Leippold, and Vanini (2003), attributes the choice of VaR by the Basel Committee as “guided by political considerations”.

- Basel Committee on Banking Supervision (1999) *Capital Requirements and Bank Behavior: the Impact of the Basle Accord*, Working Paper no.1, BIS, Basel, April.
- Basel Committee on Banking Supervision (2001) *The New Basel Capital Accord*, Consultative Document, BIS, Basel, January.
- Cambanis, Stamatis, Steel Huang, and Gordon Simon (1981) On the theory of elliptically contoured distributions, *Journal of Multivariate Analysis*, **11**, 368–385.
- Caouette, J.B., E.I. Altman and P. Narayanan (1998) *Managing Credit Risk*, Wiley, New York.
- Crouhy, Michel, Dan Galai and Robert Mark (2000) A comparative analysis of current credit risk models, *Journal of Banking and Finance*, special issue on “Credit Modelling and Regulatory Issues”, edited by Patricia Jackson and William Perraudin, **24**, 59–117.
- Crouhy, Michel, Dan Galai and Robert Mark (2001) *Risk Management*, McGraw-Hill, New York.
- Cumperayot, P.J., Jón Daníelsson, Björn N. Jorgensen and Casper G. de Vries (2000) On the (ir)relevance of value at risk regulation, in Franke, F., W. Häerde and G. Stahl (Eds), *Measuring Risk in Complex Stochastic Systems*, Springer Verlag, Berlin, pp. 99–117.
- Daníelsson, Jón, and Casper G. de Vries (1998) *Value at Risk and Extreme Returns*, London School of Economics, Financial Markets Group, Discussion Paper no. 273.
- Daníelsson, Jón, Björn N. Jorgensen and Casper G. de Vries (1998) The value at risk: statistical, financial, and regulatory considerations, Proceedings of a Conference on “Financial Services at the Crossroad: Capital Regulation in the Twenty-First Century”, *Economic Policy Review*, Federal Reserve Bank of New York, Vol. 4, No. 3.
- Daníelsson, Jón, Björn N. Jorgensen and Casper G. de Vries (2001) Incentives for effective risk management, *Journal of Banking and Finance*, Special Issue on “Statistical and Computational Problems in Risk Management: VaR, and beyond VaR”, Edited by Giorgio Szegő, **26**, 1407–1426.
- Daníelsson, Jón, Paul Embrechts, Charles Goodhart, Con Keating, Felix Muennich, Olivier Renault and Hyun Song Shin (2001) *An Academic Response to Basel II*, Special Paper no. 130, FMG and ESRC, London, May, www.riskresearch.org.
- Eberlein, Ernst, and Ulrich Keller (1995) Hyperbolic distributions in finance, *Bernoulli*, **1**, 261–299.
- Embrechts, Paul, Claudia Klüppelberg and Thomas Mikosch (1997) *Extremal Events in Finance and Insurance*, Springer Verlag, Berlin.
- Embrechts, Paul, Alexander McNeil and Daniel Straumann (1999) Correlation and dependence in risk management: properties and pitfalls, August, www.math.ethz.ch/finance.
- Embrechts, Paul, Filip Lindskog and Alexander McNeil (2001), Modelling dependence with copulas and applications to risk management, September, www.math.ethz.ch/finance.
- Franke, F., W. Häerde and G. Stahl (2000) *Measuring Risk in Complex Stochastic Systems*, Springer Verlag, Berlin.
- Frees, E.W. and E. Valdez (1998) Understanding relationships using copulas, *North American Actuarial Journal*, **2**, 1–25.
- Frey, Rüdiger, and Alexander J. McNeil (2001) Modelling dependent defaults, Presented at the Conference on “Statistical and Computational Problems in Risk Management”, University of Rome “La Sapienza”, June 14–16.
- Frey, Rüdiger, and Alexander McNeil (2001) VaR and Expected Shortfall in Credit Portfolios: Conceptual and Practical Insights, *Journal of Banking and Finance*, Special Issue on “Statistical and Computational Problems in Risk Management: VaR, and beyond VaR”, edited by Giorgio Szegő, **26**, 1317–1334.
- Frey, Rüdiger, Alexander J. McNeil and Mark A. Nyfeler (2001) Modelling dependent defaults: asset correlations are not enough, March, www.math.ethz.ch/finance.
- Frittelli, Marco (2002) Putting order in risk measures, *Journal of Banking and Finance*, Special Issue on “Statistical and Computational Problems in Risk Management: VaR, and beyond VaR”, edited by Giorgio Szegő, **26**, 1473–1486.
- Geman, Helyette (2002) Pure jump Lévy processes for asset price modelling, Presented at the Conference on “Statistical and Computational Problems in Risk Management”, University of Rome, “La Sapienza”, June 14–16. *Journal of Banking and Finance*, Special Issue on “Beyond VaR”, edited by Giorgio Szegő, **26**, 1257–1317.
- Gordy, Michael B. (2000a) A comparative anatomy of credit risk models, *Journal of Banking and Finance*, **24**, 119–149.

- Gordy, Michael B. (2000b) *A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules*, Federal Reserve Board.
- Gordy, Michael B. (2001) Calculation of higher moments in CreditRisk with applications, Presented at the Conference on “Statistical and Computational Problems in Risk Management”, University of Rome, “La Sapienza”, June 14–16.
- Graham, B., D.L. Dodd and S. Cottle (1962) *Securities Analysis: Principles and Techniques*. 4th edn, McGraw-Hill, New York.
- Jagannathan, Ravi, and Zhenyu Wang (1996) The conditional CAPM and the cross-section of expected returns, *Journal of Finance*, **51**, 3–53.
- Joe, H. (1997) *Multivariate Models and Dependence Concepts*, Chapman & Hall, London.
- Johansen, Anders, Didier Sornette and Olivier Ledoit (1999) Predicting financial crashes using discrete scalar inverse, *Journal of Risk*, **1**, 5–32. www.nbi.dk/~johansen/pub.html.
- Jones, David (2000) Emerging Problems with the Basel capital accord: regulatory capital arbitrage and related issues, *Journal of Banking and Finance*, Special Issue on “Credit Risk Modelling and Regulatory Issues”, edited by P. Jackson and W. Perraudin, **24**, 35–58.
- Jorion, Philippe (2000) *VaR: The New Benchmark for Managing Financial Risk*, McGraw-Hill, New York.
- Journal of Banking and Finance* (1991) Special Issue on “Deposit Insurance Reform”, edited by M. Berlin, A. Saunders and G. Udell, **15**, No. 4/5.
- Journal of Banking and Finance* (1995) Special Issue on “The Role of Capital in Financial Institutions”, edited by A. Berger, R. Herring and G.P. Szegö, **19**, No. 3/4.
- Journal of Banking and Finance* (1998) Special Issue on “Credit Risk Assessment and Relationship Lending”, edited by E. Altman, J. Krahnén and A. Saunders, **22**, No. 11/12.
- Journal of Banking and Finance* (2000) Special Issue on “Credit Risk Modeling and Regulatory Issues”, edited by P. Jackson and W. Perraudin, **24**, No. 1/2.
- Journal of Banking and Finance* (2001) Special Issue on “Credit Rating and the Proposed New BIS Guidelines on Capital Adequacy for Bank Credit Assets”, edited by E. Altman, **25**, No. 1/2.
- Journal of Banking and Finance* (2002a) Special Issue on “Risk Management in the Global Economy. Measurement, Management, and Macroeconomic Implications”, edited by William C. Hunter and Stephen D. Smith, **26**, No. 2/3.
- Journal of Banking and Finance* (2002b) Special Issue on “Statistical and Computational Problems in Risk Management: VaR, and beyond VaR”, edited by Giorgio Szegö, **26**, No. 7.
- JP Morgan (1994) *Riskmetrics*, New York, October and November 1995.
- JP Morgan (1997) *Credit Metrics*, New York.
- Kolmogorov, A.N., and S.V. Fomin (1957) *Elements of the Theory of Functions and Functional Analysis*, Vol. 1, Graylock Press, Rochester, NY.
- Landsman, Zinoviy, and Michael Sherris (2001) Risk measures and insurance premium principles, *Insurance Mathematics and Economics*, **29**, 103–115.
- Lee, J. (1993) Generating Random Binary Deviates Having Fixed Marginal Distributions and Specified Degrees of Association, *Statistical Computing*, **47**, 209–215.
- Leippold, Markus, and Paolo Vanini (2003) Half as many cheers – the multiplier reviewed, presented at the International conference on Modeling, Optimization, and Risk Management Finance, University of Florida, Gainesville, March 5–7, 2003.
- Levy, Haim, and Marshal Sarnat (1972) *Investment and Portfolio Analysis*, Wiley, New York.
- Manganelli, Simone, and Robert F. Engle (2001) *Value at Risk Models in Finance*, Working Paper no. 75, ECB, Frankfurt.
- Markowitz, H.M. (1952) Portfolio Selection, *Journal of Finance*, **7**, 77–91.
- Markowitz, H.M. (1959) *Portfolio Selection*, Wiley, New York.
- Merton, Robert, C. (1989) *Continuous-Time Finance*, Basil Blackwell, Oxford.
- Naldi, Marco (2003) Robust and efficient estimation of equity correlations, *Quantitative Credit Research*, Lehman Brothers, January, pp. 20–30.
- Nielsen, R.B. (1999) *An Introduction to Copulas*, Springer Verlag, Berlin.
- O’Kane, Dominique, and Lutz Schloegl (2003) An analytical portfolio credit model with tail dependence, *Quantitative Credit Finance*, Lehman Brothers, January, pp. 51–63.
- Phelan, Michael, J. (1997) Probability and statistics applied to the practice of financial risk management: the case of JP Morgan’s risk metrics, *Journal of Financial Services Research*, **12**, 175–200.

- Rockafellar, R. Tyrrell, and Stanislav Uryasev (2002) Optimization of conditional value-at-risk, *Journal of Banking and Finance*, Special Issue on “Statistical and Computational Problems in Risk Management: VaR, and beyond VaR”, edited by Giorgio Szegö, **26**, 1443–1472.
- Sharpe, W.F. (1963) A simplified model for portfolio analysis, *Management Science*, **9**, 277–293.
- Sharpe, W.F. (1970) *Portfolio Theory and Capital Markets*, McGraw-Hill, New York.
- Sklar, A. (1973) Random Variables, Joint Distributions, and Copulas, *Kybernetika*, **9**, 449–460.
- Sornette, Didier, and Y. Malevergne (2001) From Rational Bubbles to Crashes, *Physica A*, **299**, 40–59. <http://arXiv.org/abs/cond-mat/0102305>.
- Sornette, Didier, and A. Johansen (2001) Significance of log-periodic precursors to financial crashes, *Quantitative Finance*, **1**, 452–471. <http://arXiv.org/abs/cond-mat/0106520>.
- Szegö, Giorgio (1972) *Modelli Analitici di Gestione Bancaria*, Tamburini Editore, Milan.
- Szegö, Giorgio (1980) *Portfolio Theory*, Academic Press, New York.
- Szegö, Giorgio (1993) “Pseudo Risk-adjusted Capital Requirements: Many Thorns and Little Flowers”, Seminar at Harvard University, May 21.
- Szegö, Giorgio (1994) Financial regulation and multi-tier financial intermediation system, in R. D’Ecclesia and S. Zenios (Eds), *Operations Research Models in Quantitative Finance*, Proc. XIII Annual Meeting of the EURO Working Group for Financial Modeling, Nicosia, Physica-Verlag, Heidelberg, pp. 36–62.
- Szegö, Giorgio (1995) *Il Sistema Finanziario, Economia e Regolamentazione*, McGraw-Hill Italia, Milan.
- Szegö, Giorgio (1997) A critique of the Basle regulation, or how to enhance (im)moral hazards, Presented at the International Conference on “Risk Management and Regulation in Banking”, Bank of Israel, Jerusalem, May 17–19. Published in the Proceedings, Kluwer, Dordrecht, 1999.
- Szegö, Giorgio (2001) (Bad) news on bank regulation, Lecture presented at the International Conference on “Financial and Real Markets, Risk Management and Corporate Governance: Theory and International Evidence”, Sousse, Tunisia, March 15–17, 2001. Published as Towards a New Bank Regulatory Framework, *The International Journal of Finance*, **13**, 1837–1855. [w3.uniroma1/bancaefinanza](http://www.uniroma1.it/bancaefinanza).
- Szegö, Giorgio (2002) Measures of risk, *Journal of Banking and Finance*, Special Issue on “Statistical and Computational Problems in Risk Management: VaR, and beyond VaR”, edited by Giorgio Szegö, **26**, 1443–1472.
- Szegö, Giorgio, and Franco Varetto (1999) *Il Rischio Creditizio: Misura e Controllo*, UTET Libreria, Turin.
- Testuri, Carlos E., and Stanislav Uryasev (2000) *On Relation between Expected Regret and Conditional Value-at-Risk*, Mimeo, ISE University of Florida, August.
- Uryasev, Stanislav (2000) Conditional value-at-risk: optimization algorithms and applications, *Financial Engineering News*, **14**, 1–6.
- Wang, Shuan S., Virginia R. Young and Harry H. Panjer (1997) Axiomatic characterization of insurance prices, *Insurance Mathematics and Economics*, **21**, 173–183.

