Chapter 1

Assembling Your Tools

In This Chapter

- Giving names to the basic numbers
- Reading the signs — and interpreting the language
- Operating in a timely fashion

You've probably heard the word *algebra* on many occasions, and you knew that it had something to do with mathematics. Perhaps you remember that algebra has enough information to require taking two separate high school algebra classes — Algebra I and Algebra II. But what exactly *is* algebra? What is it *really* used for?

This book answers these questions and more, providing the straight scoop on some of the contributions to algebra's development, what it's good for, how algebra is used, and what tools you need to make it happen. In this chapter, you find some of the basics necessary to more easily find your way through the different topics in this book. I also point you toward these topics.

In a nutshell, *algebra* is a way of generalizing arithmetic. Through the use of *variables* (letters representing numbers) and formulas or equations involving those variables, you solve problems. The problems may be in terms of practical applications, or they may be puzzles for the pure pleasure of the solving. Algebra uses positive and negative numbers, integers, fractions, operations, and symbols to analyze the relationships between values. It's a systematic study of numbers and their relationship, and it uses specific rules.

Beginning with the Basics: Numbers

Where would mathematics and algebra be without numbers? A part of everyday life, numbers are the basic building blocks of algebra. Numbers give you a value to work with. Where would civilization be today if not for numbers? Without numbers to figure the distances, slants, heights, and directions, the pyramids would never have been built. Without numbers to figure out navigational points, the Vikings would never have left Scandinavia. Without numbers to examine distance in space, humankind could not have landed on the moon.

Even the simple tasks and the most common of circumstances require a knowledge of numbers. Suppose that you wanted to figure the amount of gasoline it takes to get from home to work and back each day. You need a number for the total miles between your home and business and another number for the total miles your car can run on a gallon of gasoline.

The different sets of numbers are important because what they look like and how they behave can set the scene for particular situations or help to solve particular problems. It's sometimes really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers do not include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

Algebra uses different sets of numbers, in different circumstances. I describe the different types of numbers here.

Aha algebra

Dating back to about 2000 B.C. with the Babylonians, algebra seems to have developed in slightly different ways in different cultures. The Babylonians were solving three-term quadratic equations, while the Egyptians were more concerned with linear equations. The Hindus made further advances in about the sixth century A.D. In the seventh century, Brahmagupta of India provided general solutions to quadratic equations and had interesting takes on 0. The Hindus regarded irrational numbers as actual numbers — although not everybody held to that belief.

The sophisticated communication technology that exists in the world now was not available then, but early civilizations still managed to exchange information over the centuries. In A.D. 825, al-Khowarizmi of Baghdad wrote the first algebra textbook. One of the first solutions to an algebra problem, however, is on an Egyptian papyrus that is about 3,500 years old. Known as the Rhind Mathematical Papyrus after the Scotsman who purchased the 1-foot-wide, 18-foot-long papyrus in Egypt in 1858, the artifact is preserved in the British Museum — with a piece of it in the Brooklyn Museum. Scholars determined that in 1650 B.C., the Egyptian scribe Ahmes copied some earlier mathematical works onto the Rhind Mathematical Papyrus.

One of the problems reads, "Aha, its whole, its seventh, it makes 19." The *aha* isn't an exclamation. The word *aha* designated the unknown. Can you solve this early Egyptian problem? It would be translated, using current algebra symbols, as: $x + \frac{x}{7} = 19$. The unknown is represented by the x, and the solution is $x = 16\frac{5}{8}$. It's not hard; it's just messy.

Really real numbers

Real numbers are just what the name implies. In contrast to imaginary numbers, they represent *real* values — no pretend or make-believe. Real numbers cover the gamut and can take on any form — fractions or whole numbers, decimal numbers that can go on forever and ever without end, positives and negatives. The variations on the theme are endless.

Counting on natural numbers

A *natural number* (also called a *counting number*) is a number that comes naturally. What numbers did you first use? Remember someone asking, "How old are you?" You proudly held up four fingers and said, "Four!" The natural numbers are the numbers starting with 1 and going up by ones: 1, 2, 3, 4, 5, 6, 7, and so on into infinity. You'll find lots of counting numbers in Chapter 6, where I discuss prime numbers and factorizations.

Wholly whole numbers

Whole numbers aren't a whole lot different from natural numbers. Whole numbers are just all the natural numbers plus a 0: 0, 1, 2, 3, 4, 5, and so on into infinity.

Whole numbers act like natural numbers and are used when whole amounts (no fractions) are required. Zero can also indicate none. Algebraic problems often require you to round the answer to the nearest whole number. This makes perfect sense when the problem involves people, cars, animals, houses, or anything that shouldn't be cut into pieces.

Integrating integers

Integers allow you to broaden your horizons a bit. Integers incorporate all the qualities of whole numbers and their opposites (called their *additive inverses*). *Integers* can be described as being positive and negative whole numbers: . . . –3, –2, –1, 0, 1, 2, 3, ...

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, it's not a fraction! This doesn't mean that answers in algebra can't be fractions or decimals. It's just that most textbooks and reference books try to stick with nice answers to increase the comfort level and avoid confusion. This is my plan in this book, too. After all, who wants a messy answer, even though, in real life, that's more often the case. I use integers in Chapters 8 and 9, where you find out how to solve equations.

Being reasonable: Rational numbers

Rational numbers act rationally! What does that mean? In this case, acting rationally means that the decimal equivalent of the rational number behaves. The decimal ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving."

Some rational numbers have decimals that end such as: 3.4, 5.77623, –4.5. Other rational numbers have decimals that repeat the same pattern, such as $3.164164\overline{164}$, or $0.66666666\overline{6}$. The horizontal bar over the 164 and the 6 lets you know that these numbers repeat forever.

In *all* cases, rational numbers can be written as fractions. Each rational number has a fraction that it's equal to. So one definition of a *rational number* is any number that can be written as a fraction, $\frac{p}{q}$, where p and q are integers (except q can't be 0). If a number can't be written as a fraction, then it isn't a rational number. Rational numbers appear in Chapter 13, where you see quadratic equations, and in Part IV, where the applications are presented.

Restraining irrational numbers

Irrational numbers are just what you may expect from their name — the opposite of rational numbers. An *irrational number* cannot be written as a fraction, and decimal values for irrationals never end and never have a nice pattern to them. Whew! Talk about irrational! For example, pi, with its never-ending decimal places, is irrational. Irrational numbers are often created when using the quadratic formula, as you see in Chapter 13.

Picking out primes and composites

A number is considered to be *prime* if it can be divided evenly only by 1 and by itself. The first prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on. The only prime number that's even is 2, the first prime number. Mathematicians have been studying prime numbers for centuries, and prime numbers have them stumped. No one has ever found a formula for producing all the primes. Mathematicians just assume that prime numbers go on forever.

A number is *composite* if it isn't prime — if it can be divided by at least one number other than 1 and itself. So the number 12 is composite because it's divisible by 1, 2, 3, 4, 6, and 12. Chapter 6 deals with primes, but you also see them in Chapters 8 and 10, where I show you how to factor primes out of expressions.

Speaking in Algebra

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. It's important to know the vocabulary in a foreign language; it's just as important in algebra.

- An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. $2x^2 + 4x$ is an example of an expression. You see distributions over expressions in Chapter 7.
- A *term,* such as $4xy$, is a grouping together of one or more *factors* (variables and/or numbers). Multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another. For example, the expression $3xy + 5x - 6$ has three *terms.*
- An *equation* uses a sign to show a relationship — that two things are equal. By using an equation, tough problems can be reduced to easier problems and simpler answers. An example of an equation is $2x^2 + 4x = 7$. See the chapters in Part III for more information on equations.
- An *operation* is an action performed upon one or two numbers to produce a resulting number. Operations are addition, subtraction, multiplication, division, square roots, and so on. See Chapter 5 for more on operations.
- A *variable* is a letter representing some unknown; a variable always represents a number, but it *varies* until it's written in an equation or inequality. (An *inequality* is a comparison of two values. For more on inequalities, turn to Chapter 15.) Then the fate of the variable is set — it can be solved for, and its value becomes the solution of the equation. By convention, mathematicians usually assign letters at the end of the alphabet to be variables (such as x, y, and z).
- A *constant* is a value or number that never changes in an equation — it's constantly the same. Five is a constant because it is what it is. A variable can be a constant if it is assigned a definite value. Usually, a variable representing a constant is one of the first letters in the alphabet. In the equation $ax^2 + bx + c = 0$, a, b, and c are constants and the x is the variable. The value of x depends on what a, b, and c are assigned to be.

- An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2. It's used to show repeated multiplication. An exponent is also called the *power* of the value. For more on exponents, see Chapter 4.

Taking Aim at Algebra Operations

In algebra today, a variable represents the unknown. (You can see more on variables in the "Speaking in Algebra" section earlier in this chapter.) Before the use of symbols caught on, problems were written out in long, wordy expressions. Actually, using letters, signs, and operations was a huge breakthrough. First, a few operations were used, and then algebra became fully symbolic. Nowadays, you may see some words alongside the operations to explain and help you understand, like having subtitles in a movie.

By doing what early mathematicians did — letting a variable represent a value, then throwing in some operations (addition, subtraction, multiplication, and division), and then using some specific rules that have been established over the years — you have a solid, organized system for simplifying, solving, comparing, or confirming an equation. That's what algebra is all about: That's what algebra's good for.

Deciphering the symbols

The basics of algebra involve symbols. Algebra uses symbols for quantities, operations, relations, or grouping. The symbols are shorthand and are much more efficient than writing out the words or meanings. But you need to know what the symbols represent, and the following list shares some of that info. The operations are covered thoroughly in Chapter 5.

- + means *add* or *find the sum, more than,* or *increased by;* the result of addition is the *sum.* It also is used to indicate a *positive number.*
- – means *subtract* or *minus* or *decreased by* or *less than;* the result is the *difference.* It's also used to indicate a *negative number.*
- × means *multiply* or *times.* The values being multiplied together are the *multipliers* or *factors;* the result is the *product.* Some other symbols meaning *multiply* can be grouping symbols: ( ), [ ], { }, ·, *. In algebra, the × symbol is used infrequently because it can be confused with the variable *x.* The dot is popular because it's easy to write. The grouping symbols are used when you need to contain many terms or a messy expression. By themselves, the grouping symbols don't mean to multiply, but if you put a value in front of a grouping symbol, it means to multiply.

- ÷ means *divide*. The number that's going into the *dividend* is the *divisor.* The result is the *quotient.* Other signs that indicate division are the fraction line and slash, /.
- $\sqrt{\ }$ means to take the *square root* of something — to find the number, which, multiplied by itself, gives you the number under the sign. (See Chapter 4 for more on square roots.)
- $|\ |$ means to find the *absolute value* of a number, which is the number itself or its distance from 0 on the number line. (For more on absolute value, turn to Chapter 2.)
- π is the Greek letter pi that refers to the irrational number: 3.14159. . . . It represents the relationship between the diameter and circumference of a circle.

Grouping

When a car manufacturer puts together a car, several different things have to be done first. The engine experts have to construct the engine with all its parts. The body of the car has to be mounted onto the chassis and secured, too. Other car specialists have to perform the tasks that they specialize in as well. When these tasks are all accomplished in order, then the car can be put together. The same thing is true in algebra. You have to do what's inside the *grouping* symbol before you can use the result in the rest of the equation.

Grouping symbols tell you that you have to deal with the *terms* inside the grouping symbols *before* you deal with the larger problem. If the problem contains grouped items, do what's inside a grouping symbol first, and then follow the order of operations. The grouping symbols are

- **Parentheses ():** Parentheses are the most commonly used symbols for grouping.
- **Brackets [ ] and braces { }:** Brackets and braces are also used frequently for grouping and have the same effect as parentheses. Using the different types of symbols helps when there's more than one grouping in a problem. It's easier to tell where a group starts and ends.
- **Radical $\sqrt{\ }$:** This is used for finding roots.
- **Fraction line (called the *vinculum*):** The fraction line also acts as a grouping symbol — everything above the line (in the *numerator*) is grouped together, and everything below the line (in the *denominator*) is grouped together.

Even though the order of operations and grouping-symbol rules are fairly straightforward, it's hard to describe, in words, all the situations that can come up in these problems. The examples in Chapters 5 and 7 should clear up any questions you may have.

Defining relationships

Algebra is all about relationships — not the he-loves-me-he-loves-me-not kind of relationship — but the relationships between numbers or among the terms of an equation. Although algebraic relationships can be just as complicated as romantic ones, you have a better chance of understanding an algebraic relationship. The symbols for the relationships are given here. The equations are found in Chapters 11 through 14, and inequalities are found in Chapter 15.

- = means that the first value *is equal to* or the same as the value that follows.
- ≠ means that the first value *is not equal to* the value that follows.
- ≈ means that one value is *approximately the same* or *about the same* as the value that follows; this is used when rounding numbers.
- ≤ means that the first value is *less than or equal to* the value that follows.
- < means that the first value is *less than* the value that follows.
- ≥ means that the first value is *greater than or equal to* the value that follows.
- > means that the first value is *greater than* the value that follows.

Taking on algebraic tasks

Algebra involves symbols, such as variables and operation signs, which are the tools that you can use to make algebraic expressions more usable and readable. These things go hand in hand with simplifying, factoring, and solving problems, which are easier to solve if broken down into basic parts. Using symbols is actually much easier than wading through a bunch of words.

- To *simplify* means to combine all that can be combined, cut down on the number of terms, and put an expression in an easily understandable form.
- To *factor* means to change two or more terms to just one term. (See Part II for more on factoring.)
- To *solve* means to find the answer. In algebra, it means to figure out what the variable stands for. (You see solving equations in Part III and solving for answers to practical applications in Part IV.)

Equation solving is fun because there's a point to it. You solve for something (often a variable, such as x) and get an answer that you can check to see whether you're right or wrong. It's like a puzzle. It's enough for some people to say, "Give me an x." What more could you want? But solving these equations is just a means to an end. The real beauty of algebra shines when you solve some problem in real life — a practical application. Are you ready for these two words: *story problems?* Story problems are the whole point of doing algebra. Why do algebra unless there's a good reason? Oh, I'm sorry — you may just like to solve algebra equations for the fun alone. (Yes, some folks are like that.) But other folks love to see the way a complicated paragraph in the English language can be turned into a neat, concise expression, such as, "The answer is three bananas."

Going through each step and using each tool to play this game is entirely possible. *Simplify, factor, solve, check.* That's good! Lucky you. It's time to dig in!