# 1 Principles and Methods of Electric Power Conversion

In this introductory chapter we provide a background for the subject of the book. The scope, tools, and applications of power electronics are outlined. The concept of generic power converter is introduced to illustrate the principles of operation of power electronic converters and types of power conversion performed. Components of voltage or current waveforms and the related figures of merit are defined. Two basic methods of magnitude control, that is, phase control and pulse width modulation, are presented. Calculation of output current waveforms is explained. Single-phase diode rectifiers are described as simple examples of practical power converters.



## 1.1 WHAT IS POWER ELECTRONICS?

Modern society, with its conveniences, relies strongly on the ubiquitous availability of electric energy. Electricity performs most of the physical labor, provides heating and lighting, activates electrochemical processes, and facilitates information collecting, processing, storage, and exchange.

Contemporary *power electronics* can be defined as a *branch of electrical engineering devoted to conversion and control of electric power using electronic converters based on semiconductor power switches*. The existing power systems deliver an alternating-current (ac) voltage of fixed frequency and magnitude. Typically, homes, offices, stores, and similar small facilities are supplied from single-phase, low-voltage power lines, and three-phase supply systems with various voltage levels are available in industrial plants and other large commercial enterprises. The 60-Hz fixed-voltage electric power used in the United States (50 Hz in most other parts of the world) can be thought of as *raw power*, which for many applications must be *conditioned*. Power conditioning involves *conversion*, from ac to dc (direct current), or vice versa, and *control* of the magnitude and/or frequency of voltages and currents. Using electric lighting as a simple example, an incandescent bulb can be supplied directly with raw power. However, a fluorescent lamp requires an electronic ballast, which starts the arc and controls the current to maintain the light output desired. The ballast is thus

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a power conditioner, necessary for proper operation of the lamp. If used in a movie theater, incandescent bulbs are supplied from an ac voltage controller that allows gradual dimming of the light just before the movie begins. Again, this controller is an example of a power conditioner, or *power converter*.

The birth of power electronics can be traced back to the dawn of the twentieth century, when the first mercury-arc rectifiers were invented. However, for conversion and control of electric power, *rotating electromachine converters* were generally used. An electromachine converter was simply an electric generator driven by an electric motor. If, for example, adjustable dc voltage was to be obtained from fixed ac voltage, an ac motor operated a dc generator with controlled output voltage. Conversely, if ac voltage was required and the supply energy came from a battery pack, a speed-controlled dc motor and an ac synchronous generator were employed. Clearly, the convenience, efficiency, and reliability of such systems were inferior to those of today's *static power electronic converters*, involving motionless and direct energy conversion.

Today's power electronics began with the development of the *silicon-controlled rectifier* (SCR), also called a *thyristor*, by the General Electric Company in 1958. The SCR is a unidirectional semiconductor power switch that can be turned on (closed) by a low-power electric pulse applied to its controlling electrode, the gate. The voltage and current ratings of SCRs are the highest of those of all types of semiconductor power switches. However, the SCR is inconvenient for use in dc-input power electronic converters because, when conducting a current, it cannot be turned off (opened) by controlling the gate. Thus, the SCR is a *semicontrolled* switch. Within the last three decades, several types of *fully controlled* semiconductor power switches turned on and off by an electric signal have been introduced to the market. These switches, as well as the SCR, are described in detail in Chapter 2.

Widespread introduction of power electronic converters to most areas of distribution and use of electric energy is common in all developed countries. The converters condition the electric power for a variety of applications, such as electric motor drives, uninterruptible power supplies, heating and lighting, electrochemical and electrothermal processes, electric arc welding, high-voltage dc transmission lines, active power filters and reactive power compensators in power systems, and high-quality supply sources for computers and other electronic equipment.

It is estimated that at least half of the electric power generated in the United States flows through power electronic converters, and an increase in this share to almost 100% is expected in the following few decades. In particular, a thorough revamping of the existing U.S. power system is envisioned within the FACTS program initiated by the Electric Power Research Institute. Introduction of power electronic converters to all stages of power generation, transmission, and distribution allows a dramatic increase in the system's capability without investment in new power plants and transmission lines. The important role of power electronics in renewable energy systems and electric and hybrid vehicles is also worth stressing. Therefore, it is safe to say that practically every electrical engineer encounters some power electronic converters in his or her professional career.

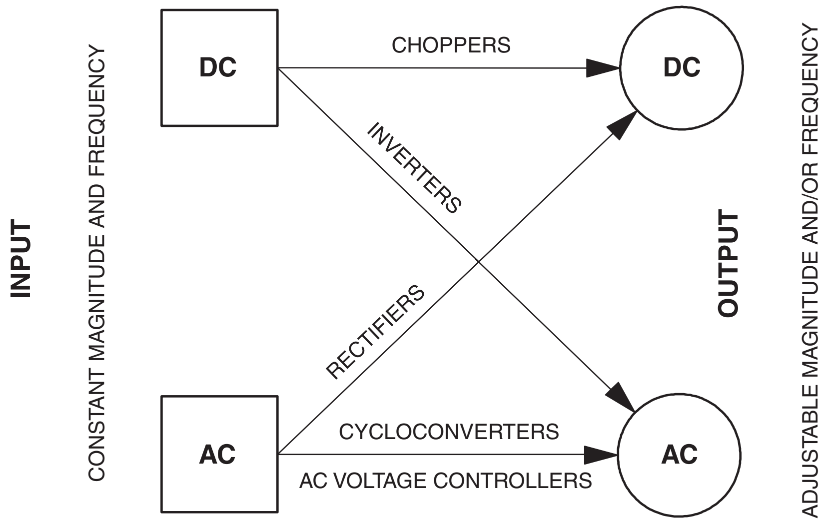


**Figure 1.1** Types of electric power conversion and corresponding power electronic converters.

Types of electric power conversion and the corresponding converters employed in the contemporary power electronics are shown in Figure 1.1. For example, ac-to-dc conversion is accomplished using rectifiers, which are supplied from an ac source whose output voltage contains a significant fixed or adjustable dc component. Individual types of power electronic converters are described and analyzed in Chapters 4 through 8. Basic principles of power conversion and control are explained in the following sections of this chapter.

## 1.2 GENERIC POWER CONVERTER

Although not a practical apparatus, the hypothetical *generic power converter* shown in Figure 1.2 is a useful teaching tool to illustrate the principles of electric power conversion and control. It is a two-port network of five switches. Switches S1 and S2 provide *direct connection* between the input (supply) terminals, I1 and I2, and the output (load) terminals, O1 and O2, respectively, while switches S3 and S4 allow *cross-connection* between these pairs of terminals. A voltage source, either dc or ac, supplies the electric power to a load through the converter. Practical loads usually contain a significant inductive component, so a resistive–inductive load (an RL load) is assumed in subsequent considerations. To ensure a closed path for the load current under any operating conditions, a fifth switch, S5, is connected between the output terminals of the converter, and closed when switches S1 through S4 are open. It is assumed that the switches open or close instantaneously.

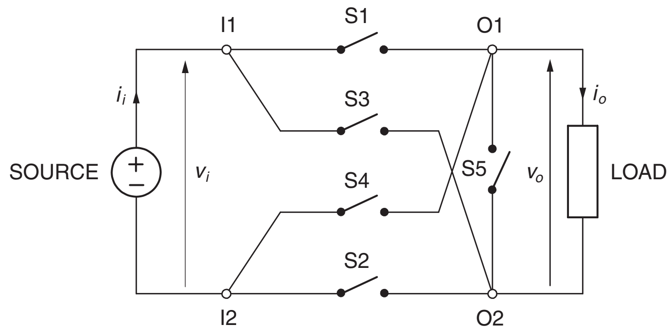


**Figure 1.2** Generic power converter.

The supply source is an ideal voltage source, and as such it may not be shorted. Also, the load current may not be interrupted. It would cause rapid release of the electromagnetic energy accumulated in the load inductance, and a high and potentially damaging overvoltage would occur. Therefore, the generic converter can only assume the following three states:

*State 0*. Switches S1 through S4 are open and switch S5 is closed, shorting the output terminals and closing a path for the lingering load current, if any. The output voltage is zero. The input terminals are cut off from the output terminals, so the input current is also zero.

*State 1*. Switches S1 and S2 are closed and the remaining switches are open. The output voltage equals the input voltage, and the output current equals the input current.

*State 2*. Switches S3 and S4 are closed and the remaining switches are open. Now the output voltage and current are reversed with respect to their input counterparts.

To illustrate voltage and current waveforms, specific values of the input voltage and the RL load of the generic converter were employed. The amplitude of the input voltage was taken as 100 V for both the ac and dc voltage considered, and the load resistance and inductance were assumed to be 1.3 Ω and 2.4 mH, respectively. These data were needed for the preparation of subsequent figures and in example calculations in the next section. However, for generality, the waveforms are shown without the magnitude scale.

Let us assume that the generic converter is to perform the ac-to-dc conversion. The sinusoidal input voltage, $v_i$, whose waveform is shown in Figure 1.3, is given by

$$v_i = V_{i,p}\sin(\omega t) \tag{1.1}$$

where $V_{i,p}$ denotes the peak value of the voltage and $\omega$ is the input radian frequency. The output voltage, $v_o$, of the converter should contain a possibly large dc component.

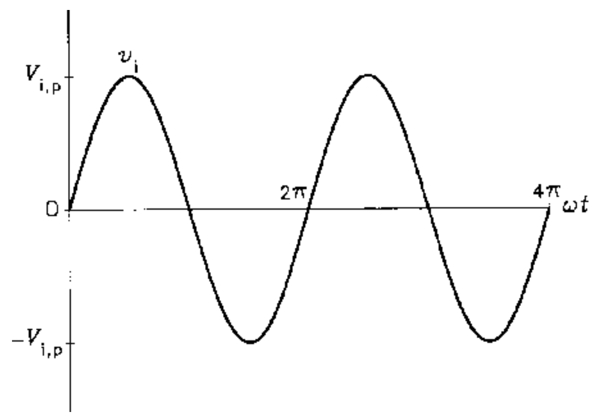


**Figure 1.3** Input ac voltage waveform.

Note that the output voltage is not expected to be of ideal dc quality, since such voltage and current are not possible to obtain in the generic converter as well as in practical power electronic converters. The same applies to the ideally sinusoidal output voltage and current in ac-output converters. If within the first half-cycle of the input voltage the converter is in state 1 and within the second half-cycle in state 2, the output voltage waveform will be as depicted in Figure 1.4; that is,

$$v_o = |v_i| = V_{i,p} |\sin(\omega t)| . \tag{1.2}$$

The dc component is the average value of the voltage. Power electronic converters performing ac-to-dc conversion are called *rectifiers*.

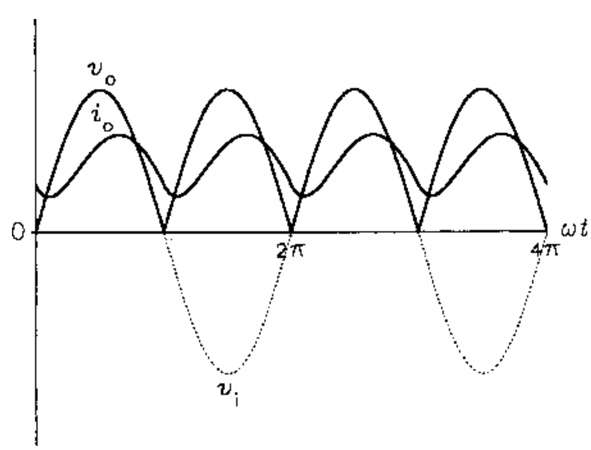


**Figure 1.4** Output voltage and current waveforms in a generic rectifier.

The output current waveform, $i_o$, can be obtained as a numerical solution of the load equation

$$L\frac{di_o}{dt} + Ri_o = v_o. \tag{1.3}$$

Techniques for analytical and numerical computation of voltage and current waveforms in power electronic circuits are described at the end of the chapter. Here, only general features of the waveforms are outlined. The output current waveform of the generic rectifier considered is also shown in Figure 1.4, and the consecutive states of the converter are indicated there. It can be seen that this waveform is closer to an ideal dc waveform than is the output voltage waveform, because of the frequency-dependent load impedance. The $k$th harmonic, $v_{o,k}$, of the output voltage produces the corresponding harmonic, $i_{o,k}$, of the output current such that

$$I_{o,k} = \frac{V_{o,k}}{\sqrt{R^2 + (k\omega_o L)^2}} \tag{1.4}$$

where $I_{o,k}$ and $V_{o,k}$ denote rms (root-mean-square) values of the current and voltage harmonics in question, respectively. In the rectifier considered, the fundamental radian frequency, $\omega_o$, of the output voltage is twice as high as the input frequency, $\omega$. The load impedance (represented by the denominator on the right-hand side of Eq. (1.4)) for individual current harmonics increases with the harmonic number, $k$. Clearly, the dc component ($k = 0$) of the output current encounters the lowest impedance, equal to the load resistance only, while the load inductance attenuates only the ac component. In other words, the RL load acts as a low-pass filter. In the next section we provide a detailed explanation of terms related to the components and harmonic spectra of waveforms.

Interestingly, if an ac output voltage is to be produced and the generic converter is supplied from a dc source, so that the input voltage is $v_i = V_i = \text{const}$, the switches are operated in the same manner as in the preceding case. Specifically, for every half-period of the desired output frequency, states 1 and 2 are interchanged. In this way, the input terminals are alternately connected and cross-connected with the output terminals, and the output voltage acquires the ac (although not sinusoidal) waveform shown in Figure 1.5. The output current is composed of growth- and decay-function segments, typical for transient conditions of an RL circuit subjected to dc excitation. Again, thanks to the attenuating effects of the load inductance, the current waveform is closer than the voltage waveform to the sinusoid desired. In practice, the dc-to-ac power conversion is performed by power electronic *inverters*. In the case described, the generic inverter is said to operate in the *square-wave mode*.

If the input or output voltage is to be a three-phase ac voltage, the topology of the generic power converter portrayed here would have to be expanded, but it still would be a network of switches. Real power electronic converters are also *networks of semiconductor power switches*. For various purposes, other elements,

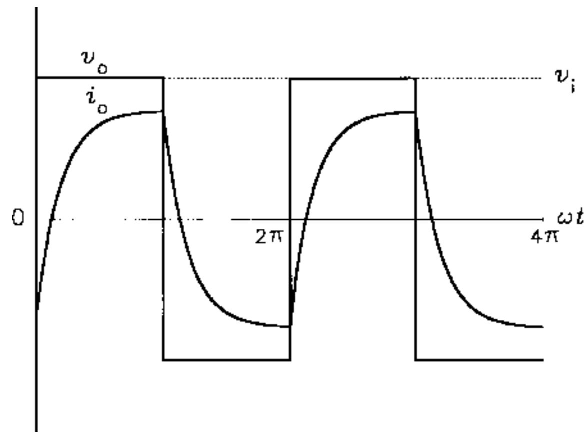


**Figure 1.5** Output voltage and current waveforms in a generic inverter.

such as inductors, capacitors, fuses, and auxiliary circuits, are employed besides the switches in power circuits of practical power electronic converters. Yet in most of these converters, the fundamental operating principle is the same as in the generic converter; that is, the input and output terminals are being connected, cross-connected, and disconnected in a specific manner and sequence required for the given type of power conversion. Typically, as in the generic rectifier and inverter presented, the load inductance inhibits the switching-related undesirable high-frequency components of the output current.

Although a voltage source has been assumed for the generic power converter, some power electronic converters are supplied from current sources. In such converters, a large inductor is connected in series with the input terminals to prevent rapid changes in the input current. Analogously, voltage-source converters usually have a large capacitor connected across the input terminals to stabilize the input voltage. Inductors or capacitors are also used at the output of some converters to smooth the output current or voltage, respectively.

According to one of the principles of circuit theory, two ideal current sources may not be connected in series, and two ideal voltage sources may not be connected in parallel. Consequently, the load of a current-source converter may not appear as a current source, while that of a voltage-source converter may not appear as a voltage source. As illustrated in Figure 1.6, this means that in a current-source power electronic converter a capacitor should be placed in parallel with the load. Apart from smoothing the output voltage, the capacitor prevents the potential hazards of connecting the input inductance conducting a certain current with a load inductance conducting a different current. In contrast, in voltage-source converters, no capacitor may be connected across the output terminals, and it is the load inductance (or an extra inductor between the converter and the load) that smoothes the output current.

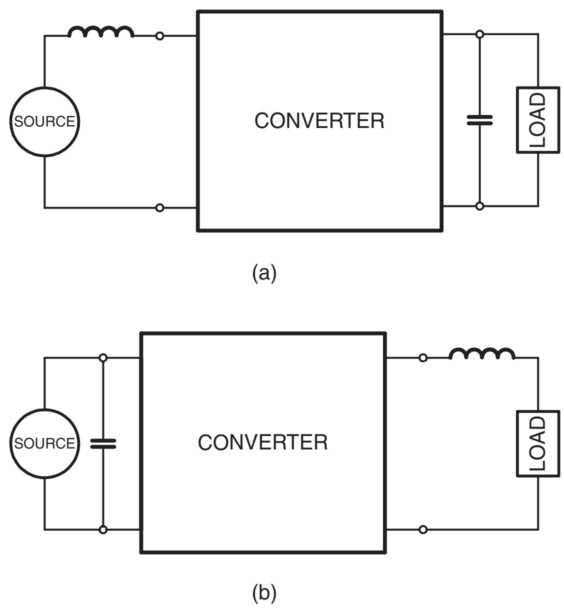


**Figure 1.6** Basic configurations of power electronic converters: (a) current-source; (b) voltage-source.

## 1.3 WAVEFORM COMPONENTS AND FIGURES OF MERIT

Terms such as the *dc component*, *ac component*, and *harmonics* used in the preceding section deserve closer examination. Knowledge of the basic components of voltage and current waveforms allows evaluation of the performance of a converter. Certain relations of these components are commonly used as performance indicators, or *figures of merit*.

A time function $\psi(t)$, here a waveform of voltage or current, is said to be *periodic* with a period $T$ if

$$\psi(t) = \psi(t + T) \tag{1.5}$$

that is, if the pattern (shape) of the waveform is repeated every $T$ seconds. In the realm of power electronics, it is often convenient to analyze voltages and currents in the *angle domain* instead of the *time domain*. The *fundamental frequency*, $f_1$, in hertz, is defined as

$$f_1 = \frac{1}{T} \tag{1.6}$$

and the corresponding *fundamental radian frequency*, $\omega_1$, in rad/s, as

$$\omega_1 = 2\pi f_1 = \frac{2\pi}{T}. \tag{1.7}$$

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Now, a periodic function $\psi(\omega_1 t)$ can be defined such that

$$\psi(\omega_1 t) = \psi(\omega_1 t + 2\pi). \tag{1.8}$$

The *rms value*, $\Psi$, of waveform $\psi(\omega_1 t)$ is defined as

$$\Psi \equiv \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \psi^2(\omega_1 t)\, d\omega_1 t} \tag{1.9}$$

and the *average value*, or *dc component*, $\Psi_{\text{dc}}$, of the waveform as

$$\Psi_{\text{dc}} \equiv \frac{1}{2\pi}\int_0^{2\pi} \psi(\omega_1 t)\, d\omega_1 t \tag{1.10}$$

When the dc component is subtracted from the waveform, the remaining waveform, $\psi_{\text{ac}}(\omega_1 t)$, is called the *ac component*, or *ripple*, that is,

$$\psi_{\text{ac}}(\omega_1 t) = \psi_{\text{ac}}(\omega_1 t) - \psi_{\text{dc}}. \tag{1.11}$$

Clearly, the ac component has an average value of zero and a fundamental frequency of $f_1$.

The rms value, $\Psi_{\text{ac}}$, of $\psi_{\text{ac}}(\omega_1 t)$ is defined as

$$\Psi_{\text{ac}} \equiv \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \psi_{\text{ac}}^2(\omega_1 t)\, d\omega_1 t} \tag{1.12}$$

and it is easy to show that

$$\Psi^2 = \Psi_{\text{dc}}^2 + \Psi_{\text{ac}}^2. \tag{1.13}$$

For waveforms of the desirable ideal dc quality, such as the load current of a rectifier, a figure of merit called a *ripple factor*, RF, is defined as

$$\text{RF} = \frac{\Psi_{\text{ac}}}{\Psi_{\text{dc}}}. \tag{1.14}$$

A low value of the ripple factor indicates the high quality of a waveform.

Before proceeding to other waveform components and figures of merit, the terms and formulas introduced so far will be illustrated using the waveform of output voltage, $v_o$, of the generic rectifier, shown in Figure 1.4. The waveform pattern is repeating itself every $\pi$ radians and, within the 0-to-$\pi$ interval, $v_o = v_i$. Therefore, the average value, $V_{o,\text{dc}}$, of the output voltage can most conveniently be determined by calculating the area under the waveform from $\omega t = 0$ to $\omega t = \pi$ and dividing it by the length, $\pi$, of the interval considered. Thus,

$$V_{o,\mathrm{dc}} = \frac{1}{\pi}\int_0^{\pi} V_{i,p}\sin\omega t\; d\omega t = \frac{2}{\pi}V_{i,p} = 0.64V_{i,p}. \tag{1.15}$$

Note that formula (1.15) differs from (1.10). Since $\omega_1 = \omega_0 = 2\omega$, the integration is performed in the 0-to-$\pi$ interval of $\omega t$ instead of the 0-to-$2\pi$ interval of $\omega_1 t$.

Similarly, the rms value, $V_o$, of the output voltage can be calculated as

$$V_0 = \sqrt{\frac{1}{\pi}\int_0^{\pi}(V_{i,p}\sin\omega t)^2\; d\omega t} = \frac{V_{i,p}}{\sqrt{2}} = 0.71V_{i,p}. \tag{1.16}$$

This result agrees with the well-known relation for a sine wave as $v_o^2 = v_i^2$.

Based on Eqs. (1.13) and (1.14), the rms value, $V_{o,\mathrm{ac}}$, of the ac component of the voltage in question can be calculated as

$$V_{o,\mathrm{ac}} = \sqrt{V_0^2 - V_{0,\mathrm{dc}}^2} = \sqrt{\left(\frac{V_{i,p}}{\sqrt{2}}\right)^2 - \left(\frac{2}{\pi}V_{i,p}\right)^2} = 0.31V_{i,p} \tag{1.17}$$

and the ripple factor, $\mathrm{RF}_V$, of the voltage as

$$\mathrm{RF}_V = \frac{V_{0,\mathrm{ac}}}{V_{0,\mathrm{dc}}} = \frac{0.31V_{i,p}}{0.64V_{i,p}} = 0.48. \tag{1.18}$$

Decomposition into the dc and ac components of the waveform being analyzed is shown in Figure 1.7.

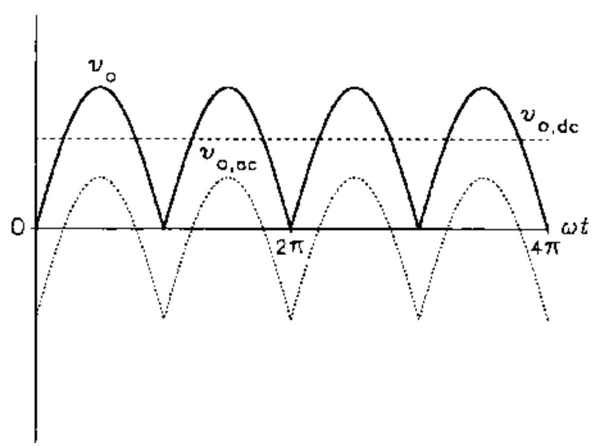


**Figure 1.7** Decomposition of an output voltage waveform in a generic rectifier.

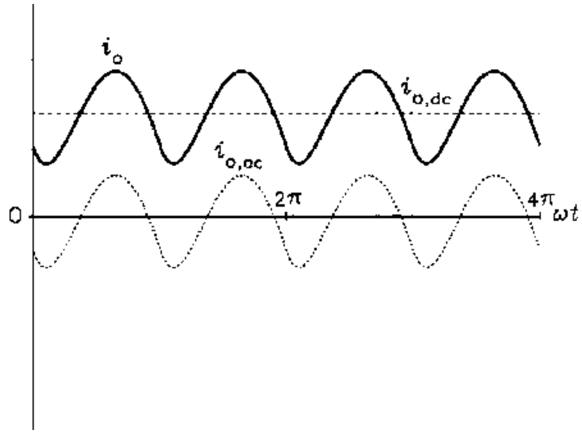


**Figure 1.8** Decomposition of the output current waveform in a generic rectifier.

To determine the ripple factor, $RF_I$, of the output current analytically, the output current waveform, $i_o(\omega t)$, would have to be expressed in a closed form. Instead, numerical computations were performed on the waveform in Figure 1.4 and $RF_I$ was found to equal 0.31. This value is 36% lower than that of the output voltage. This is an example only, but output currents in power electronic converters are indeed of higher quality than output voltages. It is worth mentioning that the value of $RF_I$ obtained is poor. Practical high-quality dc current waveforms have a ripple factor on the order of a few percentage points, and below the 5% level, the current is considered to be of practically ideal dc quality. The current ripple factor depends on the type of converter, and it decreases with an increase in the inductive component of a load. Components of the current waveform evaluated are shown in Figure 1.8.

The ripple factor is of no use for quality evaluation of ac waveforms such as the output current of an inverter, which ideally should be a pure sinusoid. However, as already mentioned and exemplified by the waveforms in Figure 1.5, purely sinusoidal voltages and currents cannot be produced by switching power converters. Therefore, an appropriate figure of merit must be defined as a measure of deviation of a practical ac waveform from its ideal counterpart.

Following the theory of Fourier series (see Appendix B), the ac component, $\psi_{ac}(t)$, of a periodic function, $\psi(t)$, can be expressed as an infinite sum of the harmonics, that is, sine waves whose frequencies are multiples of the fundamental frequency, $f_1$, of $\psi(t)$. In the angle domain,

$$\psi_{ac}(\omega_1 t) = \sum_{k=1}^{\infty} \psi_k k\omega_1 t = \sum_{k=1}^{\infty} \Psi_{k,p} \cos(k\omega_1 t + \varphi_k) \tag{1.19}$$

where $k$ is the *harmonic number* and $\Psi_{k,p}$ and $\varphi_k$ denote the peak value and phase angle of the $k$th harmonic, respectively. The first harmonic, $\psi_1(\omega_1 t)$, is usually called a *fundamental*. The terms *fundamental voltage* and *fundamental current* are used throughout the book to denote the fundamental of a given voltage or current.

The peak value, $\Psi_{1,p}$, of the fundamental of a periodic function, $\psi(\omega_1 t)$, is calculated as

$$\Psi_{1,p} = \sqrt{\Psi_{1,c}^2 + \Psi_{1,s}^2} \tag{1.20}$$

where

$$\Psi_{1,c} = \frac{1}{\pi} \int_0^{2\pi} \psi(\omega_1 t) \cos \omega_1 t \; d\omega_1 t \tag{1.21}$$

$$\Psi_{1,s} = \frac{1}{\pi} \int_0^{2\pi} \psi(\omega_1 t) \sin \omega_1 t \; d\omega_1 t \tag{1.22}$$

and the rms value, $\Psi_1$, of the fundamental is

$$\Psi_1 = \frac{\Psi_{1,p}}{\sqrt{2}}. \tag{1.23}$$

Since the fundamental of a function does not depend on the dc component of the function, the ac component, $\psi_{\text{ac}}(\omega_1 t)$, can be used in Eqs. (1.21) and (1.22) in place of $\psi(\omega_1 t)$.

When the fundamental is subtracted from the ac component, the *harmonic component*, $\psi_h(\omega_1 t)$, is obtained as

$$\psi_h(\omega_1 t) = \psi_{\text{ac}}(\omega_1 t) - \psi_1(\omega_1 t). \tag{1.24}$$

The rms value, $\Psi_h$, of $\psi_h(\omega_1 t)$, called a *harmonic content* of function $\psi(\omega_1 t)$, can be calculated as

$$\Psi_h = \sqrt{\Psi_{\text{ac}}^2 - \Psi_1^2} = \sqrt{\Psi^2 - \Psi_{\text{dc}}^2 - \Psi_1^2} \tag{1.25}$$

and used for calculation of the *total harmonic distortion*, THD, defined as

$$\text{THD} \equiv \frac{\Psi_h}{\Psi_1}. \tag{1.26}$$

The concept of total harmonic distortion is widely employed in practice, also outside power electronics, as, for example, in characterization of the quality of audio equipment. Conceptually, the total harmonic distortion constitutes an ac counterpart of the ripple factor.

Using as an example the generic inverter whose output waveforms are shown in Figure 1.6, the rms value, $V_o$, of the output voltage is equal to the dc input voltage, $V_i$. Since $v_o$ is either $V_i$ or $-V_i$, then $v_0^2 = V_i^2$. The peak value, $v_{o,1,p}$, of the fundamental output voltage is

$$V_{o,1,p} = V_{0,1,s} \tag{1.27}$$

because the waveform in question has odd symmetry (see Appendix B). Consequently,

$$V_{o,1,p} = \frac{2}{\pi}\int_0^{\pi} V_i \sin\omega t \; d\omega t = \frac{4}{\pi}V_i = 1.27V_i. \tag{1.28}$$

In this case, $\omega$ denotes the fundamental output frequency. Now, the fundamental output voltage, $v_{o,1}(\omega t)$, can be expressed as

$$v_{o,1}(\omega t) = V_{o,1,p}\sin\omega t = \frac{4}{\pi}V_i \sin\omega t. \tag{1.29}$$

The rms value, $V_{o,1}$, of the fundamental output voltage is

$$V_{o,1} = \frac{V_{0,1,p}}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi}V_i = 0.9V_i \tag{1.30}$$

and the harmonic content, $v_{o,h}$, is

$$V_{o,h} = \sqrt{V_0^2 - V_{0,1}^2} = \sqrt{V_i^2 - \left(\frac{2\sqrt{2}}{\pi}V_i\right)^2} = 0.44V_i. \tag{1.31}$$

Thus, the total harmonic distortion of the output voltage, $\text{THD}_V$, is

$$\text{THD}_V = \frac{V_{o,h}}{V_{o,1}} = \frac{0.44V_i}{0.9V_i} = 0.49. \tag{1.32}$$

The high value of $\text{THD}_V$ is not surprising since the output voltage waveform of a generic inverter operating in the square-wave mode differs strongly from a sine wave. Decomposition of the waveform analyzed is illustrated in Figure 1.9.

The numerically determined total harmonic distortion, $\text{THD}_I$, of the output current, $i_o$, is 0.216 in this example, that is, less than that of the output voltage by as much as 55%. Indeed, as shown in Figure 1.10, which shows decomposition of the current waveform, the harmonic component is quite small compared with the fundamental. As in a generic rectifier, it shows the attenuating influence of the load inductance on the output current. In practical inverters, the output current is considered to be of high quality if $\text{THD}_I$ does not exceed 0.05 (5%).

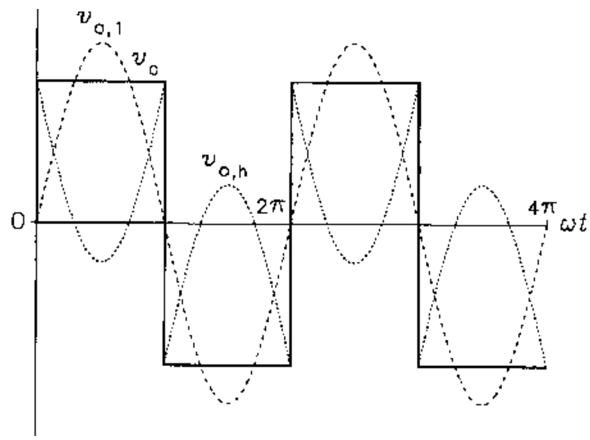


**Figure 1.9** Decomposition of the output voltage waveform in a generic inverter.

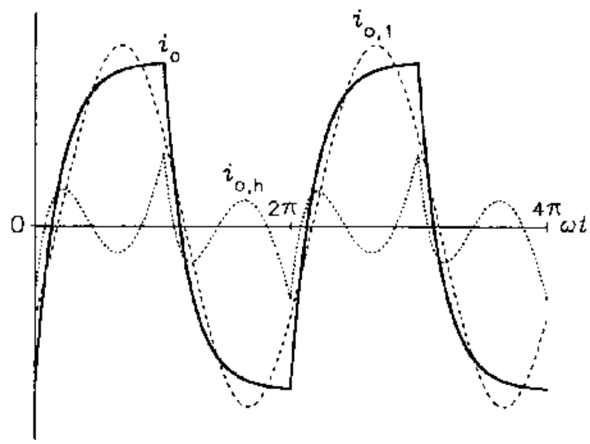


**Figure 1.10** Decomposition of the output current waveform in a generic inverter.

Other figures of merit often employed for performance evaluation of power electronic converters are:

1. *The power efficiency*, $\eta$, of the converter, defined as

$$\eta \equiv \frac{P_0}{P_i} \tag{1.33}$$

where $P_o$ and $P_i$ denote the output and input powers of the converter, respectively.

2. *The conversion efficiency*, $\eta_c$, of the converter, defined as

$$\eta_c \equiv \frac{P_{o,\text{dc}}}{P_i} \tag{1.34}$$

for dc output converters, and

$$\eta_c \equiv \frac{P_{o,1}}{P_i} \tag{1.35}$$

for ac output converters. The symbol $P_{o,\text{dc}}$ denotes the dc output power, that is, the product of the dc components of the output voltage and current, while $P_{o,1}$ is the ac output power carried by the fundamental components of the output voltage and current.

3. *The input power factor*, PF, of the converter, defined as

$$\text{PF} = \frac{P_i}{S_i} \tag{1.36}$$

where $S_i$ is the apparent input power. The power factor can also be expressed as

$$\text{PF} = K_d K_\Theta. \tag{1.37}$$

Here $K_d$ denotes the *distortion factor* (not to be confused with the total harmonic distortion, THD), defined as the ratio of the rms fundamental input current, $I_{i,1}$, to the rms input current, $I_i$, and $K_\Theta$ is the *displacement factor*, that is, the cosine of the phase shift, $\Theta$, between the fundamentals of input voltage and current.

The power efficiency, $\eta$, of a converter simply indicates what portion of the power supplied to the converter reaches the load. In contrast, the conversion efficiency, $\eta_c$, expresses the relative amount of *useful* output power and, therefore constitutes a more valuable figure of merit than the power efficiency. Since the input voltage to a converter is usually constant, the power factor serves mainly as a measure of utilization of the input current, drawn from the source that supplies the converter. With a constant power consumed by the converter, a high power factor implies a low current and, consequently, low power losses in the source. The reader is probably already familiar with the term *power factor* as the cosine of the phase shift between the voltage and current waveforms, as used in the theory of ac circuits. However, it must be stressed that it is true for purely sinusoidal waveforms only, and the general definition of the power factor is that given by Eq. (1.36).

In an ideal power converter, all three figures of merit defined above would equal unity. To illustrate the relevant calculations, the generic rectifier will again be employed. Since ideal switches have been assumed, no losses are incurred in the generic

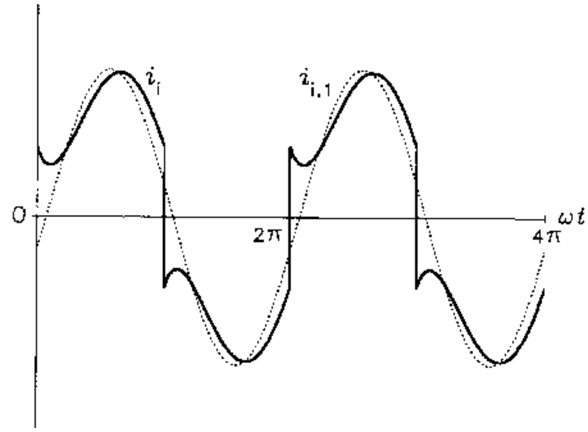


**Figure 1.11** Decomposition of the output current waveform in a generic inverter.

converter, so that the input power, $P_i$, and output power, $P_o$, are equal and the power efficiency, $\eta$, of the converter is always unity.

For the RL load assumed for the generic rectifier, it can be shown that the conversion efficiency, $\eta_c$, is a function of the current ripple factor, $\mathrm{RF}_I$. Specifically,

$$\begin{aligned}\eta_c &= \frac{P_{o,\mathrm{dc}}}{P_i} = \frac{P_{o,\mathrm{dc}}}{P_0/\eta} = \eta\frac{RI_{o,\mathrm{dc}}^2}{RI_o^2} \\ &= \eta\frac{I_{o,\mathrm{dc}}^2}{I_{o,\mathrm{dc}}^2 + I_{o,\mathrm{ac}}^2} = \frac{\eta}{1+(I_{o,\mathrm{ac}}/I_{o,\mathrm{dc}})^2} = \frac{\eta}{1+(RF_1)^2}\end{aligned} \tag{1.38}$$

where $R$ is the load resistance. The ripple factor for the current in Figure 1.4 has already been found to be 0.307. Hence, with $\eta = 1$, the conversion efficiency is $1/(1 + 0.307^2) = 0.914$.

The input current waveform, $i_i(t)$, of the rectifier is shown in Figure 1.11 with the fundamental found numerically, $i_{i,1}(t)$. The converter either passes the input voltage and current directly to the output or inverts them. Therefore, the rms values of input voltage, $V_i$, and current, $I_i$, are equal to those of the output voltage, $V_o$, and current, $I_o$. The apparent input power, $S_i$, is a product of the rms values of input voltage and current. Hence,

$$\mathrm{PF} = \frac{P_i}{S_i} = \frac{P_o/\eta}{V_i I_i} = \frac{RI_o^2}{\eta V_o I_o} = \frac{RI_o}{\eta V_o} \tag{1.39}$$

and specific values of $R$, $I_o$, and $V_o$ are needed to determine the power factor. As mentioned in the preceding section, the peak input voltage, $V_{i,p}$, in the example rectifier described is 100 V and the load resistance, $R$, is 1.3 Ω. Thus, according to Eq. (1.16), the dc output voltage, $V_o$, is 70.7 V. The rms output current, $I_o$, computed numerically is 51.3 A. Consequently, PF = (1.3 × 51.3)/(1 × 70.7) = 0.943 (lag), "lag" indicating that the fundamental input current lags the fundamental input voltage (compare Figures 1.1 and 1.11).

## 1.4 PHASE CONTROL

Based on the idea of a generic converter whose switches connect, cross-connect, or disconnect the input and output terminals, the principles of ac-to-dc and dc-to-ac power conversion were explained in Section 1.2. However, the question of how to control the magnitude of the output voltage and, consequently, that of the output current, has not yet been answered.

The reader is likely to be familiar with electric transformers and autotransformers that allow magnitude regulation of ac voltage and current. They are heavy and bulky, designed for a fixed frequency and impractical for wide-range magnitude control. Moreover, their principle of operation inherently excludes transformation of dc quantities. In the early days of electrical engineering, adjustable resistors were employed predominantly for voltage and current control. Today, the *resistive control* can still be encountered in relay-based starters for electric motors and obsolete adjustable-speed drive systems. Small rheostats and potentiometers are, of course, still used widely in low-power electric and electronic circuits, in which the issue of power efficiency is not of major importance.

Resistive control does not have to involve real resistors. Actually, any of the existing transistor-type power switches could serve this purpose. Between the state of saturation, in which a transistor offers minimum resistance in the collector–emitter path, and the blocking state, resulting in practically zero collector and emitter currents, a wide range of intermediate states are available. Therefore, such a switch can be viewed as a controlled resistor, and one may wonder if, for example, the transistor switches used in power electronic converters could be operated in the same way as are transistors in low-power analog electronic circuits.

To show why the resistive control *should not* be used in high-power applications, two basic schemes, depicted in Figure 1.12, will be considered. For simplicity it is assumed that the circuits shown are to provide control of a dc voltage supplied to a resistive load. The dc input voltage, $V_i$, is constant, while the output voltage, $V_o$, is to be adjustable within the zero-to-$V_i$ range. Generally, for power converters with a controlled output quantity (voltage or current), the *magnitude control ratio*, $M$, can be defined as

$$M \equiv \frac{\Psi_{o,\mathrm{adj}}}{\Psi_{o,\mathrm{adj(max)}}} \tag{1.40}$$

where $\Psi_{o,\mathrm{adj}}$ denotes the value of the adjustable component of the output quantity: for example, the dc component of the output voltage in a controlled rectifier or the fundamental output voltage in an inverter, while $\Psi_{o,\mathrm{adj(max)}}$ is the maximum available

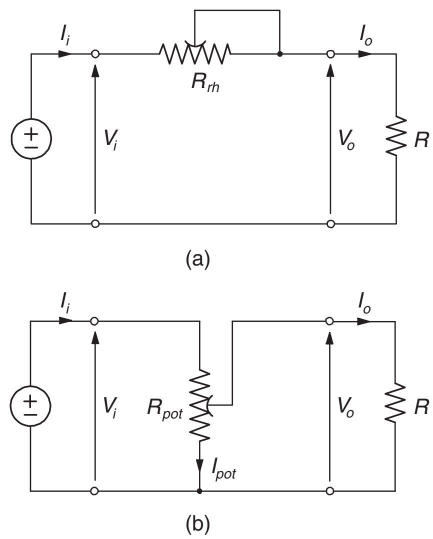


**Figure 1.12** Resistive control schemes: (a) rheostatic control; (b) potentiometric control.

value of this component. Usually, it is required that $M$ be adjustable from a certain minimum value to unity.

The magnitude control ratio should not be confused with the *voltage gain*, $K_V$, which generally represents the ratio of the output voltage to the input voltage. Specifically, the average value is used for dc voltages and the peak value of the fundamental for ac voltages. For example, the voltage gain of a rectifier is defined as the ratio of the dc output voltage, $V_{o,\mathrm{dc}}$, to the peak input voltage, $V_{i,p}$. In the resistive control schemes considered, $K_V = V_o/V_i$, and since the maximum available value of the output voltage equals $V_i$, the voltage gain equals the magnitude control ratio, $M$.

Figure 1.12a illustrates *rheostatic control*. The active part, $R_{\mathrm{rh}}$, of the controlling rheostat forms a voltage divider with the load resistance, $R$. Here,

$$M = \frac{V_o}{V_i} = \frac{R}{R_{\mathrm{rh}} + R} \tag{1.41}$$

and since the input current, $I_i$, equals the output current, $I_o$, the efficiency, $\eta$, of the power transfer from the source to the load is

$$\eta = \frac{RI_o^2}{(R_{\mathrm{rh}} + R)I_i^2} = \frac{RI_i^2}{(R_{\mathrm{rh}} + R)I_i^2} = \frac{R}{R_{\mathrm{rh}} + R} = M. \tag{1.42}$$

The identity relation between $\eta$ and $M$ is a serious drawback of rheostatic control, since decreasing the output voltage causes an equal reduction in efficiency.

*Potentiometric control*, shown in Figure 1.12b and based on the principle of current division, fares even worse. Note that the input current, $I_i$, is greater than the output current, $I_o$, by the amount of the potentiometer current, $I_{pot}$. The power efficiency, $\eta$, is

$$\eta = \frac{V_o I_o}{V_i I_i} = M \frac{I_o}{I_i} \tag{1.43}$$

that is, less than $M$.

It can be seen that the principal trouble with resistive control is that the output current flows through the controlling resistance. As a result, power is lost in that resistance, and the power efficiency is reduced to a value equal to, or less than, the magnitude control ratio. In practical power electronic systems, this is totally unacceptable. Imagine, for example, a 100-kVA converter (not excessively large against today's standards) that at $M = 0.5$ loses as much power in the form of heat as do 25 typical 2-kW domestic heaters! Efficiencies of power electronic converters are seldom lower than 90% in low-power converters and exceed 95% in high-power converters. Apart from economical considerations, large power losses in a converter would require an extensive cooling system. Even in contemporary high-efficiency power conversion schemes, cooling is often quite a problem since the semiconductor power switches are of relatively small size and, consequently, of limited thermal capacity. Therefore, they tend to get overheated quickly if the cooling is inadequate.

The resistive control allows adjustment of the *instantaneous values* of voltage and current, which is important in many applications: for example, those requiring amplification of analog signals, such as those of radio, TV, and tape recorders. There, transistors and operational amplifiers operate on the principle of resistive control, and because of the low levels of power involved, the low efficiency is of minor concern. As illustrated later, in power electronic converters it is sufficient to control the *average value* of dc waveforms and the *rms value* of ac waveforms. This can be accomplished by periodic application of state 0 of the converter (see Section 1.2), in which the connection between the input and output is broken and the output terminals are shorted. In this way the output voltage is made zero within specified intervals of time and, depending on the length of zero intervals, its average or rms value is more or less reduced compared with that of the full waveform.

Clearly, the mode of operation described can be implemented by appropriate use of switches of the converter. Note that there are no power losses in ideal switches, because when a switch is on (closed), there is no voltage drop across it, whereas when it is off (open), there is no current through it. For this reason, both the power conversion and control in power electronic converters are accomplished by means of switching. Analogously to the switches in a generic power converter, the semiconductor devices used in practical converters are allowed to assume two states only. The device is either fully conducting, with a minimum voltage drop between its main electrodes (on-state), or fully blocking, with a minimum current passing between these electrodes (off-state). That is why the term semiconductor power *switches* is used for the devices employed in power electronic converters.

The only major difference between the ideal switches in a hypothetical generic converter and practical semiconductor switches lies in the unidirectionality of the latter devices. In the on-state, a current in the switch can flow in only one direction: for example, from the anode to the cathode in an SCR. Therefore, an idealized semiconductor power switch can be thought of as the connection of an ideal switch and an ideal diode in series.

Historically, for a major part of the past century, only semicontrolled power switches such as mercury-arc rectifiers, gas tubes (thyratrons), and SCRs were available for power conditioning purposes. As mentioned in Section 1.1, once turned on ("fired"), a semicontrolled switch cannot be turned off ("extinguished") as long as the current conducted drops below a certain minimum level for a sufficient amount of time. This condition is required for extinguishing the arc in thyratrons and mercury-arc rectifiers, or for SCRs to recover their blocking capability. If a switch operates in an ac circuit, the turn-off occurs naturally when the current changes polarity from positive to negative. After turn-off, a semicontrolled switch must be refired in every cycle when the anode–cathode voltage becomes positive, that is, when the switch becomes *forward biased*.

Because the forward bias of a switch in an ac-input power electronic converter lasts a half-cycle of the input voltage, firing can be delayed by up to a half-cycle from the instant when the bias changes from reverse to forward. This creates an opportunity for control of the average or rms value of the output voltage of the converter. To demonstrate this control method, the generic power converter will again be employed. For simplicity, the firing delay of the converter switches, previously assumed zero, is now made to equal a quarter of the period of the ac input voltage, that is, 90° in the angle domain, $\omega t$.

Controlled ac-to-dc power conversion is illustrated in Figure 1.13. As explained in Section 1.2, when switches S1 through S4 of a generic converter are open, switch S5 must close to provide a path for the load current, which because of the inductance

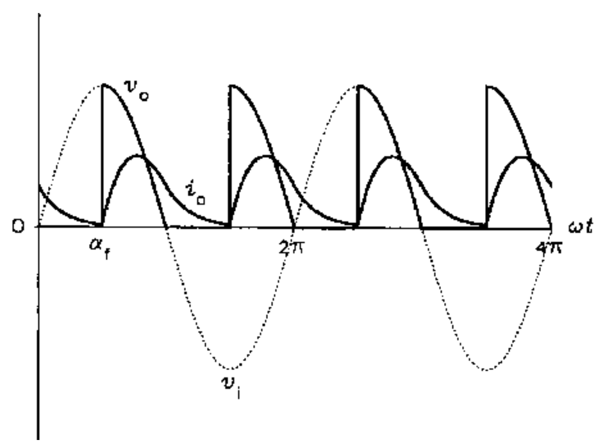


**Figure 1.13** Output voltage and current waveforms in a generic rectifier with a firing angle of 90°.

of the load may not be interrupted. Thus, states 1 and 2 of the converter are separated by state 0. It can be seen that the sinusoidal half-waves of the output voltage in Figure 1.4 have been replaced by quarter-waves. As a result, the dc component of the output voltage has been reduced by 50%. Clearly, a longer delay in closing switches S1–S2 and S3–S4 would reduce this component further, until, with a delay of 180°, it would drop to zero. The generic power converter now operates as a *controlled rectifier*. However, as shown in Chapter 4, practical controlled rectifiers based on SCRs do not need to employ an equivalent of switch S5. As already explained, an SCR cannot be turned off when conducting a current. Therefore, state 1 can be terminated only by switching to state 2, and the other way around, so that one pair of switches takes over the current from another pair. Both these states provide a closed path for the output current. State 0, if any, occurs only when the output current has already died out.

In the angle domain, the firing delay is referred to as a *delay angle*, or *firing angle*, and the method of output voltage control described is called *phase control*, since the firing occurs at a specified phase of the input voltage waveform. In practice, phase control is limited to power electronic converters based on SCRs. Fully controlled semiconductor switches allow more effective control by means of *pulse width modulation*, described in the next section.

The voltage control characteristic, $V_{o,\mathrm{dc}}(\alpha_f)$, of a generic controlled rectifier can be determined as

$$V_{o,\mathrm{dc}}(\alpha_f) = \frac{1}{\pi}\int_0^{\pi} v_0(\omega t)\,d\omega t = \frac{1}{\pi}\int_{\alpha_f}^{\pi} V_{i,p}\sin\omega t\; d\omega t = \frac{V_{i,p}}{\pi}(1+\cos\alpha_f). \tag{1.44}$$

If $\alpha_f = 0$, Eq. (1.44) becomes identical to Eq. (1.15). The control characteristic, which is nonlinear, is shown in Figure 1.14.

Ac-to-ac conversion performed by the generic converter for adjustment of the rms value of an ac output voltage is illustrated in Figure 1.15. Power electronic converters used for this type of power conditioning are called *ac voltage controllers*. Practical ac voltage controllers are based primarily on *triacs*, whose internal structure is equivalent to two SCRs connected antiparallel. Phase-controlled ac voltage controllers do not require a counterpart of switch S5.

The voltage control characteristic, $V_o(\alpha_f)$, of a generic ac voltage controller, given by

$$\begin{aligned} V_o(\alpha_f) &= \sqrt{\frac{1}{\pi}\int_0^{\pi} v_0^2(\omega t)\,d\omega t} \\ &= \sqrt{\frac{1}{\pi}\int_{\alpha_f}^{\pi} (V_{i,p}\sin\omega t)^2\,d\omega t} = V_{i,p}\sqrt{\frac{1}{2\pi}\left(\pi - \alpha_f + \frac{\sin 2\alpha_f}{2}\right)} \end{aligned} \tag{1.45}$$

is shown in Figure 1.16. Again, the characteristic is nonlinear. The fundamental output voltage, $V_{o,1}$, can also be shown to depend nonlinearly on the firing angle.

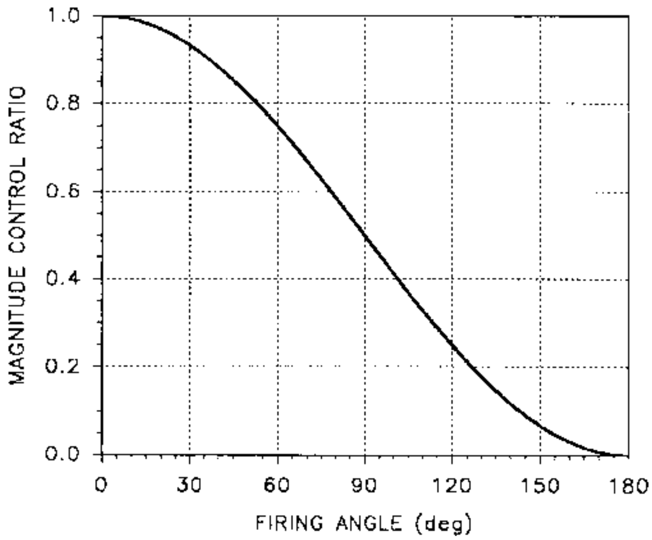


**Figure 1.14** Control characteristic of a generic phase-controlled rectifier.

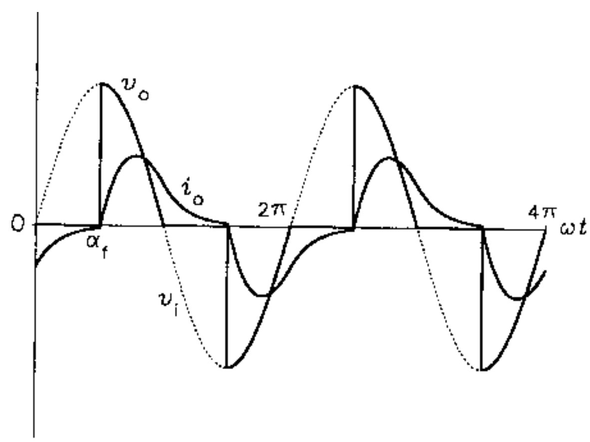


**Figure 1.15** Output voltage and current waveforms in a generic ac voltage controller with a firing angle of 90°.

## 1.5 PULSE WIDTH MODULATION

As explained in the preceding section, control of the output voltage of a power electronic converter by means of an adjustable firing delay has been dictated primarily by the operating properties of semicontrolled power switches. Phase control, although conceptually simple, results in serious distortion of the output current of a converter, which is greater the longer that delay is used. Clearly, the distorted current is a

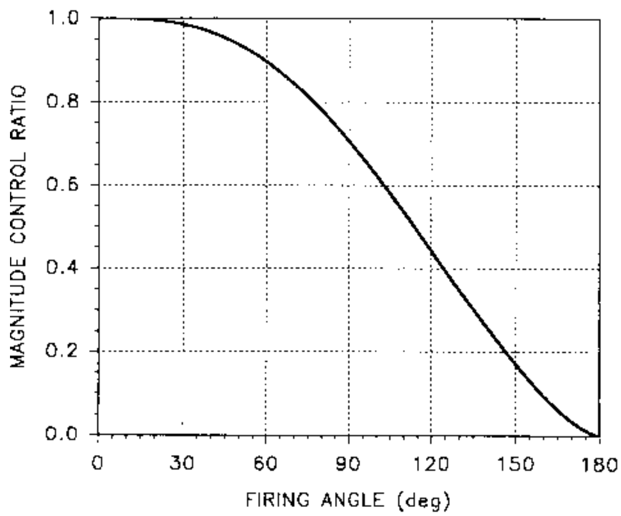


**Figure 1.16** Control characteristic of a phase-controlled generic ac voltage controller.

consequence of the distorted voltage. However, the shape of the output voltage waveforms in power electronic converters is of much less practical concern than that of the output current waveforms. Plainly speaking, *it is the current that does the job*, whether it is the torque production in an electric motor or the electrolytic process in an electrochemical plant.

As mentioned earlier, most practical loads contain inductance. Since such a load constitutes a low-pass filter, high-order harmonics of the output voltage of a converter have a weaker impact on the output current waveform than do low-order harmonics. Therefore, the quality of the current depends strongly on the amplitudes of low-order harmonics in the frequency spectrum of the output voltage. Such spectra for a phase-controlled generic rectifier and an ac voltage controller are illustrated in Figure 1.17. Amplitudes of the harmonics are expressed in the per-unit format, with the peak value, $V_{i,p}$, of input voltage taken as the base voltage. Both spectra display several high-amplitude low-order harmonics.

In ac-input converters, distortion of the *input* current is of equal importance. Distorted currents drawn from the power system cause *harmonic pollution* of the system, resulting in faulty operation of system protection relays and electromagnetic interference (EMI) with communication systems. To alleviate these problems, utility companies require that input filters be installed, which raises the total cost of power conversion. The lower the frequency of harmonics to be attenuated, the larger and more expensive the filters that are needed. The alternative method of voltage and current control, by *pulse width modulation* (PWM), results in better spectral characteristics of converters and smaller filters. Therefore, pulse width modulation schemes are used increasingly in modern power electronic converters.

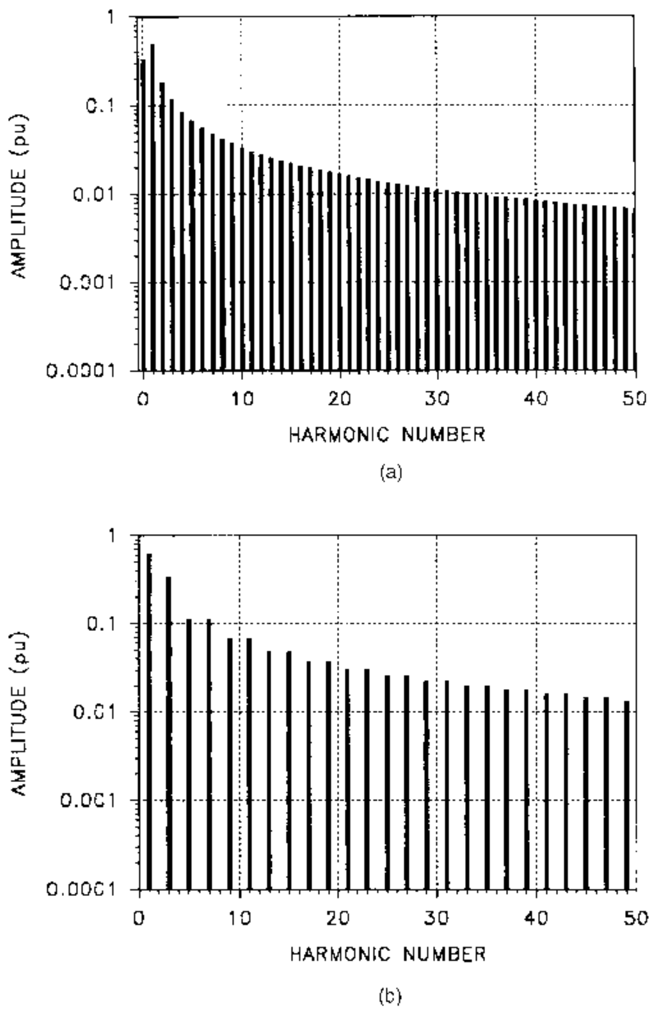


**Figure 1.17** Harmonic spectra of the output voltage with a firing angle of 90° in (a) a phase-controlled generic rectifier, (b) a phase-controlled generic ac voltage controller.

The principle of pulse width modulation can best be explained by considering the dc-to-dc power conversion performed by a generic converter supplied with fixed dc voltage. The converter controls the dc component of the output voltage. As shown in Figure 1.18, this is accomplished by using the converter switches such that the output voltage consists of a train of pulses (state 1 of the generic converter) interspersed with notches (state 0). Fittingly, the respective practical power electronic converters are called *choppers*. Low-power converters used in power supplies for electronic equipment are usually referred to as *dc voltage regulators* or, simply, *dc-to-dc converters*.

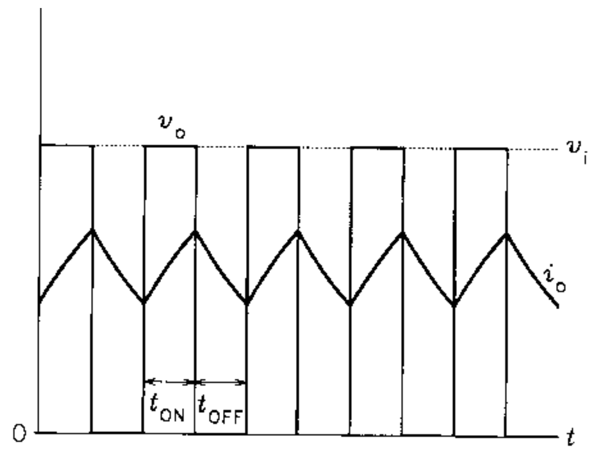


**Figure 1.18** Output voltage and current waveforms in a generic chopper.

In the case illustrated, the pulses and notches are of equal duration; that is, switches S1 and S2 operate with a *duty ratio* of 0.5. A duty ratio, $d$, of a switch is defined as

$$d \equiv \frac{t_{ON}}{t_{ON} + t_{OFF}} \tag{1.46}$$

where $t_{ON}$ denotes the on-time, that is, the interval within which the switch is closed, and $t_{OFF}$ is the off-time, that is, the interval within which the switch is open. Here, switch S5 also operates with a duty ratio of 0.5. However, if the duty ratio of switches S1 and S2 were, for example, 0.6, the duty ratio of switch S5 would have to be 0.4. Switches S3 and S4 of the generic converter are not utilized (unless a reversal of polarity of the output voltage is required), so their duty ratio is zero.

It is easy to see that the average value (dc component), $V_{o,\mathrm{dc}}$, of the output voltage is proportional to the fixed value, $V_i$, of the input voltage and to the duty ratio, $d_{12}$, of switches S1 and S2; that is,

$$V_{o,\mathrm{dc}} = d_{12} V_i. \tag{1.47}$$

Since the range of a duty ratio is zero (switch open all the time) to unity (switch closed all the time), adjusting the duty ratio of appropriate switches allows setting $V_{o,\mathrm{dc}}$ at any level between zero and $V_i$. As follows from Eq. (1.47), the voltage control characteristic, $V_{o,\mathrm{dc}} = f(d_{12})$, of the generic chopper is linear.

Although the *switching frequency*, $f_{\mathrm{sw}}$, defined as

$$f_{SW} \equiv \frac{1}{t_{ON} + t_{OFF}} \tag{1.48}$$

does not affect the dc component of the output voltage, the quality of the output current depends strongly on $f_{\mathrm{sw}}$. As illustrated in Figure 1.19, if the number of pulses

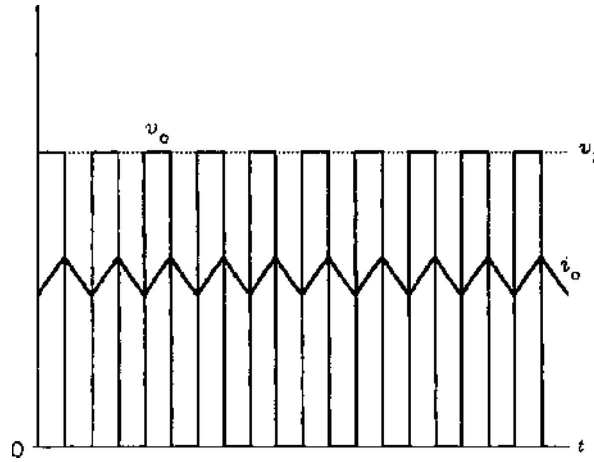


**Figure 1.19** Output voltage and current waveforms in a generic chopper with a switching frequency twice as high as that in Figure 1.18.

per second is doubled compared with that in Figure 1.18, the increased switching frequency results in reduction of the current ripple by about 50%. The time between consecutive state changes of the converter is simply short enough to prevent significant current changes between consecutive “jumps” of the output voltage.

The reduction of current ripple can also be explained by the harmonic analysis of the output voltage. Note that the fundamental output frequency equals the switching frequency. As a result, the harmonics of the ac component of the voltage appear around frequencies that are integer multiples of $f_{sw}$. The corresponding inductive reactances of the load are proportional to these frequencies. Therefore, if the switching frequency is sufficiently high, the ac component of the output current is so strongly attenuated that the current is practically of ideal dc quality.

Voltage control using PWM can be employed in all the other types of electric power conversion described in this chapter. Instead of taking out solid chunks of the output voltage waveforms by the firing delay as shown in Figures 1.13 and 1.15, a large number of narrow segments can be removed in the PWM operating mode illustrated in Figure 1.20. Here, the ratio, $N$, of the switching and input frequencies is 12. This is also the number of pulses of output voltage per cycle. The output current waveforms are of significantly higher quality than those in Figures 1.13 and 1.15, exemplifying the operational superiority of pulse width–modulated converters over phase-controlled converters.

It should be mentioned that, for clarity, in most examples of PWM converters throughout the book, the switching frequencies employed in preparing the figures are lower than practical frequencies. Typically, depending on the type of power switches, switching frequencies are on the order of a few kilohertz, seldom less than 1 kHz and, in *supersonic converters*, higher than 20 kHz. Therefore, if, for example, a switching frequency in a 60-Hz PWM ac voltage controller is 3.6 kHz, the output voltage is “sliced” into 60 segments per cycle instead of the 12 segments shown in Figure 1.20b.

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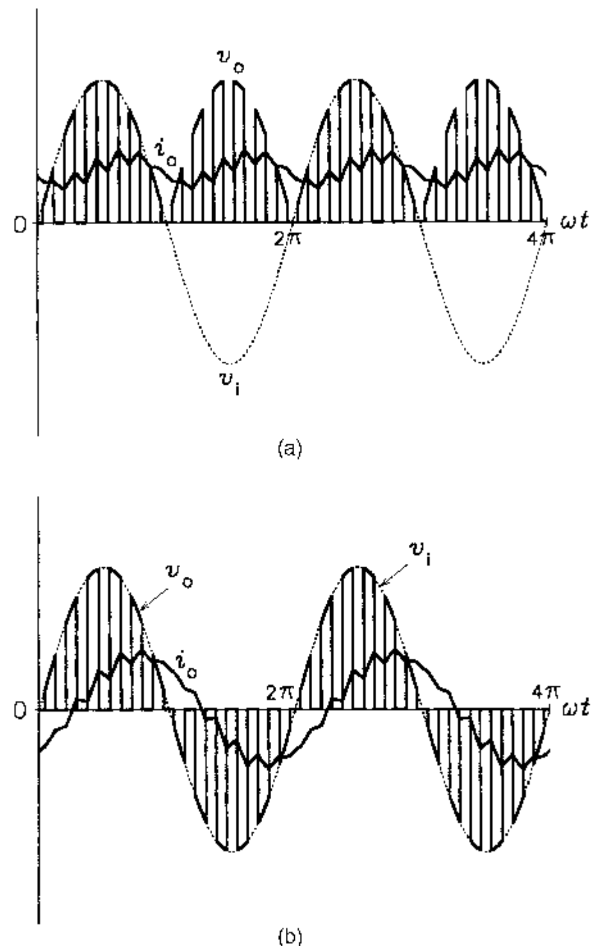


**Figure 1.20** Output voltage and current waveforms in (a) a generic PWM rectifier; (b) a generic PWM ac voltage controller ($N = 12$).

Generally, a switching frequency should be several times higher than the reciprocal of the dominant (the longest) time constant of the load.

It can be shown that the linear relation (1.46) pertaining to the chopper can be extended to other types of PWM converters operating with fixed duty ratios of switches, such as rectifiers and ac voltage controllers. In all these converters,

$$V_{o,\mathrm{adj}} = dV_{o,\mathrm{adj(max)}} \tag{1.49}$$

that is, the magnitude control ratio, $M$, equals the duty ratio, $d$, of switches connecting the input and output terminals. Depending on the type of converter, the symbol $V_{o,\mathrm{adj}}$ represents the adjustable dc component or fundamental ac component of the output voltage, while $V_{o,\mathrm{adj(max)}}$ is the maximum available value of this component. However, concerning the rms value, $V_o$, of the output voltage, the dependence on the duty ratio is radical, that is,

$$V_o = \sqrt{d}\, V_{o,\max} \tag{1.50}$$

where $V_{o(\max)}$ is the maximum available rms value of output voltage. Specific values of $V_{o,\mathrm{adj(max)}}$ and $V_{o(\max)}$ depend on the type of converter and the magnitude of the input voltage. The control characteristics of a generic PWM rectifier [Eq. (1.47)] and an ac voltage controller [Eq. (1.50)] are shown in Figure 1.21 for comparison with those of phase-controlled converters depicted in Figures 1.14 and 1.16.

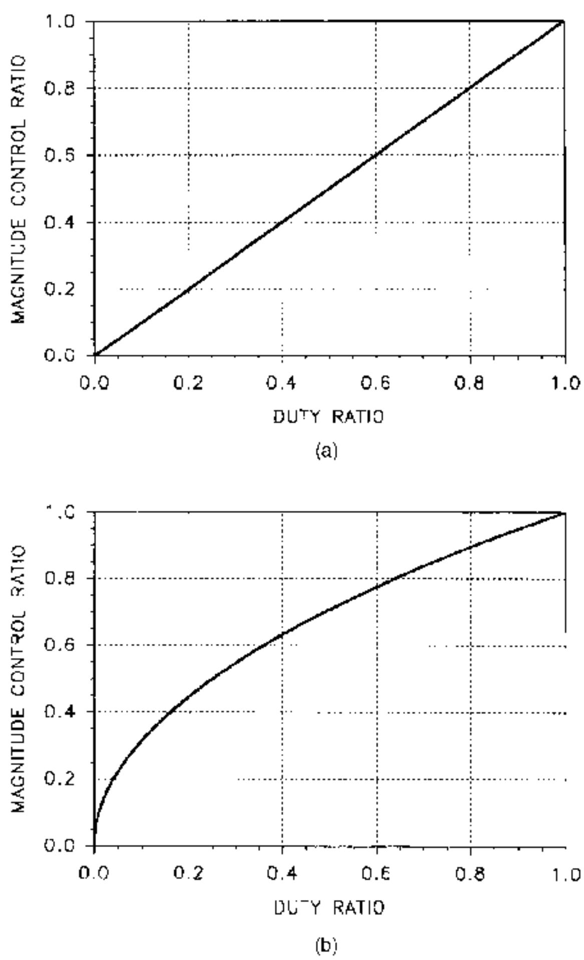


**Figure 1.21** Control characteristics of (a) a generic PWM rectifier; (b) a generic PWM ac voltage controller.

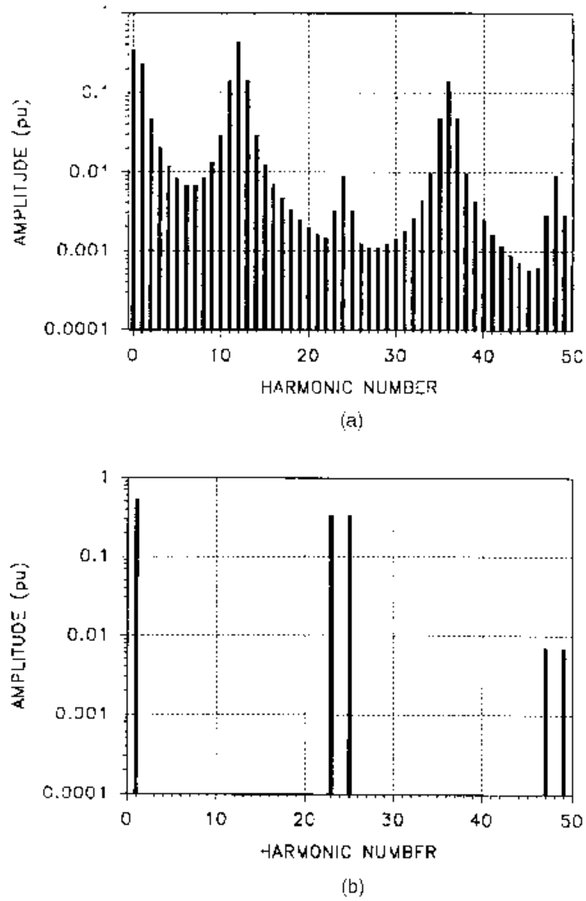


**Figure 1.22** Harmonic spectra of output voltage in (a) a generic PWM rectifier; (b) a generic PWM ac voltage controller ($N = 24$).

Harmonic spectra of the output voltage of a generic PWM rectifier and an ac voltage controller are shown in Figure 1.22, again in the per-unit format. Here the switching frequency is 24 times higher than the input frequency; that is, the output voltage has 24 pulses per cycle. The duty ratio of converter switches is 0.5. Comparing these spectra with those in Figure 1.17, essential differences can be observed. The amplitudes of the low-order harmonics have been suppressed, especially in the spectrum for the ac voltage controller. Higher harmonics appear in clusters centered about integer multiples of 12 for the rectifier and 24 for the ac voltage controller. The reduced amplitudes of low-order harmonics of the output voltage translate into enhanced quality of the output current waveforms.

It should be pointed out that the simple pulse width modulation described, with a constant duty ratio of switches, has only been used to illustrate the basic principles of PWM converters. In practice, constant duty ratios are typical for choppers and common for ac voltage controllers. PWM rectifiers and inverters employ more sophisticated PWM techniques, in which the duty ratios of switches change throughout the cycle of the output voltage. Such techniques can also be used in PWM ac voltage controllers.

PWM techniques characterized by variable duty ratios of converter switches are explained in detail in Chapters 4 and 7. Waveforms of the output voltage and current of a generic inverter with $N = 10$ voltage pulses per cycle and with variable duty ratios are shown in Figure 1.23. The magnitude control ratio, $M$, pertaining to the amplitude of the fundamental output voltage, is 1 in Figure 1.23a and 0.5 in Figure 1.23b. The varying widths of the voltage pulses and the proportionality of these widths to the magnitude control ratio can easily be observed. The output current waveforms, although rippled, are much closer to ideal sine waves than those in the square-wave

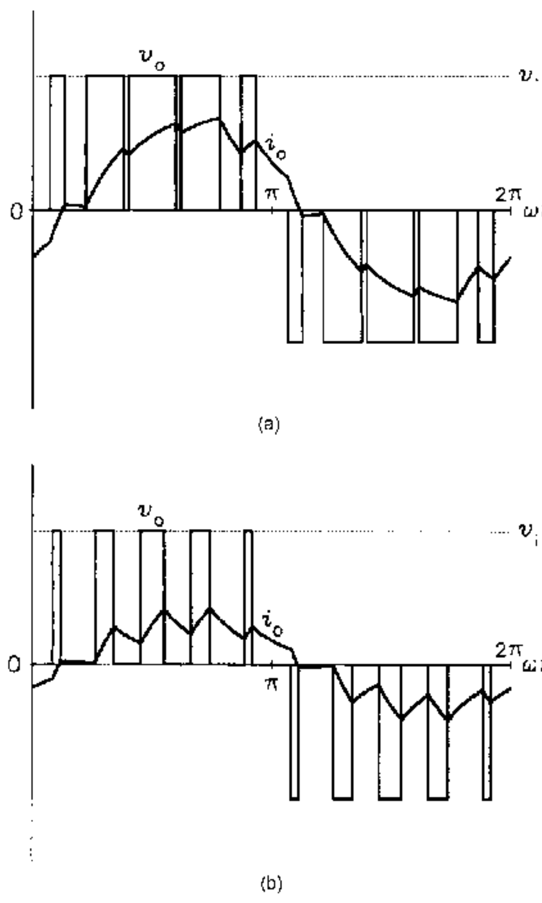


**Figure 1.23** Output voltage and current waveforms in a generic PWM inverter: (a) $M = 1$; (b) $M = 0.5$ ($N = 10$).

operating mode of the inverter (see Figure 1.5). The harmonic content of the current would decrease further with an increase in the switching frequency.

All the examples of PWM converters presented in this section show that the higher the switching frequency, the better the quality of the output current. However, the allowable switching frequency in practical power electronic converters is limited by several factors. First, any power switch requires a certain amount of time for transitions from the on-state to the off-state, and vice versa. Therefore, the operating frequency of a switch is restricted, the maximum value depending on the type and ratings of the switch. Second, the control system of a converter has a limited operating speed. Finally, as explained in Chapter 2, the *switching losses* in practical switches increase with the switching frequency, reducing the efficiency of power conversion. Therefore, the switching frequency employed in a PWM converter should represent a sensible trade-off between the quality and efficiency of operation of the converter.

## 1.6 CALCULATION OF CURRENT WAVEFORMS

Practical sources of raw electric power are predominantly voltage sources. Such a source can be converted into a current source only by providing closed-loop control, maintaining the load current at a steady level. This is a rather complicated arrangement, so most power electronic converters are supplied from voltage sources. Since, as already demonstrated, a static power converter is simply a network of switches, waveforms of voltages are easy to determine from the converter's principle of operation. It is not so with currents, which depend on both the voltages and the load. Typically, however, once one current, usually the output current, is known, the operating algorithm of the converter can again be employed to find the other currents.

As exemplified by the generic converter, power electronic converters operate most of the time in *quasi-steady state*, which is a sequence of transient states. The converters are variable-topology circuits, and each change of topology initiates a new transient state. Final values of currents and voltages in one topology become initial values in the next topology. Since linear electric circuits can be described by ordinary linear differential equations, the task of finding a current waveform becomes a classic initial value problem. As shown later, in place of differential equations, difference equations can be applied conveniently to converters operating in the PWM mode.

If the load of a converter is linear and the output voltage can be expressed in closed form, the output current waveform corresponding to individual states of the converter can also be expressed in closed form. However, when a computer is used for converter analysis, numerical algorithms can be employed to compute consecutive points of the current waveform. Both the analytical and numerical approaches are illustrated in subsequent sections.

### 1.6.1 Analytical Solution

To show the typical way of finding closed-form expressions for the output current in a power electronic converter, the generic rectifier, generic inverter, and generic PWM

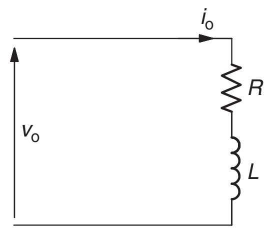


**Figure 1.24** Resistive–inductive (RL) load circuit.

ac voltage controller are considered. In all three cases, a resistive–inductive (RL) load is assumed.

The RL load circuit of a generic converter is shown in Figure 1.24. If the converter operates as a rectifier, the input and output terminals are cross-connected when the ac input voltage, given by Eq. (1.1), is negative. Consequently, the output voltage, $v_o(t)$, and current, $i_o(t)$, are periodic with a period of $T/2$, where $T$ is the period of the input voltage, equal to $2\pi/\omega$. Therefore, all subsequent consideration of the generic rectifier is limited to the interval 0 to $T/2$.

Kirchhoff's voltage law for the circuit in Figure 1.24 can be written as

$$Ri_o(t) + L\frac{di_o(t)}{dt} = V_{i,p}\sin\omega t. \tag{1.51}$$

Equation (1.51) can be solved for $i_o(t)$ using the Laplace transformation or, less tediously, taking advantage of the known property of linear differential equations, allowing $i_o(t)$ to be expressed as

$$i_o(t) = i_{o,F}(t) + i_{o,N}(t) \tag{1.52}$$

where $i_{o,F}(t)$ and $i_{o,N}(t)$ denote the *forced* and *natural components* of $i_o(t)$, respectively. The convenience of this approach lies in the fact that the forced component constitutes the steady-state solution of Eq. (1.51), that is, the steady-state current excited in the circuit in Figure 1.25 by the sinusoidal voltage $v_i(t)$. As is well known to every electrical engineer,

$$i_{o,F}(t) = \frac{V_{i,p}}{Z}\sin(\omega t - \varphi) \tag{1.53}$$

where

$$Z = \sqrt{R^2 + (\omega L)^2} \tag{1.54}$$

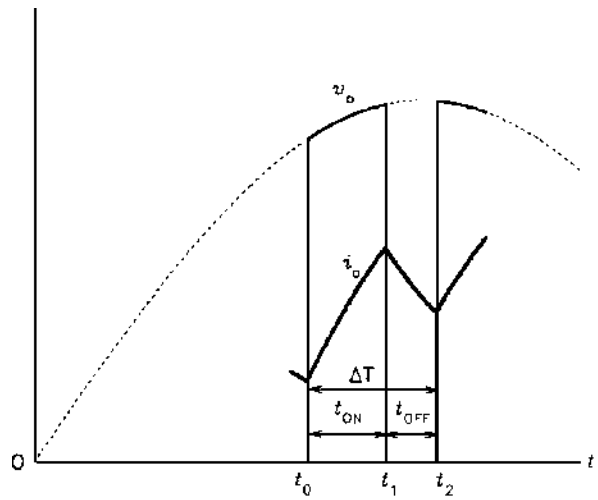


**Figure 1.25** Fragments of output voltage and current waveforms in a generic PWM ac voltage controller.

and

$$\varphi = \tan^{-1} \frac{\omega L}{R} \tag{1.55}$$

are the impedance and phase angle of the RL load, respectively. The shortcut from Eq. (1.51) to the forced solution (1.53) saves a lot of work.

The natural component, $i_{o,N}(t)$, represents a solution of the homogeneous equation

$$Ri_{o,N}(t) + L\frac{di_{o,N}(t)}{dt} = 0 \tag{1.56}$$

obtained from Eq. (1.51) by equating the right-hand side (the excitation) to zero. It can be seen that $i_{o,N}(t)$ must be such that the linear combination of it and its derivative is zero. Clearly, an exponential function

$$i_{o,N}(t) = Ae^{BT} \tag{1.57}$$

is the most likely candidate for the solution. Substituting $i_{o,N}(t)$ in Eq. (1.56) yields

$$RAe^{BT} + LABe^{BT} = 0 \tag{1.58}$$

from which $B = -R/L$ and

$$i_{o,N}(t) = Ae^{-(R/L)t} \tag{1.59}$$

Thus, based on Eqs. (1.52) and (1.53),

$$t_o(t) = \frac{V_{i,p}}{Z}\sin(\omega t - \varphi) + Ae^{-(R/L)t}. \tag{1.60}$$

To determine the constant $A$, note that in the quasi-steady state of the rectifier, $i_o(0) = i_o(T/2) = i_o(\pi/\omega)$ (see Figure 1.4). Hence,

$$\frac{V_{i,p}}{Z}\sin(-\varphi) + A = \frac{V_{i,p}}{Z}\sin(\pi - \varphi) + Ae^{-R\pi/L\omega} \tag{1.61}$$

where

$$-\frac{R\pi}{L\omega} = -\frac{\pi}{\tan\varphi}. \tag{1.62}$$

Equation (1.61) can now be solved for $A$, yielding

$$A = \frac{2V_{i,p}\sin\varphi}{Z(1 - e^{-\pi/\tan\varphi})} \tag{1.63}$$

and

$$i_o(t) = \frac{V_{i,p}}{Z}\left[\sin(\omega t - \varphi) + \frac{2\sin\varphi}{1 - e^{-\pi/\tan\varphi}}e^{-(R/L)t}\right]. \tag{1.64}$$

The approach that we have shown to derivation of the rather complex relation (1.64) takes much less time and effort than that required when the Laplace transformation is employed. The reader is encouraged to confirm this observation by his or her own computations.

When the generic power converter works as an inverter, the output voltage equals $V_i$ when the converter is in state 1 and $-V_i$ in state 2. If $T$ now denotes the period of the output voltage, it can be assumed that state 1 lasts from 0 to $T$/2 and state 2 from $T$/2 to $T$. Because it is only the polarity of output voltage that changes every half-cycle, the output current, $i_{o(2)}(t)$, in the second half-cycle equals $-i_{o(1)}(t-T/2)$, where $i_{o(1)}(t)$ is the output current in the first half-cycle. Consequently, it is sufficient to determine $i_{o(1)}(t)$ only.

The forced component, $i_{o,F(1)}(t)$, of current $i_{o(1)}(t)$, excited in the *RL* load by the dc voltage $v_o = V_i$, is given by Ohm's law as

$$i_{o,F(1)} = \frac{V_i}{R}. \tag{1.65}$$

The natural component, which depends solely on the load, is the same as that of the output current of the generic rectifier. Thus,

$$i_{o(1)}(t) = \frac{V_i}{R} + Ae^{-R/L}. \tag{1.66}$$

Constant $A$ can be found from the condition $i_{o(1)}(0) = -i_{o(1)}(T/2)$ (see Figure 1.5), that is,

$$\frac{V_i}{R} + A = -\left(\frac{V_i}{R} + Ae^{-\pi/\tan\varphi}\right) \tag{1.67}$$

which yields

$$A = \frac{2}{1 + e^{-\pi/\tan\varphi}}\frac{V_i}{R}. \tag{1.68}$$

This gives

$$i_o(t) = \begin{cases} \dfrac{V_i}{R}\left[1 - \dfrac{2}{1 + e^{-\pi/\tan\varphi}}e^{-(R/L)t}\right] & \text{for } 0 < t \leq \dfrac{T}{2} \\ -\dfrac{V_i}{R}\left[1 - \dfrac{2}{1 + e^{-\pi/\tan\varphi}}e^{-(R/L)(t-T/2)}\right] & \text{for } \dfrac{T}{2} < t \leq T \end{cases}. \tag{1.69}$$

In the final example, a PWM converter, specifically the PWM ac voltage controller, is dealt with. Within a single cycle of output voltage, the converter undergoes a large number of state changes, and within each corresponding interval of time, the waveform of output current is described by a different equation. Therefore, to express the waveform in a useful format, iterative formulas are employed. The general form of such a formula is $i_o(t + \Delta t) = f[i_o(t),t]$, where $\Delta t$ is a small increment of time, meaning that consecutive values of the output current are calculated from the previous values.

In developing the formulas, two simplifying assumptions were made; first, that the current waveform is piecewise linear; second, that the initial value, $i_o(0)$, of the output current is equal to that of a hypothetical sinusoidal current that would be generated in the same load by the fundamental of the actual output voltage of the converter. Thus, if the input voltage of the generic ac voltage controller is given by Eq. (1.1), the initial value of the output current is

$$i_o(0) = M\frac{V_{i,p}}{Z}\sin(\omega t - \varphi)\bigg|_{t=0} = -M\frac{V_{i,p}}{Z}\sin\varphi. \tag{1.70}$$

The output voltage waveform of the controller, shown in Figure 1.20b, is a chopped sinusoid. A single switching cycle of that waveform is depicted in Figure 1.25, which also shows the coinciding fragment of the output current waveform. The time interval,

$t_0$ to $t_2$, corresponding to the switching cycle is called a *switching interval*. From $t_0$ to $t_1$, the controller is in state 1 and the output voltage equals the input voltage, while from $t_1$ to $t_2$, the output voltage is zero, the controller assuming state 0. Denoting the length of the switching interval, $t_2 - t_0$, by $\Delta T$, the on-time, $t_{\mathrm{ON}}$, is $M\ \Delta T$, and the off-time, $t_{\mathrm{OFF}}$, is $(1 - M)\ \Delta T$.

The differential equation of the load circuit is

$$Ri_o(t) + L\frac{di_o(t)}{dt} = v_o(t) \tag{1.71}$$

which for the instant $t = t_0$ can be rewritten as

$$Ri_o(t_o) + L\frac{\Delta i_o}{\Delta t} = V_{i,p} \sin \omega t_0 \tag{1.72}$$

where $\Delta i_o = i_o(t_1) - i_o(t_0)$ and $\Delta t = t_1 - t_0$, that is, as a difference equation. Taking into account that $\Delta t = M\ \Delta T$, Eq. (1.72) can be solved for $i_o(t_0)$ to yield

$$t_o(t_1) = i_o(t_0) + \frac{M}{L}[V_{i,p} \sin \omega t_0 - Ri_o(t_0)]\ \Delta T. \tag{1.73}$$

Similarly, at $t = t_1$, the load circuit equation

$$Ri_o(t_1) + L\frac{\Delta i_o}{\Delta t} = 0 \tag{1.74}$$

where $\Delta i_o = i_o(t_2) - i_o(t_1)$ and $\Delta t = t_2 - t_1$, can be rearranged to

$$i_o(t_2) = i_o(t_1)\left[1 - \frac{R}{L}(1 - M)\ \Delta T\right]. \tag{1.75}$$

Formulas (1.70), (1.73), and (1.75) allow computation of consecutive segments of the piecewise linear waveform of the output current. If, as is typical in practical PWM converters, the switching frequency, $f_{\mathrm{sw}} = 1/\Delta T$, is at least one order of magnitude higher than the input or output frequency, the accuracy of those approximate relations is sufficient for all practical purposes.

### 1.6.2 Numerical Solution

When a computer program is used to simulate a converter, the output voltage is calculated as a series of values, $v_{o,0}$, $v_{o,1}$, $v_{o,2}, \ldots$, for the consecutive instants, $t_0$, $t_1$, $t_2, \ldots$. These instants should be close, so that $t_{n+1} - t_n < \tau$, where $\tau$ denotes the shortest time constant of the system simulated. Then the respective values of the output current, $i_{o,1}$, $i_{o,2}$, $i_{o,3}, \ldots$, can be computed as responses of the load circuit to step excitations $v_{o,0}u(t - t_0)$, $v_{o,1}u(t - t_1)$, $v_{o,2}u(t - t_2), \ldots$, where $u(t - t_n)$ denotes a unit step function at $t = t_n$.

For example, for the RL load considered in the preceding section, the differential equation of the load circuit for $t \geq t_n$ can be written as

$$Ri_o(t) + L\frac{di_o(t)}{dt} = v_{o,n}u(t - t_n) \tag{1.76}$$

where $u(t - t_n) = 1$. The forced component, $i_{o,F}(t)$, of the solution, $i_o(t)$, of Eq. (1.76), is given by

$$i_{o,F}(t) = \frac{v_{o,n}}{R} \tag{1.77}$$

and the natural component, $i_{o,N}(t)$, by

$$i_{o,N}(t) = Ae^{-(R/L)t}. \tag{1.78}$$

Hence,

$$i_o(t) = \frac{v_{o,n}}{R} + Ae^{-(R/L)t} \tag{1.79}$$

where the constant $A$ can be determined from

$$i_o(t_n) = i_{o,n} = \frac{v_{o,n}}{R} + Ae^{-(R/L)t_n} \tag{1.80}$$

as

$$A = \left(i_{o,n} - \frac{v_{o,n}}{R}\right)e^{-(R/L)t_n}. \tag{1.81}$$

Substituting $t = t_{n+1}$ and Eq. (1.81) in Eq. (1.79) and denoting $i_o(t_n + 1)$ by $i_{o,n+1}$ yields

$$i_{o,n+1} = \frac{v_{o,n}}{R} + \left(i_{o,n} - \frac{v_{o,n}}{R}\right)e^{-(R/L)(t_{n+1}-t_n)} \tag{1.82}$$

which is an iterative formula, $i_{o,n+1} = f(i_{o,n}, t_{n+1} - t_n)$, that allows easy computation of consecutive points of the output current waveform.

The other common loads and corresponding numerical formulas for the output current are:

*Resistive load* (*R load*):

$$i_{o,n} = \frac{v_{o,n}}{R}. \tag{1.83}$$

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*Inductive load* (*L load*):

$$i_{o,n+1} = i_{o,n} + \frac{v_{o,n}}{L}(t_{n+1} - t_n). \tag{1.84}$$

*Resistive–EMF load* (*RE load*):

$$i_{o,n} = \frac{v_{o,n} - E_o}{R} \tag{1.85}$$

where $E_n$ denotes the value of load EMF at $t = t_n$. The EMF, in series with resistance $R$, is assumed to oppose the output current.

*Inductive–EMF load* (*LE load*):

$$i_{o,n+1} = i_{o,n} + \frac{v_{o,n} - E_n}{L}(t_{n+1} - t_n) \tag{1.86}$$

where the load EMF is connected in series with inductance $L$.

*Resistive–inductive–EMF load* (*RLE load*):

$$i_{o,n+1} = \frac{v_{o,n} - E_n}{R} + \left(i_{o,n} - \frac{v_{o,n} - E_n}{R}\right) e^{-(R/L)(t_{n+1}-t_n)} \tag{1.87}$$

where the load is represented by a series connection of resistance $R$, inductance $L$, and EMF $E$.

The considerations presented can be adapted for analysis of the less common current-source converters. There, those are currents that are easily determinable from the operating principles of a converter, whereas voltage waveforms require analytical or numerical solution of appropriate differential equations.

In engineering practice, specialized computer programs are used for modeling and analysis of power electronic converters. The popular software package PSpice for simulation of electronic circuits is called for in most of the computer assignments in this book (see Appendix A). Other commercial versions of UC Berkeley's Spice, such as the Electronic Bench, are also available on the software market. Many advanced simulation programs, of which Saber is the best known example, have been developed in several countries specifically for modeling of switching power converters. Program EMTP, used primarily by utilities for power system analysis, allows simulation of power electronic converters, which is particularly useful for studies of the impact of converters on power system operation. General-purpose dynamic simulators, such as Simulink, Simplorer, or ACSL, can be used successfully for analysis not only of converters themselves but also of entire systems that include converters, such as electric motor drives.

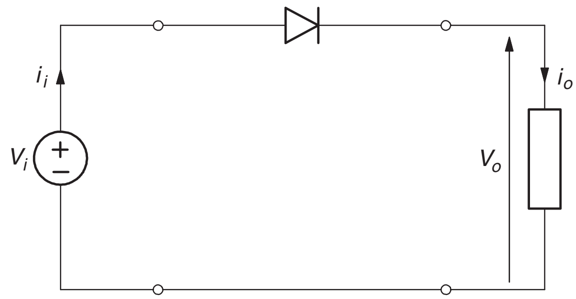


**Figure 1.26** Single-pulse diode rectifier.

### 1.6.3 Practical Examples: Single-Phase Diode Rectifiers

To illustrate the considerations presented in the preceding two sections and to start introducing practical power electronic converters, single-phase diode rectifiers are described. They are the simplest static converters of electrical power. The power diodes employed in the rectifiers can be thought of as uncontrolled semiconductor power switches. A diode in a closed circuit begins conducting (turns on) when its anode voltage becomes higher than the cathode voltage, that is, when the diode is *forward biased*. The diode ceases to conduct (turns off) when its current changes polarity.

A *single-pulse* (single-phase half-wave) diode rectifier is shown in Figure 1.26. Functionally, it is analogous to a generic power converter with only switches S1 and S2 and the provision that they stay closed as long as the current flows from terminal I1 to terminal O1. If the load is purely resistive (R load), and the sinusoidal input voltage is given by Eq. (1.1), the output current waveform resembles that of the output voltage, as shown in Figure 1.27. Comparing Figures 1.27 and 1.4, it can easily be seen that the average output voltage, $V_{o,\mathrm{dc}}$, in a single-pulse rectifier is only half as large as that in the generic rectifier described in Section 1.2, that is, $V_{o,\mathrm{dc}} = V_{i,p}/\pi \approx 0.32V_{i,p}$ [see Eq. (1.15)]. The single "pulse" of output current per cycle of the input voltage explains the name of the rectifier.

The voltage gain is even lower when the load contains an inductance (RL load). Equation (1.59) can be used to describe the output current, but only until it reaches zero at $\omega t = \alpha_e$, where $\alpha_\mathrm{e}$ is called an *extinction angle*. Thus,

$$i_o(0) = \frac{V_{i,p}}{Z}\sin(-\varphi) + A = 0 \tag{1.88}$$

from which

$$A = \frac{V_{i,p}}{Z}\sin\varphi \tag{1.89}$$

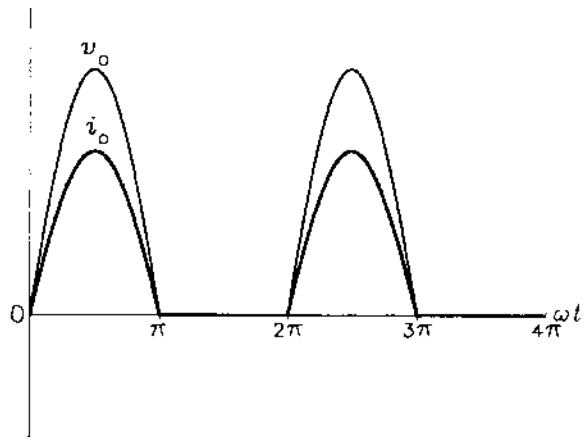


**Figure 1.27** Output voltage and current waveforms in a single-pulse diode rectifier with an R load.

and

$$i_o(t) = \frac{V_{i,p}}{Z}\left[\sin(\omega t - \varphi) + e^{-(R/L)t}\sin\varphi\right] \tag{1.90}$$

when $i_o > 0$, that is, when $0 < \omega t \leq \alpha_e$. Substituting $\alpha_e/\omega$ for $t$ and zero for $i_o$, the extinction angle can be found from

$$\frac{V_{i,p}}{Z}\left[\sin(\alpha_e - \varphi) + e^{-\alpha_e/\tan\varphi}\sin\varphi\right] = 0. \tag{1.91}$$

Clearly, no closed-form expression exists for $\alpha_e$, and the extinction angle, which is a function of the load angle $\varphi$, can only by found numerically. This problem is analyzed in greater detail in Chapter 4.

The output voltage and current waveforms of a single-pulse rectifier with an RL load are shown in Figure 1.28. As the extinction angle is greater than 180°, the output voltage is negative in the interval $\omega t = \pi$ to $\omega t = \alpha_e$, and its average value is lower than that with a resistive load. This can be remedied by connecting the *freewheeling diode*, DF, across the load, as shown in Figure 1.29.

The freewheeling diode, which corresponds to switch S5 in the generic power converter, shorts the output terminals when the output voltage reaches zero and provides a path for the output current in the interval $\pi$ to $\alpha_e$. Until the voltage reaches zero, the waveform of output voltage is the same as that in Figure 1.28, and that of output current is described by Eq. (1.90). Later, the current dies out while freewheeled

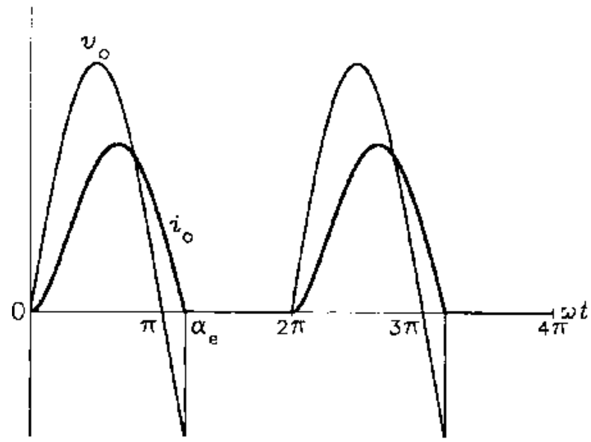


**Figure 1.28** Output voltage and current waveforms in a single-pulse diode rectifier with an RL load.

by diode DF. As $v_o = 0$ now, the equation of the current is simply

$$i_o(t) = Ae^{-(R/L)t} \tag{1.92}$$

where constant $A$ can be found by equaling currents given by Eqs. (1.90) and (1.92) at $\omega t = \pi$, that is,

$$\frac{V_{i,p}}{Z}(1 - e^{-\pi/\tan\varphi})\sin\varphi = Ae^{-\pi/\tan\varphi}. \tag{1.93}$$

From Eq. (1.93),

$$A = \frac{V_{i,p}}{Z}(e^{\pi/\tan\varphi} - 1)\sin\varphi \tag{1.94}$$

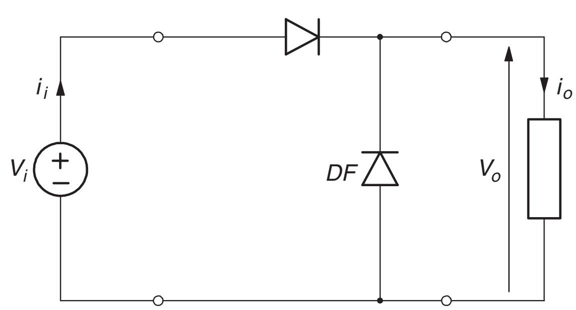


**Figure 1.29** Single-pulse diode rectifier with a freewheeling diode.

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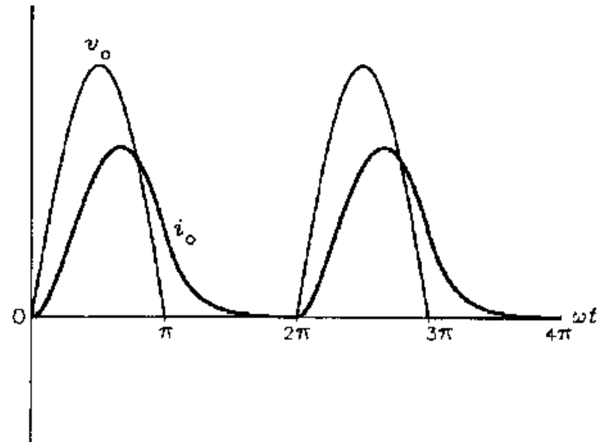


**Figure 1.30** Output voltage and current waveforms in a single-pulse diode rectifier with a freewheeling diode and an RL load.

and

$$i_o(t) = \frac{V_{i,p}}{Z}(e^{\pi/\tan\varphi} - 1)e^{-(R/L)t}\sin\varphi. \tag{1.95}$$

Waveforms of the output voltage and current of a single-pulse rectifier with the freewheeling diode are shown in Figure 1.30.

More radical enhancement of the single-pulse rectifier shown in Figure 1.31 consists of connecting a large capacitor across the output terminals. The capacitor charges up when the input voltage is high and discharges through the load when the input voltage drops below a specific level that depends on the capacitor and load. Typical waveforms of the output voltage, $v_o$, and capacitor current, $i_C$, with a resistive load are shown in Figure 1.32. Advanced readers are encouraged to try and obtain analytical expressions for these waveforms using the approach sketched in this chapter.

Practical single-pulse rectifiers do not belong in the realm of true power electronics, as the output capacitors would have to be excessively large. The average output

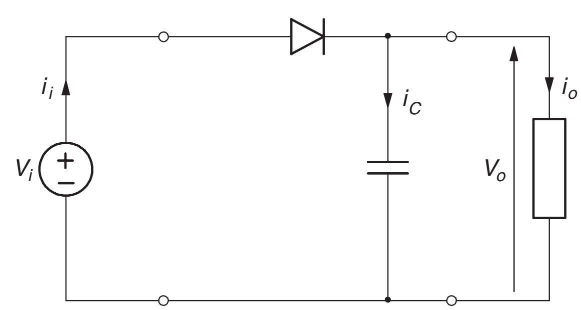


**Figure 1.31** Single-pulse diode rectifier with an output capacitor.

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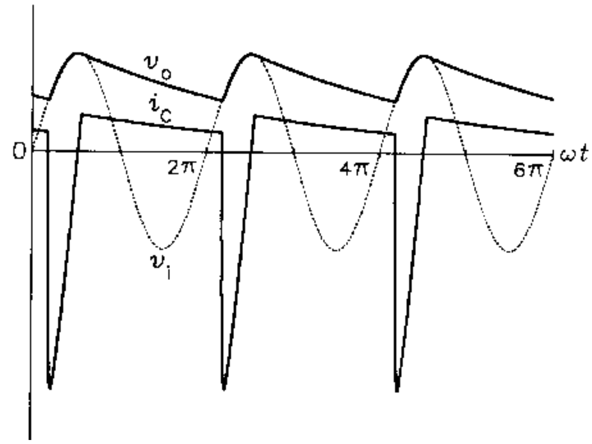


**Figure 1.32** Output voltage and current waveforms in a single-pulse diode rectifier with an output capacitor and an RL load.

voltage, $V_{o,\text{dc}}$, can be increased to $2v_{i,p}/\pi \approx 0.64v_{i,p}$ in the two-pulse (single-phase full-wave) diode rectifier in Figure 1.33. Diodes D1 through D4 correspond to the respective switches in the generic converter, connecting directly and cross-connecting the input terminals with the output terminals in dependence on the polarity of the input voltage. As a result, the output voltage and current waveforms are the same as in the generic converter (see Figure 1.4). In low-power rectifiers, a capacitor can be placed across the load, similar to the single-pulse rectifier in Figure 1.31, to smooth out the output voltage and increase its dc component. Generally, however, single-phase rectifiers have operating parameters distinctly inferior to those of three-phase rectifiers, presented in Chapter 4.

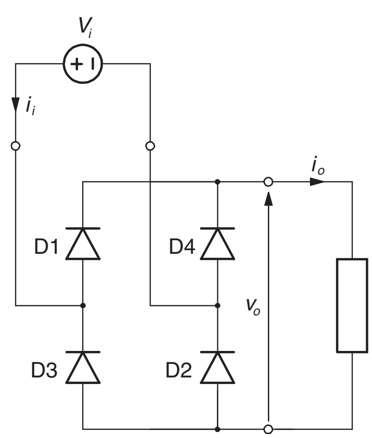


**Figure 1.33** Two-pulse diode rectifier.

## 1.7 SUMMARY

Power conversion and control are performed in power electronic converters that are networks of semiconductor power switches. Most practical converters are supplied from dc or ac voltage sources. Such sources have also been assumed for a generic converter, introduced here for instructional purposes. Three basic states of a voltage-source converter can be distinguished (although in some converters only two states suffice): the input and output terminals are directly connected, cross-connected, or separated. In the last state, the output terminals are shorted to maintain a closed path for the output current. An appropriate sequence of states results in conversion of the given input (supply) voltage to the output (load) voltage desired.

Current-source converters are also feasible, although less common than voltage-source converters. A load of a current-source converter must appear as a voltage source, and that of a voltage-source converter as a current source. In practice, the current- and voltage-source requirements imply a series-connected inductor and a parallel-connected capacitor, respectively.

Control of the output voltage in voltage-source power electronic converters is realized by periodic use of state 0 of the converter. This results in removal of certain portions of the output voltage waveform. Two approaches are employed: phase control and pulse width modulation. The latter technique, promoted by the availability of fast fully controlled semiconductor power switches, is employed increasingly in modern power electronics. The PWM results in higher power conversion and control quality than that of phase control.

The output voltage waveforms in voltage-source converters are usually easy to determine. Current waveforms, however, require analytical or numerical solution of differential or, for PWM converters, difference equations describing quasi-steady-state operation of the converters. A dual approach is applicable to current-source converters. Simulation software is used extensively in engineering practice.

Single-phase diode rectifiers are the simplest ac-to-dc power converters. However, their voltage gain, especially that of a single-pulse rectifier, is low, and ripple factors of the output voltage and current are high. These values can be improved by installing an output capacitor, but that solution is feasible only in low-power rectifiers.

## EXAMPLES

***Example 1.1*** A generic power converter is supplied from a 120-V 50-Hz ac voltage source and controlled so that its output voltage has the waveform shown in Figure 1.34. It can be seen that the converter performs ac-to-ac conversion, resulting in the output fundamental frequency reduced with respect to the input frequency. This type of power conversion is characteristic for power electronic converters called *cycloconverters*. The generic cycloconverter in this example operates in a simple *trapezoidal mode*, in which the ratio of the input frequency to output fundamental frequency is an integer.

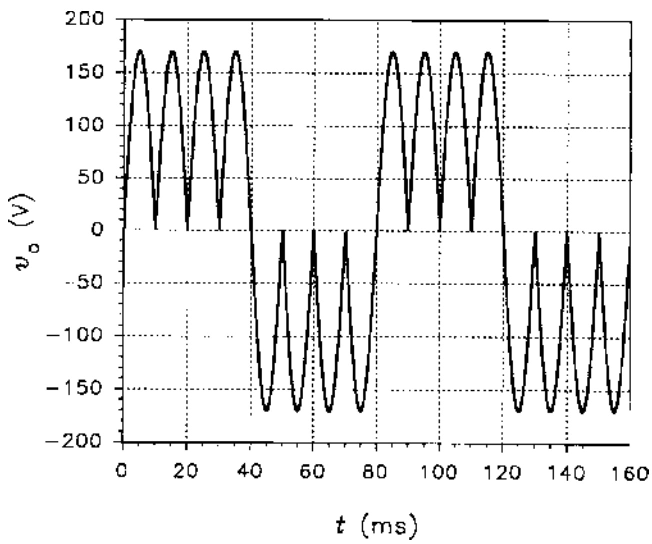


**Figure 1.34** Output voltage waveform in the generic cycloconverter in Example 1.1.

What is the fundamental frequency of the voltage in Figure 1.34? Sketch the timing diagram of the converter switches.

*Solution:* The output fundamental frequency in question is four times lower than the input frequency of 50 Hz, that is, 12.5 Hz. The timing diagram of the generic cycloconverter switches is shown in Figure 1.35. Switch S5 is open all the time since either switches S1 and S2 or S3 and S4 are closed.

***Example 1.2*** For the output voltage waveform in Figure 1.34, find:

a. The rms value, $V_o$
b. The peak value, $V_{o,1,p}$, and rms value, $V_{o,1}$, of the fundamental
c. The harmonic content, $V_{o,h}$
d. The total harmonic distortion, $\mathrm{THD_V}$.

*Solution:* The peak value of the input voltage is $\sqrt{2} \times 120 = 170$ V. Denoting the fundamental output radian frequency by $\omega$, the output voltage waveform can be expressed as

$$v_o(\omega t) = \begin{cases} |170| \sin 4\omega t & \text{for } 2n\dfrac{\pi}{4} \le \omega t < (2n+1)\dfrac{\pi}{4} \\ -|170| \sin 4\omega t & \text{for } (2n+1)\dfrac{\pi}{4} \le \omega t < (2n+2)\dfrac{\pi}{4} \end{cases}$$

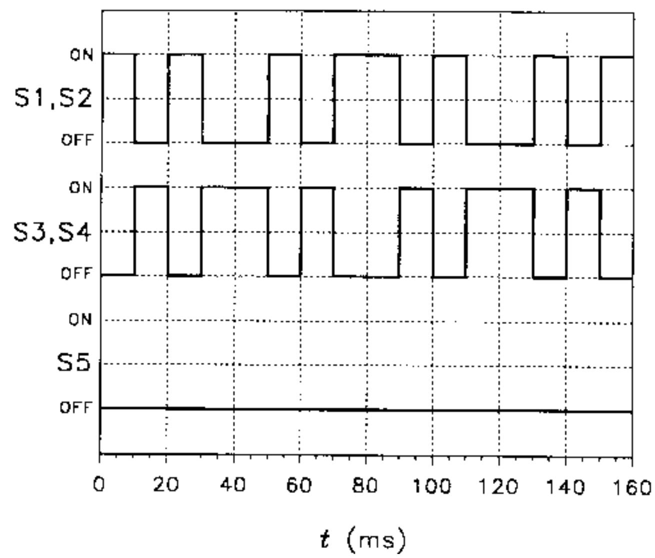


**Figure 1.35** Timing diagram of switches in the generic cycloconverter in Example 1.1.

where $n = 0, 1, 2, \ldots$, and the rms value, $V_o$, of the voltage is the same as that of the input voltage, that is, 120 V. The waveform has the odd and half-wave symmetries, so the peak value, $V_{o,1,p}$, of the fundamental is

$$
\begin{aligned}
V_{o,1,p} &= \frac{4}{\pi}\int_0^{\pi/2} v_o(\omega t)\sin\omega t \; d\omega t \\
&= \frac{4}{\pi}\int_0^{\pi/4} 170\sin 4\omega t \; d\omega t - \frac{4}{\pi}\int_{\pi/4}^{\pi/2} 170\sin 4\omega t \; d\omega t \\
&= \frac{4\times 170}{\pi}\left\{\left[\frac{\sin 3\omega t}{6} - \frac{\sin 5\omega t}{10}\right]_0^{\pi/4}\right. \\
&= \left. -\left[\frac{\sin 3\omega t}{6} - \frac{\sin 5\omega t}{10}\right]_{\pi/4}^{\pi/2}\right\} = 139\text{ V.}
\end{aligned}
$$

The rms value of the fundamental is

$$V_{o,1} = \frac{V_{o,1,p}}{\sqrt{2}} = \frac{139}{\sqrt{2}} = 98\text{ V}$$

and since there is no dc component, the harmonic content is

$$V_{o,k} = \sqrt{V_o^2 - V_{o,1}^2} = \sqrt{120^2 - 98^2} = 69\text{ V.}$$

Thus, the total harmonic distortion is

$$\mathrm{THD}_V = \frac{V_{o,h}}{V_{o,1}} = \frac{69}{98} = 0.7$$

***Example 1.3*** For a generic phase-controlled rectifier, find the relation between the ripple factor of the output voltage, $\mathrm{RF}_V$, and the firing angle, $\alpha_f$.

*Solution:* Equation (1.44) gives the relation between the dc component, $V_{o,\mathrm{dc}}$, of the output voltage in question and the firing angle, while Eq. (1.45), concerning the rms value, $V_o$, of the output voltage of a generic ac voltage controller, can be applied directly to the generic rectifier (why?). Consequently,

$$\begin{aligned}\mathrm{RF}_V(\alpha_f) &= \frac{V_{o,\mathrm{ac}}(\alpha_f)}{V_{o,\mathrm{dc}}(\alpha_f)} = \frac{\sqrt{V_o^2(\alpha_f) - V_{o,\mathrm{dc}}^2(\alpha_f)}}{V_{o,\mathrm{dc}}^2(\alpha_f)} \\ &= \sqrt{\frac{V_o^2(\alpha_f)}{V_{o,\mathrm{dc}}^2(\alpha_f)} - 1} = \sqrt{\frac{(1/2\pi)\left[\pi - \alpha_f + (\sin 2\alpha_f/2)\right]}{(1/\pi^2)(1+\cos\alpha_f)^2} - 1} \\ &= \sqrt{\frac{\pi}{2}\frac{\pi - \alpha_f + (\sin 2\alpha_f/2)}{(1+\cos\alpha_f)^2} - 1}.\end{aligned}$$

Graphical representation of the relation derived is shown in Figure 1.36. It can be seen that when the firing angle exceeds 150°, the voltage ripple increases rapidly because the dc component approaches zero.

***Example 1.4*** A generic converter is supplied from a 100-V dc source and operates as a chopper with a switching frequency, $f_{\mathrm{sw}}$, of 2 kHz. The average output voltage, $V_{o,\mathrm{dc}}$, is −60 V. Determine the duty ratios of all switches of the converter and the corresponding on- and off-times, $t_{\mathrm{ON}}$ and $t_{\mathrm{OFF}}$.

*Solution:* The negative polarity of the output voltage implies that switches S3, S4, and S5 perform the modulation, and switches S1 and S2 are permanently open (turned off). Therefore, the duty ratio, $d_{12}$, of the latter switches is zero. Adapting Eq. (1.47) to switches S3 and S4, the duty ratio, $d_{34}$, of these switches is

$$d_{34} = \frac{V_{o,\mathrm{dc}}}{V_1} = \frac{60}{100} = 0.6.$$

The negative sign at 60 V is omitted since the very use of switches S3 and S4 causes reversal of the output voltage (obviously, a duty ratio can only assume values between zero and unity). Switch S5 is turned on when the other switches are off. Hence, its

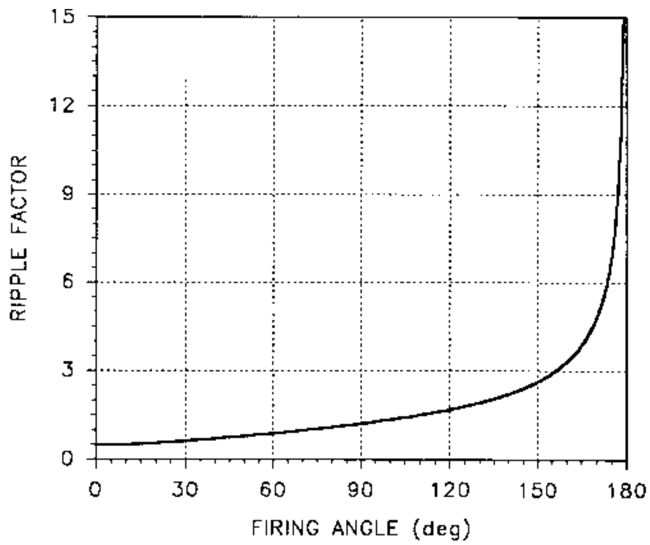


**Figure 1.36** Voltage ripple factor versus firing angle in the generic rectifier in Example 1.3.

duty ratio, $d_5$, is

$$d_5 = 1 - d_{34} = 1 - 0.6 = 0.4.$$

From Eq. (1.48),

$$t_{\text{ON}} + t_{\text{OFF}} = \frac{1}{f_{\text{sw}}} = \frac{1}{2 \times 10^3} = 0.0005 \text{ s} = 0.5 \text{ ms}$$

and from Eq. (1.45),

$$t_{\text{ON},34} = d_{34}(t_{\text{ON}} + t_{\text{OFF}}) = 0.6 \times 0.5 = 0.3 \text{ ms}$$

while

$$t_{\text{ON},5} = d_5(t_{\text{ON}} + t_{\text{OFF}}) = 0.4 \times 0.5 = 0.2 \text{ ms}.$$

Consequently,

$$t_{\text{OFF},34} = 0.5 - 0.3 = 0.2 \text{ ms}$$

and

$$t_{\text{OFF},5} = 0.5 - 0.2 = 0.3 \text{ ms}.$$

***Example 1.5*** A generic converter is supplied from a 120-V 60-Hz ac voltage source and operates as a PWM rectifier with an RL load, where $R = 2\ \Omega$ and $L = 5$ mH. The switching frequency is 720 Hz and the magnitude control ratio is 0.6. Develop iterative formulas for the output current and calculate values of that current at the switching instants within one cycle of the output voltage.

*Solution:* Inspecting similar formulas, (1.73) and (1.75), for the generic PWM ac voltage controller, it can be seen that they are easy to adapt to the PWM rectifier by replacing the $\sin \omega t_0$ term in Eq. (1.73) with $|\sin \omega t_0|$. Then the output current of the rectifier can be computed from the equations

$$i_o(t_1) = i_o(t_0) + \frac{M}{L}\left[V_{i,p}\,|\sin \omega t_0| - Ri_o(t_0)\right]\ \Delta T$$

and

$$i_o(t_2) = i_o(t_1)\left[1 - \frac{R}{L}(1 - M)\ \Delta T\right].$$

The input frequency, $\omega$, is $2\pi \times 60 = 377$ rad/s, and the length, $\Delta T$, of a switching interval is 1/720 s. Thus, the general iterative formulas above give

$$\begin{aligned} i_o(t_1) &= i_o(t_0) + \frac{0.6}{5 \times 10^{-3}}\left[\sqrt{2} \times 120\,|\sin(377t_0)| - 2i_o(t_0)\right] \times \frac{1}{729} \\ &= 0.667i_o(t_0) + 28.3\,|\sin(377t_0)| \end{aligned}$$

and

$$i_o(t_2) = i_o(t_1)\left[1 - \frac{2}{5 \times 10^{-3}}(1 - 0.6)\frac{1}{720}\right] = 0.778i_o(t_1).$$

As shown in Figure 1.20a, the period of the output voltage of the rectifier equals half of that of the input voltage, that is, 1/120 s = 8.333 ms. Comparing it with the switching frequency, the number of switching intervals per cycle of the output voltage turns out to be six. With $M = 0.6$, the on-time, $t_{\mathrm{ON}}$, equal to $t_1 - t_0$, is 0.6/720 s = 0.833 ms, and the off-time, $t_{\mathrm{OFF}}$, equal to $t_2 - t_1$, is 0.4/720 s = 0.556 ms.

To start the computations, the initial value, $i_o(0)$, is required as $i_o(t_0)$ in the first switching interval. It can be estimated as the dc component, $I_{o,\mathrm{dc}}$, of the output current, given by

$$I_{o,\mathrm{dc}} = \frac{M(2/\pi)V_{i,p}}{R} = \frac{0.6 \times 2/\pi \times \sqrt{2} \times 120}{2} = 32.4\ \mathrm{A}.$$

Now, the computations of $i_o(t)$ can be performed for consecutive switching intervals, the $t_2$ instant for a given interval being the $t_0$ instant for the next interval.

*First interval* ($t_0 = 0$, $t_1 = 0.833$ ms, $t_2 = 1.389$ ms):

$$i_o(0.833\text{ ms}) = 0.667 \times 32.4 + 28.3\,|\sin(377 \times 0)| = 21.6\text{ A}$$

and

$$i_o(1.389\text{ ms}) = 0.778 \times 21.6 = 16.8\text{ A.}$$

*Second interval* ($t_0 = 1.389$ ms, $t_1 = 2.222$ ms, $t_2 = 2.778$ ms):

$$i_o(2.222\text{ ms}) = 0.667 \times 16.8 + 28.3\left|\sin(377 \times 1.389 \times 10^{-3})\right| = 25.4\text{ A}$$

and

$$i_o(2.778\text{ ms}) = 0.778 \times 25.4 = 19.8\text{ A.}$$

*Third interval* ($t_0 = 2.778$ ms, $t_1 = 3.611$ ms, $t_2 = 4.167$ ms):

$$i_o(3.611\text{ ms}) = 0.667 \times 19.8 + 28.3\left|\sin(377 \times 2.778 \times 10^{-3})\right| = 37.7\text{ A}$$

and

$$i_o(4.167\text{ ms}) = 0.778 \times 37.7 = 29.3\text{ A.}$$

*Fourth interval* ($t_0 = 4.167$ ms, $t_1 = 5$ ms, $t_2 = 5.556$ ms):

$$i_o(4.167\text{ ms}) = 0.667 \times 29.3 + 28.3\left|\sin(377 \times 4.167 \times 10^{-3})\right| = 47.8\text{ A}$$

and

$$i_o(5.556\text{ ms}) = 0.778 \times 47.8 = 37.2\text{ A.}$$

*Fifth interval* ($t_0 = 2.778$ ms, $t_1 = 3.611$ ms, $t_2 = 4.167$ ms):

$$i_o(6.389\text{ ms}) = 0.667 \times 37.2 + 28.3\left|\sin(377 \times 5.556 \times 10^{-3})\right| = 49.3\text{ A}$$

and

$$i_o(6.944\text{ ms}) = 0.778 \times 49.3 = 38.4\text{ A.}$$

*Sixth interval* ($t_0 = 6.944$ ms, $t_1 = 7.778$ ms, $t_2 = 8.333$ ms):

$$i_o(7.778\text{ ms}) = 0.667 \times 38.4 + 28.3\left|\sin(377 \times 6.944 \times 10^{-3})\right| = 39.8\text{ A}$$

and

$$i_o(8.333 \text{ ms}) = 0.778 \times 39.8 = 31.0 \text{ A}.$$

As the rectifier is assumed to operate in the quasi-steady state, the last, final value, $i_o(8.333 \text{ ms})$, should equal the initial value, $i_o(0)$. It is not exactly so, which implies an incorrect assumption of the initial value of 32.4 A. Note that the impact of the initial value on the final value is minimal, since at each step of the computations the previous value of the output current is multiplied by either 0.667 or 0.778. Thus, after the 12 steps resulting from the six switching intervals, the initial value error, $\Delta i_o(0)$, is translated into the respective final value error, $\Delta i_o(8.333 \text{ ms})$, of $(0.667 \times 0.778)^6 \, \Delta i_o(0) \approx 0.02 \, \Delta i_o(0)$ only. Therefore, it is a safe guess that the final value of 31 A obtained is only slightly greater than the actual initial value. Assuming $i_o(0)$ to be 30.9 A and repeating the iterative calculations yields the following points of the output current waveform:

| | |
|---|---|
| 0 | 30.9 A |
| 0.833 ms | 20.6 A |
| 1.389 ms | 16.0 A |
| ⋮ | ⋮ |
| 6.944 ms | 38.3 A |
| 7.778 ms | 39.7 A |
| 8.333 ms | 30.9 A |

Waveforms of the output voltage and current are shown in Figure 1.37. Clearly, the calculations presented can easily be computerized, greatly reducing the amount of work involved.

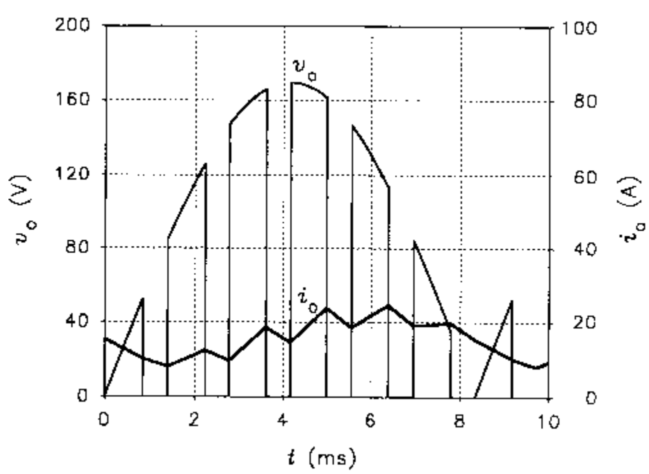


**Figure 1.37** Output voltage and current waveforms in the generic PWM rectifier in Example 1.5.

## PROBLEMS

**P1.1** Refer to Figure 1.13 and sketch the waveforms of the output voltage of the generic rectifier with a firing angle of:

**(a)** 36°
**(b)** 72°
**(c)** 108°
**(d)** 144°

**P1.2** Refer to Figure 1.15 and sketch the waveforms of the output voltage of the generic ac voltage controller with a firing angle of:

**(a)** 36°
**(b)** 72°
**(c)** 108°
**(d)** 144°

**P1.3** From the harmonic spectrum in Figure 1.17a, find:

**(a)** The per-unit value of the dc component of the output voltage of a generic rectifier with a firing angle of 90° [use Eq. (1.44) to verify the result]
**(b)** The peak and rms per-unit values of the fundamental output voltage with the same firing angle as in part (a)

**P1.4** From the harmonic spectrum in Figure 1.17b, read the peak per-unit value of the fundamental output voltage of the generic ac voltage controller with a firing angle of 90°, and find:

**(a)** The rms per-unit value of the output voltage [use Eq. (1.44)]
**(b)** The per-unit harmonic content of the output voltage
**(c)** The total harmonic distortion of the output voltage

**P1.5** For a generic inverter in the square-wave operating mode, derive an equation for the peak per-unit value of the *k*th harmonic of the output voltage (take the dc input voltage, $V_i$, as the base voltage). Calculate the peak per-unit values of the first 10 harmonics.

**P1.6** A generic rectifier is supplied from a 115-V 60-Hz ac source and operates with a firing angle of 30°. For the output voltage of the rectifier, find:

**(a)** The dc component
**(b)** The rms value of the ac component
**(c)** The ripple factor

**P1.7** For the generic rectifier in Problem 1.6, find the fundamental frequency of the output voltage.

**P1.8** For a generic ac voltage controller, derive an equation for the peak per-unit value of the *k*th harmonic of the output voltage as a function of the firing angle (take the peak value, $V_{i,p}$, of the input voltage as the base voltage). Use the spectrum in Figure 1.17b to verify the equation for a firing angle of 90° and the five lowest-order harmonics.

**P1.9** Review the output voltage waveforms in Figures 1.13 and 1.15 and the corresponding harmonic spectra in Figure 1.17. Which harmonics present in the spectrum for the generic phase-controlled rectifier are absent in the spectrum for the generic ac voltage controller? Why?

**P1.10** A generic converter operates as a chopper and is supplied from a 100-V dc source. What is the magnitude control ratio of the chopper, and what are the duty ratios of individual switches if the average output voltage is 70 V?

**P1.11** Repeat Problem 1.10 for an output voltage of −35 V.

**P1.12** A generic converter operates as a chopper with a magnitude control ratio of 0.4. The duration of a pulse of the output voltage is 0.2 ms. Find the switching frequency.

**P1.13** A generic converter operates as a chopper with 1250 pulses of output voltage per second and with a magnitude control ratio of 0.3. Find the duration of a pulse and a notch of the output voltage.

**P1.14** A generic PWM rectifier operates with the average output voltage reduced by 20% with respect to the maximum available value of this voltage. Find the magnitude control ratio and the ratio of the width of the pulse of the output voltage to the notch width.

**P1.15** A generic PWM rectifier operates as in Problem 1.14 with 50 pulses of the output voltage per period of the 60-Hz input voltage. Find the width of a pulse and the switching frequency.

**P1.16** A generic PWM ac voltage controller is supplied from a 60-Hz ac voltage source. The fundamental output voltage is reduced by two-thirds with respect to the supply voltage, and the switching frequency is 1.5 kHz. Find:

**(a)** The number of pulses of the output voltage per cycle of this voltage
**(b)** The duty ratios of switches S1, S2, and S5
**(c)** The pulse width

**P1.17** Refer to Figure 1.23a and determine the states (on or off) of all the switches of the generic PWM inverter at:

**(a)** $\omega t = \pi/2$ rad
**(b)** $\omega t = \pi$ rad
**(c)** $\omega t = 3\pi/2$ rad

**P1.18** Waveforms in Figure 1.23 for the generic PWM inverter, with 10 pulses of the output voltage per cycle, were obtained in the following way:

1. The 360° period of the voltage was divided into 10 equal switching intervals of 36° each.
2. Denoting by $\alpha_n$ the central angle of the $n$th switching interval ($\alpha_1 = 18°$, $\alpha_2 = 54°$, etc.), the duty ratio, $D_n$, of operating switches S1–S2 or S3–S4 for this interval was calculated as

$$D_n = M\,|\sin\alpha_n|$$

where $M$ denotes the magnitude control ratio.

For 8 pulses per cycle and $M = 0.75$, find the widths (in degrees) of individual pulses of the output voltage and sketch to scale the resulting voltage waveform, $v_o(\omega t)$. The voltage pulses should be located in the middle of switching intervals.

**P1.19** Refer to Figure 1.34 and sketch the output voltage waveform of a generic phase-controlled cycloconverter with a magnitude control ratio of 0.5 and an input/output frequency ratio of 2. Identify the states of the converter.

**P1.20** Refer to Figure 1.34 and sketch the output voltage waveform of a generic PWM cycloconverter with an input/output frequency ratio of 3 and a magnitude control ratio of 0.5 (for convenience, assume a low number of pulses of the output voltage).

**P1.21** A generic PWM rectifier is supplied from a 240-V 60-Hz ac line and operates with a magnitude control ratio of 0.5 and a switching frequency of 720 Hz. The rectifier feeds a dc motor, which under the given operating conditions can be represented as an RLE load with $R = 0.5\ \Omega$, $L = 12$ mH, and $E = 90$ V. Determine the piecewise linear waveform of the output current for one cycle of output voltage.

**P1.22** A generic PWM ac voltage controller with an RL load is supplied from a 115-V 50-Hz source and operates with 10 switching intervals per cycle and a magnitude control ratio of 0.75. The resistance and inductance of the load are 10 Ω and 25 mH, respectively. Determine the piecewise linear waveform of the output current for one cycle of the output voltage.

## COMPUTER ASSIGNMENTS

A generic power converter, which represents an idealized theoretical concept, cannot be modeled precisely using PSpice, which is designed for simulation of practical circuits. In contrast to the ideal, infinitely fast switches assumed for the generic converter, PSpice switch models have finite time of transition from one state to another. To avoid interruptions of the output current, the output terminals of the converter must therefore be shorted by switch S5 just before the output is separated from the input by opening switches S1–S2 or S3–S4. As a result, the supply source is temporarily shorted, too, albeit very briefly, and large impulses ("spikes") of the input current appear in the Probe oscillograms. Such short-circuit currents are not present in practical correctly designed and controlled power electronic converters. However, the output voltage and current in the generic converter can be simulated quite accurately.

To calculate the figures of merit, make use of the avg($x$) (average value of $x$) and rms($x$) (rms value of $x$) functions. Also, employ the *Fourier* option for the $X$-axis to obtain harmonic spectra of voltages and currents. Refer to Appendix A and the book by Rashid and Rashid [4] for instructions on PSpice simulations.

Assignments involving PSpice files accompanying the book are identified by an asterisk.

***CA1.1** Run PSpice file *Gen_Ph-Contr_Rect.cir* for a generic phase-controlled rectifier. Perform simulations for firing angles of 0° and 90° and find for the output voltage of the converter:

**(a)** The dc component

**(b)** The rms value

**(c)** The rms value of the ac component

**(d)** The ripple factor

Observe oscillograms of the input and output voltages and currents. Explain what causes the spikes in the oscillograms of the input quantities and what limits the amplitude of the current spikes.

**CA1.2** Develop a PSpice circuit file for a generic inverter operating in the square-wave mode with an adjustable fundamental output frequency. Perform a simulation for an output frequency of 50 Hz and find for the output voltage of the inverter:

**(a)** The rms value

**(b)** The rms value of the fundamental (from the harmonic spectrum)

**(c)** The harmonic content

**(d)** The total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

**CA1.3** Develop a PSpice circuit file for a generic ac voltage controller. Perform simulations for firing angles of 0° and 90° and find for the output voltage of the converter:

**(a)** The rms value

**(b)** The rms value of the fundamental (from the harmonic spectrum)

**(c)** The harmonic content

**(d)** The total harmonic distortion.

Observe oscillograms of the input and output voltages and currents.

**CA1.4** Refer to Example 1.1 and Problem P1.19 and develop a PSpice circuit file for a generic phase-controlled cycloconverter with an input frequency of

P1: OTA/XYZ P2: ABC
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50 Hz and a fundamental output frequency of 25 Hz. Perform simulations for firing angles of 0° and 90° and find for the output voltage of the converter:

**(a)** The rms value
**(b)** The rms value of the fundamental (from the harmonic spectrum)
**(c)** The harmonic content
**(d)** The total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

**CA1.5** Develop a PSpice circuit file for a generic chopper. Perform simulations for a switching frequency of 1 kHz and magnitude control ratios of 0.6 and −0.3, and find for the output voltage of the converter:

**(a)** The dc component
**(b)** The rms value
**(c)** The rms value of the ac component
**(d)** The ripple factor

Observe oscillograms of the input and output voltages and currents.

***CA1.6** Run PSpice program *Gen_PWM_Rect.cir* for a generic PWM rectifier. Perform simulations for 12 pulses of the output voltage per cycle of the input voltage and a magnitude control ratio of 0.5, and find for the output voltage of the converter:

**(a)** The dc component
**(b)** The rms value
**(c)** The rms value of the ac component
**(d)** The ripple factor

Observe oscillograms of the input and output voltages and currents. Explain what causes the spikes in the oscillograms of the input quantities and what limits the amplitude of the current spikes.

**CA1.7** Develop a PSpice circuit file for a generic PWM ac voltage controller. Perform a simulation for 12 pulses of the output voltage per cycle and a magnitude control ratio of 0.5, and find for the output voltage of the converter:

**(a)** The rms value
**(b)** The rms value of the fundamental (from the harmonic spectrum)
**(c)** The harmonic content
**(d)** The total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

**CA1.8** Refer to Example 1.1 and Problem P1.20 and develop a PSpice circuit file for a generic PWM cycloconverter. Perform a simulation for an input frequency of 50 Hz, a fundamental output frequency of 25 Hz, a magnitude control ratio of 0.5, and 10 pulses of output voltage per cycle of the input voltage. Find for the output voltage of the converter:

**(a)** The rms value

**(b)** The rms value of the fundamental (from the harmonic spectrum)

**(c)** The harmonic content

**(d)** The total harmonic distortion

Observe oscillograms of the input and output voltages and currents.

**CA1.9** Develop a computer program for calculation of the harmonics of a given periodic waveform, $\psi(\omega t)$. Data points that represent one cycle of the waveform are stored in an ASCII file in the $\omega t$, $\psi(\omega t)$ format. Generate and store the voltage waveform in Figure 1.13 (generic phase-controlled rectifier) and use your program to obtain the harmonic spectrum of the waveform. Compare the results with those in Figure 1.17a.

**CA1.10** Develop a computer program for calculation of the following parameters of a given periodic waveform, $\psi(\omega t)$:

**(a)** The rms value

**(b)** The dc component

**(c)** The rms value of the ac component

**(d)** The rms value of the fundamental

**(e)** The harmonic content

**(f)** The total harmonic distortion

Data points that represent one cycle of the waveform are stored in an ASCII file in the $\omega t$, $\psi(\omega t)$ format. Generate and store the voltage waveform in Figure 1.15 (generic phase-controlled ac voltage controller) and apply the program to compute parameters (a) through (f).

**CA1.11** Develop a computer program for the determination of a current waveform generated in a given load by a given voltage. The load can be of the R, RL, RE, LE, or RLE type, and the voltage waveform, $v(t)$, is given as either a closed-form function of time, $v = f(t)$, or as an ASCII file of ($t$, $v$) pairs.

***CA1.12** Run PSpice program *Diode_Rect_1P.cir* for a single-pulse diode rectifier. Repeat the simulation for the rectifier with a freewheeling diode and with an output capacitor ("comment out" the unused components). In each case, determine the average output voltage and ripple factor of that voltage.

## LITERATURE

[1] Agrawal, J. P., *Power Electronic Systems: Theory and Design*, Prentice Hall, Upper Saddle River, NJ, 2001, Chap. 1.

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