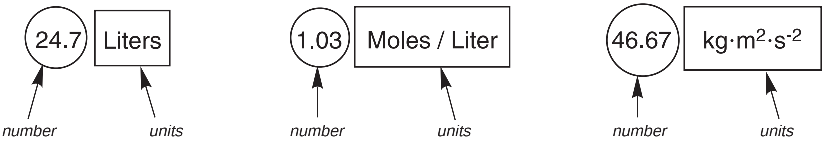
CHAPTER 1

NUMBERS AND UNITS

If you flip through your textbook and look at the kinds of problems you will be solving in each chapter, you will find that the answer to almost every problem is a number. You might see an occasional question asking you to explain some principle or concept, but in general, most problems require you to *calculate* an answer. Whether it is the pH of a solution or the enthalpy of formation for a reaction, you will be spending a lot of time learning how to use equations to calculate answers. But even if you know what equations to use and you know how to calculate the answer, it is still possible to get the answer wrong.

There are TWO equally important parts of any answer: (1) the number, and (2) the units. Here are some simple examples:



If you incorrectly report either one of these two parts, then you will mess up your answer. You need to learn how to report each part properly. This might seem like simple stuff, but little mistakes can ruin your answer (and you can lose precious points on an exam). So let's try to avoid that, and let's get some practice. Most of the problems in this chapter are very simple and won't take long to do, so you should be able to breeze through this chapter pretty quickly. In fact, much of the material in this chapter is just a review of what your intuition already knows. The main point of the chapter is to reinforce the *skills* you need to report an answer (so that you won't have to rely on your intuition during an exam).

In our discussion of ***units*** toward the end of this chapter, we will see a very important method that helps us ensure that our answers have the correct units. This method, ***the factor-label method***, will be used throughout the entire course, so we need to *master* this method early on.

We will start our discussion with the first part of every answer: the number.

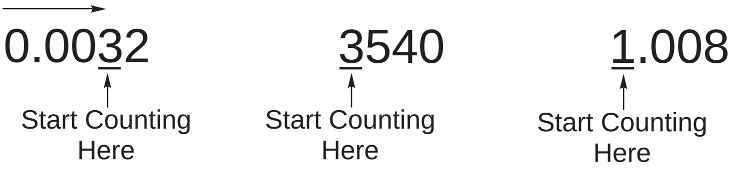
1.1 SIGNIFICANT FIGURES

To understand the importance of significant figures, consider the following example. The current estimate for world population is *about* 6.5 billion. Notice the important term “about”. No one really knows the exact number, so we must use an estimate. Now imagine dividing up the world into three regions with equal populations. The number of people in each region is just 6.5 billion divided by 3. If you key that into your calculator, you will get 2,166,167 people. Even though the calculator gives this exact answer, we know that each region will NOT have *exactly* that number of people, because the starting number (6.5 billion) was just an estimate. So, when we divide by 3, our answer must also be an estimate. The starting number had only two significant digits (the 6 and the 5), so our answer cannot have more than two significant digits either. Our answer must be 2.2 billion people in each region.

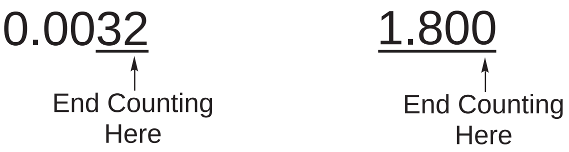
If we want to have an answer with more significant figures, then we will have to start with an estimate that has more significant figures. For example, if we learn that the newest estimate for world population is 6.532 billion, then we have a starting number with four significant figures. So, our answer (6.532 billion divided by 3) will also have four significant figures. If you key that into your calculator and round your answer to the fourth digit, your answer will be 2.177 billion.

Now comes the important part—if the answer you write on an exam has too many OR too few significant figures, then the answer is WRONG. If you write down 2.1773333 billion instead of 2.177 billion, then the answer is wrong, ***even if the calculator says 2.1773333***. First we will learn how to count significant figures, and then we will learn how to determine how many significant figures belong in an answer.

Here is the way to count significant figures: Start at the left side of the number and move toward the right until you get to the first non-zero digit. Start counting at that digit. Examples:



Once you figured out where to start counting, then you have to find out where to end counting. If the number has a decimal in it, then you count until the very end:



If the number does *not* have a decimal, then we generally count until the last non-zero number:

$$\underline{35}\underset{\substack{\uparrow\\ \text{End Counting}\\ \text{Here}}}{\underline{4}}0 \qquad\qquad \underline{800}\underset{\substack{\uparrow\\ \text{End Counting}\\ \text{Here}}}{\underline{7}}00$$

This last set of examples points out a difficulty. Suppose you estimate the number of people at a baseball game to be *approximately* 8500. Your number is just an estimation (there could be 8527 or 8496 people). The way you report your number indicates that you have estimated the number to the nearest hundred—that's why you show two significant figures in your number (the 8 and the 5). If you had rounded to the nearest thousand, you might have estimated 9000 people (which would be just one significant number). However, what if you count each and every person at the baseball game and the number comes out to be exactly 8500 people? How do you report that your number is exact and not just an estimation? If you just say 8500, everyone will assume that you are talking about *two* significant figures. We will show how to solve this issue in the next section. For now, we will consider the number 8500 as having two significant figures.

Let's get practice counting significant figures, so that we can get comfortable using them properly in our answers.

EXERCISE 1.1. Count the significant figures that you see in the number 0.007520.

Answer: Look at the far left of the number, and move right until you see the first non-zero number. That's where you start counting. In this case, you start counting with the 7:

$$\overrightarrow{0.00}\underset{\substack{\uparrow\\ \text{Start Counting}\\ \text{Here}}}{\underline{7}}520$$

Next you have to figure out how far to count. In this case, there is a decimal point, so you count until the very end:

$$0.00\underline{752}\underset{\substack{\uparrow\\ \text{End Counting}\\ \text{Here}}}{\underline{0}}$$

As you can see, there are four significant figures.

Let's get a little bit of practice before we move on. For each of the following problems, how many significant figures do you see? Remember that you always start with the first non-zero digit, and you decide where to stop counting based on whether there is a decimal. If there is a decimal, count all the way to the end of the number. If there is no decimal, count until the last non-zero digit.

1.2. 0.0713200
1.3. 7843000
1.4. 1.4800
1.5. 100
1.6. 100.0
1.7. 894.003
1.8. 89400
1.9. 0.03000
1.10. 74.000

Now that we have seen how to count significant figures, we will use that skill to multiply and divide two or more numbers when one of the numbers has less significant figures than the other(s). For example, if you had to multiply 0.034 and 127, you should notice that 0.034 has two significant figures and 127 has three significant figures. When it comes to multiplying and dividing, the rule goes like this: look at the number with the fewest significant figures, and your answer should have the same amount of significant figures. In our case, the number with the fewest significant figures is 0.034, which has only two significant figures. So, in this example, your answer should have only two significant figures. If you use your calculator to multiply 0.034 and 127, you will get 4.318. But you must fight the temptation to write that down as your answer. You must round this number to only two significant figures. So the answer is 4.3.

The same rule applies when you divide two numbers. You just choose the number with the fewest significant figures, and that is how many significant figures should be in your answer.

In each of the following problems, determine how many significant figures should be present in the answer:

1.11. 472×101
1.12. 4600×0.005
1.13. 36.0×4752
1.14. $\dfrac{45.08}{36.2}$
1.14. $\dfrac{0.003}{472}$
1.15. $\dfrac{1.003}{8500}$
1.16. $\dfrac{0.003}{472} \times 12$
1.17. $\dfrac{3.003}{475.0} \times \dfrac{0.30}{524}$
1.18. 0.3005×4.1

So far, we have seen how to multiply and divide numbers that have a different amount of significant figures. Now we need to see how to add and subtract, because the rules are a bit different. For adding and subtracting, we count the decimal places. For example, 472.44 has two decimal places (there are two digits appearing after the decimal); 890 has no decimal places. Look at the number with the fewest decimal places, and make sure that your answer has the same amount of decimal places. So, if you add 45.62 and 3.7, your calculator will say 49.32. But don't write that down as your answer. One of the starting numbers (3.7) only had one decimal place, so your answer must also have only one decimal place. Thus, round your answer off to 49.3. Similarly, if you are adding 45.62 and 24, then your answer should be 69 (with no decimal at all), because 24 has no decimal places.

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The same rule applies when subtracting numbers. You just choose the number with the fewest decimal places, and that is how many decimal places should be in your answer.

In each of the following problems, determine how many decimal places should be present in the answer:

1.19. $23.56 + 24.983$
1.20. $4.78 - 2.892$
1.21. $46.83 - 0.03$
1.22. $34.892 + 5.0$
1.23. $134.033 - 0.02$
1.24. $48.2 - 46$

So far, we have seen the rules to determine how many digits to put in our answer. But to actually write down the answer, you will need to round the answer displayed on your calculator screen, so that the answer you write has the correct number of digits. For the most part, rounding is intuitive, but there is one part that is definitely not intuitive. We'll explore this now.

When rounding off a number, we look at the digit that is directly beyond the last significant digit. For example, if the answer on your calculator is 34.27, and you have determined that you must round off to three significant figures, then you look at the fourth digit and ask if it is more than 5. In this case (34.2*7*), the 7 is the first digit that is not significant. Since 7 is more than 5, we round up, and round off our answer to 34.3. Similarly, if the first nonsignificant digit is less than 5, then we round down. For example, if your calculator says 45.782, and you have determined that you can only have 4 significant figures, then look at the fifth digit (which is a 2) and round down to 45.78.

Now you know what to do if your first nonsignificant digit is more than 5 or less than 5. But what if it is exactly 5? Then what? It's easy if something comes after that 5. For example, if we have the number 43.501 and we have to round off to 2 significant figures, then we would round up and our answer is 44. But what about a case where nothing comes after the 5? Here is the part where it is not so intuitive. Your intuition probably tells you to always round up. For example, you might say that 43.5 should be rounded up to 44, and 44.5 should be rounded up to 45. But chemists have come up with a different way of doing things. It goes like this: When you have a 5 right after the last significant figure, then look at the last significant figure, and ask whether it is odd or even. In the case of 43.5, the digit before the 5 is odd. In the case of 44.5, the digit before the 5 is even. Believe it or not, the convention is to treat these differently. We round UP when the last significant figure is odd, but we round DOWN when the last significant figure is even. That's right. As crazy as it might sound, 43.5 will get rounded UP to 44, because the last significant figure (3) is an odd digit. We treat 0 as an even number, so 40.5 would get rounded DOWN to 40.

Let's get some practice. Using the rules you just learned, round off the following numbers to three significant digits:

1.25. 34.78 ____________
1.26. 24.33 ____________
1.27. 17.51 ____________
1.28. 17.50 ____________
1.29. 18.50 ____________
1.30. 20.5 ____________
1.31. 45.50001 ____________
1.32. 45.5000 ____________ QU1
1.32. 25.49 ____________ QU1

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We have now seen a lot of rules, and we are ready to put them all together to do some calculations. Let's just quickly review everything we have seen:

1. When counting significant figures, start from the left at the first non-zero number. If there is a decimal in the number, then count until the end. If the there is no decimal, then count until the last non-zero number.
2. When multiplying or dividing numbers, choose the number with the fewest significant figures, and that is how many significant figures should be in your answer.
3. When adding or subtracting numbers, choose the number with the fewest decimal places, and that is how many decimal places should be in your answer.
4. When rounding off an answer, look at the first nonsignificant number. If it is less than 5, round down. If it is more than 5, round up. If it *is* 5, then look to see if there are any non-zero digits after the 5. If so, round up (e.g., 42.500001 will round up to 43). If there are no non-zero digits after the 5, then you must look at the digit before the 5—round up if odd; round down if even (e.g., 42.50000 will round down).

EXERCISE 1.33. Make the following calculation and report your answer with the correct amount of significant figures.

$$8.410 \times 5.00 = \underline{\hspace{3cm}}$$

Answer: Begin by counting the significant figures in each of the starting numbers. In the first number (8.410), begin with the 8. This number has a decimal, so count all the way until the end, including the zero. You'll note that it has *four significant figures*. In the second number (5.00), begin with the 5. This number also has a decimal, so by counting until the end, including both zeros, you see that there are *three significant figures*. Therefore, your answer should have three significant figures.

Now multiply the numbers on your calculator: 8.410 × 5.00. Your calculator should say 42.05. We just determined that we have to round off to three significant figures, so we look at the fourth digit to see whether to round up or down. The fourth digit is a 5, and this is situation where it is not so intuitive so we have to use the rules. Look back to the last significant digit, which is a zero. Remember that a zero counts like an even number, and when the last significant digit is an even number, then the 5 gets rounded DOWN. So, the answer is 42.0.

Now let's get some practice. For each of the following mathematical operations, calculate the answer and record your answer with the correct number of significant digits.

1.34. $34.03 \times 0.0072 = \underline{\hspace{3cm}}$

1.35. $\dfrac{3.003}{475.0} = \underline{\hspace{3cm}}$

1.36. $67.75 - 7.2 = \underline{\hspace{3cm}}$

1.37. $356.50 + 6.0 = \underline{\hspace{3cm}}$

1.38. $356.50 + 12 = \underline{\hspace{3cm}}$

1.39. $356.50 + 13 = \underline{\hspace{3cm}}$

1.40. $\dfrac{3.003}{475.0} \times 0.0322 =$ __________

1.41. $1.003 \times 5.0 =$ __________

1.42. $46.7 - 0.4 =$ __________

1.43. $\dfrac{12.4}{1.7} =$ __________

Before we move on to the next section, there are two important points to make:

1. When you are making a series of calculations, you should wait to round off your answer until the very end. Do all of your calculations without any rounding off, and then just round once at the end. If you are working on a long problem and you do round your answers after each calculation in a series of calculations, then you might find that the last digit of your answer might be different from the answer in the back of your textbook.
2. Whenever you count discrete objects, significant figures don't apply. For example, if you count that there are 6 chairs around the table, then the number 6 is an exact number. It is not an estimate. We don't treat this number as if it has one significant figure. Rather, we treat it as if it has an infinite number of significant figures. For example, if you know that each chair weighs 2.34 pounds, and you want to know how much 6 chairs will weigh, then you need to multiply 6 times 2.34. In order to determine the amount of significant figures to put in your answer, look at the number with the fewest significant figures. When looking at the two numbers, 6 and 2.34, we would normally say that 6 has the fewest significant numbers. But in this case, the number 6 is counting discrete objects, so it is treated as if it has an infinite number of significant digits (6.00000 . . .). So, in this case, the number with the fewest significant figures will be 2.34. Therefore, your answer should have three significant figures in it.

1.2 ORDERS OF MAGNITUDE

Most of the numbers that you will use in this course are either very large or very small. It gets a little bit crazy to talk about numbers like 4,782,000,000,000,000,000,000 molecules. Similarly, it is crazy to talk about numbers like 0.0000000000843 grams. There are so many zeros in each of these numbers that it is impossible to read them and get a sense of their magnitude. All we can say is that the first number is very big and the second number is very small. To get a real sense of *how* big and *how* small, we express numbers in *scientific notation*—a system based on powers of 10. You have probably seen this system already in high school, but let's just review quickly. Scientific notation is expressed like this: 10^1, 10^2, 10^3, 10^4, etc. The superscript above the 10 tells us the magnitude of the number. So,

$$
\begin{aligned}
10^2 &= 100\\
10^3 &= 1000\\
10^4 &= 10{,}000\\
\text{and } 10^{23} &= 100{,}000{,}000{,}000{,}000{,}000{,}000{,}000
\end{aligned}
$$

If we want to express a number like 4,782,000,000,000,000,000,000, then we can just say 4.782×10^{21}.

In a moment we will review how to convert numbers into scientific notation, we will revisit significant figures as applied to scientific notation, and we will see the subtleties of performing arithmetic operations on these numbers (multiplying and dividing). But first, we should take a moment to reflect on the magnitude that is conveyed when we use powers of 10.

As an example, consider the age of the universe—13.7 billion years. Try to guess how many seconds are in 13.7 billion years. 10 to the power of what? 10^{100}? 10^{1000}? Would you believe that it is just 10^{17}? Just multiply the following numbers: (13.7 billion) × (365) × (24) × (60) × (60) and your calculator will give you the answer: 432,043,200,000,000,000. When you convert this answer into scientific notation and express the appropriate significant figures (we will see how to do this very soon), you get 4.32×10^{17} seconds. Now we can begin to appreciate that 10^{17} is a REALLY BIG number. We cannot even begin to relate to a number that large. But consider this: 10 times that number is 10^{18}, and 10 times that is 10^{19}, and 10 times that, and 10 times that, and 10 times that, and 10 times that—is roughly the same number as the number of molecules in a teaspoon of water (10^{23}). *Now that is truly mind boggling!* Clearly, it would be a gross understatement to say that there are billions of water molecules in that teaspoon of water. That would be like saying that Bill Gates is worth a few pennies. The magnitude would be totally off.

The *order of magnitude* (the exponent on the 10) tells us how big or small a number is. Compare 10^9 and 10^6. 10^9 is three orders of magnitude larger than 10^6. That means that 10^9 is 1000 times larger than 10^6. It is important to realize the scope of these numbers, because many numbers in chemistry rely on your ability to appreciate orders of magnitude. Exponents are often used throughout the course, and so are logarithms (logarithms are just a fancy way of repackaging exponents). The last chapter (Chapter 9) of this book provides a more detailed explanation of exponents and logarithms.

The problems that you will solve in your chemistry course will often have answers that must be expressed using exponents. Students often get confused when we start manipulating exponents. So, we need to get some practice converting regular numbers into numbers with exponents, and vice versa. Here's how we do it.

Consider the number 362. We know that we can get this number if we multiply 36.2 × 10. We also get the same number if we multiply 3.62 × 100. Notice that we have moved the decimal place so that there is now only one digit to the left of the decimal. This is exactly how far we will always move the decimal. Now we express the number in terms of powers of 10, like this: (3.62 × 100) is the same as 3.62×10^2. When we express a number in this format, we call it ***scientific notation***.

So we see that the procedure for converting a number into scientific notation is fairly simple. We count how far we need to move the decimal place so that there will be one non-zero digit to the left of the decimal. Then we multiply by a power of 10 that tells us how far we moved the decimal. Let's see an example:

EXERCISE 1.44. Express the following number in scientific notation: 400374.2.

Answer: Begin by counting how far to the left you need to move the decimal in order for there to be exactly one digit to the left of the decimal:

You’ll see that you need to move the decimal place 5 times to the left. If you did not multiply by a power a 10, then you would be making the number much smaller by moving the decimal over 5 places. So, to compensate, multiply the new number by 10^5. Your answer is: **4.003740×10^5**. Make sure that you don’t drop the last zero. It is a significant figure, so it must stay in your answer.

For each problem below, express the number in scientific notation:

1.45. 1435.27 = ____________ 1.46. 243.1 = ____________

1.47. 3801 = ____________ 1.48. 274.30 = ____________

Sometimes, you have a number where there is no decimal place. In cases like this, just place a decimal at the end of the number, and then count how far you have to move the decimal place. Let’s see an example.

EXERCISE 1.49. Express the following number in scientific notation: 4642.

Answer: Place a decimal at the end of the number, and then count how far to the left you need to move the decimal in order for there to be exactly one digit to the left of the decimal:

You’ll see that you need to move the decimal to the left 3 times. So, you need to multiply the new number by 10^3. And your answer is: **4.642×10^3**.

For each problem below, express the number in scientific notation:

1.49. 4,892 = ____________ 1.50. 243 = ____________

1.51. 372,567 = ____________ 1.52. 28,942 = ____________

Now we can answer a question asked in the previous section when we were discussing significant figures. We considered a number like 8500. And we wanted to know how many significant figures it has. If we use the rules that were laid out, we would find that there are two significant figures (the 8 and the 5). But what if you are counting people at a baseball game and you find that there are exactly 8500? How do you indicate that this is not an estimate, and that the zeros are significant figures? If we use scientific notation, this issue goes away. If we want to say that 8500 is just an estimate, rounded to the nearest hundred, then we would say 8.5×10^3. Notice that there are only two significant figures here. But if we want to show that 8500 is an exact number and the zeros are significant, then we would say 8.500×10^3. Notice that this number shows that all four digits are significant. In scientific notation, counting significant figures is easy. You just count every digit. If you go back to the rules outlined in the previous section, you will see why. In scientific notation, the first digit will always be the first significant digit (by definition), and scientific notation always has a decimal place (so you always count all digits until you get to the end of the number).

For each problem below, assume that the zeros at the end are not significant, and express the number in scientific notation:

1.53. 4890 = ________ 1.54. 240 = ________

1.55. 3700 = ________ 1.56. 28,900 = ________

So far, we have seen how to express a large number in scientific notation. Now, we turn our attention to expressing small numbers. Consider the number 0.000043. In order to move the decimal place so that there is one non-zero digit to the left of the decimal, you will have to move the decimal five times *to the right*.

We express this by using a negative exponent: 4.3×10^{-5}. Let's see why. First we need to understand what a negative exponent means. A negative exponent is used for small numbers. We already saw that $1000 = 10^3$. So 1 / 1000 would just be 1 / 10^3. That is where negative exponents come in to the picture:

$$\frac{1}{10^3} = 10^{-3}$$

So, when we use a negative power of 10, we are just saying that number is a small number. In the example above, 4.3×10^{-5} is a small number.

If you want, you can memorize that moving the decimal place to the left gives a positive exponent, and moving the decimal place to the right gives a negative exponent. But this method will confuse you later on when you practice converting from scientific notation back into regular numbers. In that case, everything gets reversed, and you will get confused if you are just memorizing. So, it is much better to think about the size of the number. You can do this with the following example.

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EXERCISE 1.56. Express the following number in scientific notation: 0.008620.

Answer: First count how far you need to move the decimal in order for there to be exactly one non-zero digit to the left of the decimal:

You'll see that you need to move the decimal three times. So, you need to decide whether the exponent will be 3 or −3. Instead of memorizing which way you moved the decimal (left or right), compare your original number (0.008620) to your new number after you move the decimal (8.620). Now, ask yourself which number is larger. Clearly, the new number (8.620) is larger. But the new number can't be different than the old number. When we express a number in scientific notation, we are not changing the number; we are just writing it differently. So we must multiply a power of 10 that makes our new number (8.620) much smaller so that it will be the same as 0.008620. And so, the exponent needs to be negative, which means your answer is: $\mathbf{8.620 \times 10^{-3}}$. Don't forget to keep the zero at the end of the number so that we have the correct number of significant figures

For each problem below, express the number in scientific notation. In each case, you will have to decide whether the exponent should be positive or negative:

1.57. 0.000567 = ____________

1.58. 2400.7 = ____________

1.59. 370.24 = ____________

1.60. 0.00000564 = ____________

1.61. 0.00432 = ____________

1.62. 0.03840 = ____________

Now that you have seen how to convert regular numbers into scientific notation, the reverse process is just the opposite of this. Let's see an example.

EXERCISE 1.63. Consider the following number expressed in scientific notation: 4.634×10^{-3}. Express this number in regular notation.

Answer: Once again, do not memorize which way to move the decimal based on whether the exponent is positive or negative. If you try to memorize, you will get confused because you have to take into account whether you are going from scientific notation to regular notation or vice versa. Let's just figure it out based on the size of the number. 10^{-3} means that the number we are talking about is a small number. Therefore, you need to move the decimal three spaces to the left, which will give you **0.004634**.

Let's double check to make sure this was done correctly. If you had moved the decimal the other way, you would have gotten 4634. That can't be the answer

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because you can see from the negative exponent (4.634×10^{-3}) that the number is a small number.

For each problem below, express the number in regular notation. In each case, you will have to decide which way to move the decimal. Try to figure it out based on analyzing the relative size of the number:

1.64. $2.48 \times 10^3 =$ ____________

1.65. $4.64 \times 10^{-3} =$ ____________

1.66. $7.924 \times 10^{-2} =$ ____________

1.67. $7.924 \times 10^2 =$ ____________

1.68. $8.456 \times 10^{-4} =$ ____________

1.69. $3.84 \times 10^5 =$ ____________

So far, we have seen how to write numbers in scientific notation. Now, we need to see how to multiply numbers that are expressed in scientific notation. Suppose we need to multiply the following $(3.45 \times 10^2)(8.9 \times 10^4)$. The way we do this is by adding the exponents (for a more detailed explanation of this, see Chapter 9), which will give us $3.45 \times 8.9 \times 10^{(2+4)}$. So, we have a power of 10 that is now 10^6. We just added the exponents (2 and 4). But there are a couple of things to be careful about:

- Don't forget about significant figures. You can't ignore everything you learned in the previous section. The first number (3.45) has three significant figures, and the second number (8.9) has only two significant figures. Therefore, your answer can only have two significant figures. When you multiply 3.45×8.9, your calculator says 30.705. But that is too many digits. You must round to two significant figures. And that gives you 31.
- Based on this, you might want to write your answer as 31×10^6. But that is not correct. In scientific notation, there should only be one number to the left of the decimal. So, you need to express your answer as 3.1×10^7. Make sure that you understand why your answer is 10^7 and not 10^5. You should think about it until it makes sense to you. Once again, think about the size of the number that you are talking about. If you make 31 smaller (by turning it into 3.1), then you must make 10^6 bigger in order to compensate.

Let's see another example:

EXERCISE 1.70. Multiply the following numbers:

$$(3.01 \times 10^7)(4.644 \times 10^{-3}) = ???$$

Answer: Once again, just add the exponents, so you have $3.01 \times 4.644 \times 10^{(7-3)}$. The power of 10 is therefore 10^4. Now you need to multiply 3.01×4.644, and your calculator should say 13.97844. Clearly, that is too many significant figures. Your answer can only have three significant figures (because 3.01 only had three significant figures), so round up to 14.0.

So far, you have 14.0×10^4. But you need to change this number so that there is only one number in front of the decimal place. So, your answer is 1.40×10^5.

Let's get some practice. For each of the following problems, multiply the two numbers:

1.71. $(2.43 \times 10^{3})(8.273 \times 10^{6}) =$ ____________

1.72. $(7.89 \times 10^{-12})(4.6 \times 10^{-22}) =$ ____________

1.73. $(4.65 \times 10^{8})(5.432 \times 10^{-4}) =$ ____________

1.74. $(2.59 \times 10^{-3})(2.0 \times 10^{-5}) =$ ____________

1.75. $(9.4 \times 10^{7})(3.45 \times 10^{4}) =$ ____________

We just saw how to multiply two numbers that are expressed in scientific notation. Now we need to see how to divide two numbers. Recall that previously we saw that an exponent in the denominator is the same as a negative exponent:

$$\frac{1}{10^{3}} = 10^{-3}$$

So, instead of adding the exponents, we need to subtract. Let's see an example:

EXERCISE 1.76. Divide the following numbers:

$$\frac{3.64 \times 10^{5}}{4.2 \times 10^{2}}$$

Answer: Subtract the exponents. This gives you $\frac{3.64}{4.2} \times 10^{(5-2)}$. The power of 10 is therefore 10^{3}. Now divide $\frac{3.64}{4.2}$ and your calculator should say 0.866666667. Clearly, that is too many significant figures. Your answer can only have two significant figures (because 4.2 only has two significant figures), so round up to 0.87.

So far, you have 0.87×10^{3}. But you need to change this number so that there is one non-zero number in front of the decimal place. By changing 0.87 into 8.7, you are making a bigger number, so make 10^{3} a smaller number in order to compensate. Your answer is 8.7×10^{2}.

Now let's get some practice dividing with scientific notation. For each of the following problems, divide the two numbers:

1.77. $(2.43 \times 10^{3}) / (8.273 \times 10^{6}) =$ ____________

1.78. $(7.89 \times 10^{-12}) / (4.6 \times 10^{-22}) =$ ____________

1.79. $(4.65 \times 10^{8}) / (5.432 \times 10^{-4}) =$ ____________

1.80. $(2.59 \times 10^{-3}) / (2.0 \times 10^{-5}) =$ ____________

1.81. $(9.4 \times 10^{7}) / (3.45 \times 10^{4}) =$ ____________

1.3 UNITS

In the beginning of this chapter, it was said that there are two parts to every numerical answer: (1) the number, and (2) the units. So far, our discussions have focused on the number. Now we turn our attention to the units.

To express the importance of using units, let's consider the following situation. Suppose you wanted to know how long it took me to write this book. What if I told you: "12". You would probably say, "12 what?" Hours? Days? Months? What if I respond by saying "12 meters." You would wonder what I was talking about, and then you would say, "How long did it take? How much *time*?" Imagine at that point I respond by saying: "Oh, you want my answer to have units of time in it. OK. It took me exactly 12 meters *per second* to write the book." At this point, you would probably not want to talk to me anymore because there is something clearly wrong with my communication skills. I used the wrong units. A number with the wrong units is meaningless. If the answer is supposed to be in units of time, then *only* those units will make sense.

Students are often given a problem where they have to calculate something, like the mass of a compound, and they will write down an answer like this: "12". Without units, your answer is wrong. This may not seem like a big deal, but you must make it part of your habit to consider units as being *just as important* as the numerical part of your answer.

There are a few situations where we deal with unitless numbers in chemistry. For example, equilibrium constants can sometimes have no units. Whenever you come across a concept that involves a number with no units, you should be surprised, and you should try to understand why there are no units attached. For most of the calculations that you will do in this course, your answers must have units.

Now that you see why you must include the correct units in order to communicate properly, it should be pointed out that units can often be used to double check your answer. If you need to calculate a volume, and your answer comes out having units of m^2 instead of m^3, then you will have a clue that you made a mistake somewhere in your calculation. Whenever you calculate an answer to a problem, always look at your units and make sure that they make sense to you.

For any quantity that you measure, you will have a choice of acceptable units that would make sense. For example, if you were measuring the length of an object, you could record your measurement in inches, feet, miles, meters, yards, etc. All of these units correctly represent length. When measuring length, you can theoretically use any one of these units and you would be properly communicating. But having so many different types of units can be confusing for calculations, so to make things easier, the world has agreed upon certain ***base units*** that are the "preferred" units to use. These units are called SI units (*Le Système International d'Unités*), and they go like this:

Measure *length* in meters (m)

Measure *mass* in kilograms (kg)

Measure *time* in seconds (s)

1 long

Measure *temperature* in Kelvin (K)

Measure *amounts* in moles (mol)

There are two other SI base units, but we really don't use them in chemistry (electric current and luminous intensity), so let's just focus on the five above.

You might think that there should be more than just these five base units. What about area or volume? What about pressure? Well, it turns out that anything else you would want to measure will just be some combination of base units. Area is just $m \times m$, or m^2. Volume is just $m \times m \times m$, or m^3. Even energy is measured as a combination of the base units given above. Energy is measured with the following units:

$$\frac{kg \times m^2}{s^2}$$

Later on, we will see why energy is measured in precisely these units. This will make sense to you later. But for now, you should notice that all of the units used to measure energy (kg, m, and s) are just base units of the SI system.

We will need to see how to convert units, and we will need to take a closer look at what it means for a unit to have an exponent (like that shown above in the units for energy). But first, we need to see the prefixes that are often placed in front of the base units. Examples are ***milli***gram, and ***micro***second, etc. These prefixes are called *decimal multipliers*, because they involve the use of powers of 10. For example, a milligram is 10^{-3} grams. And a microsecond is 10^{-6} seconds. Instead of saying 2.4 microseconds, we could say 2.4×10^{-6} seconds, but it is faster and easier to use the term microseconds. There are many decimal multipliers, but only those commonly used by chemists are listed here:

Prefix	***Power of 10***	***Symbol***
kilo	10^3	k
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n

In the chart above, the last column tells you the symbol we use. A nanometer is expressed as nm. A millisecond is expressed as ms. The only symbol you might not be familiar with is μ, which is a Greek letter (*called mu*). A micrometer is expressed as μm.

To make sure that you get comfortable with the decimal multipliers above, let's do some quick and easy problems. Try to do these problems without using the chart. Take a good look at the chart for a few moments, and *then* try the problems. Express your answers using scientific notation:

1.82. 34 km ______________ m

1.83. 2 μs ______________ s

1.84. 482 cm ______________ m

1.85. 24.5 m ______________ dm

1.86. 37.2 nm ______________ m

1.87. 1.487 kg ______________ g

1.88. 2.46 mg ______________ g

1.89. 3.7 ms ______________ s

1 long

As your course moves along, you will generally see problems where the information you get is in SI units. The most common exception will be temperature. You will often be given a temperature in Celsius, and you MUST convert it to Kelvin before you can do your calculation. If you don't do the conversion, you will get the wrong answer (this is a common mistake). So, make sure you know the following conversion: whenever you are given a temperature in °C, you MUST add 273.15 to get the temperature in Kelvin. If you are told that a process takes place at 100° C, then you must convert this to 373 K.

Why didn't we convert this to 373**.15** K? Don't forget about significant figures: 100 + 273.15 = 373. Because the number 100 does not have any decimal places, our answer cannot have any decimal places either. So, 25° C is just 298 K (this is a common temperature in problems). You may as well memorize the conversion number right now (+273.15), because you are going to use it a lot.

1.4 CONVERTING UNITS USING THE FACTOR-LABEL METHOD

We will soon see how to convert any units, but first we need to know how to treat units when doing calculations. Units should be treated the same way that numbers are treated. When we multiply fractions, we just multiply all of the numerators, and we multiply all of the denominators, like this:

$$\frac{3}{5} \times \frac{4}{6} = \frac{3 \times 4}{5 \times 6}$$

If there is a number that is not a fraction, then we treat it like a numerator:

$$\frac{3}{5} \times 4 = \frac{3 \times 4}{5}$$

Units are treated exactly the same way. For example:

$$kg \times \frac{m}{s} \times \frac{m}{s} = \frac{kg \times m \times m}{s \times s}$$

To make our answers shorter, we use exponents to show when we multiply more than one of the same unit. For example:

$$\frac{kg \times m \times m}{s \times s} = \frac{kg \times m^2}{s^2}$$

In the above example we see that m^2 can be used in place of $m \times m$. We did the same thing with $s \times s$. When discussing orders of magnitude earlier in this chapter, we saw that a negative exponent means it is in the denominator. The same is true here, so:

$$s^{-2} \text{ is the same as } \frac{1}{\mathrm{s}^2}$$

Using this concept, we can rewrite the units above, so:

$$\frac{kg \times m^2}{s^2} = kg \times m^2 \times s^{-2}$$

As a side note, we do ***not*** use exponents when we *add or subtract* numbers with units. For example, 42 m + 10 m = 52 m (**not** 52 m^2). We only use m^2 when we are *multiplying* two numbers with units of meters.

Before we move on, let's get some practice. For each of the problems below, multiply all of the units and express them using exponents, just as we did in the example above. Your answer should be written in the following type of format: $kg \times m^2 \times s^{-2}$.

1.90. $\frac{g}{cm \times cm \times cm} =$

1.91. $kg \times \frac{m}{s \times s} =$

1.92. $\frac{mol}{m \times m \times m} =$

1.93. $g \times \frac{cm \times cm}{s \times s} =$

Sometimes, you will find that you have the same units in the denominator and numerator. For example:

$$kg \times \frac{m \times m}{s \times s} \times \frac{1}{m}$$

Notice that we have an m in the numerator and an m in the denominator. Since $\frac{m}{m} =$ 1, we can cross off one m in the numerator and one m in the denominator, like this:

$$kg \times \frac{\cancel{m} \times m}{s \times s} \times \frac{1}{\cancel{m}} = kg \times \frac{m}{s \times s}$$

After crossing off the units, we can then multiply like we did earlier. This gives us $kg \times m \times s^{-2}$.

Now let's get some practice crossing off units. For each of the problems below, cross off the units that appear in both the numerator and the denominator. Then multiply the units and express all together using exponents. Your answer should be written in the following type of format: $kg \times m^2 \times s^{-2}$.

1.94. $\frac{g}{mol} \times \frac{mol}{cm^3} =$

1.95. $g \times \frac{kg}{g} =$

1.96. $\frac{g}{cm^3} \times \frac{cm^3}{m^3} =$

1.97. $\frac{ft}{s} \times \frac{in.}{ft} \times \frac{cm}{in.} =$

1.98. $ft \times \frac{in.}{ft} \times \frac{cm}{in.} =$

1.99. $\frac{g}{mol} \times \frac{kg}{g} =$

Now we can explore the method that chemists use for converting units. For example, you will often have to convert grams into kilograms, or seconds into microseconds. This might seem like a simple thing to do. But as time goes on, you will encounter problems that require you to do conversions that are not so easy to just do in your head. So, let's take a close look at the method.

Our method for converting units is called the *factor-label method*. Sometimes it is referred to as *dimensional analysis,* because the method involves analyzing the dimensions (which is just a fancy term for units). This method is based on two very simple math principles:

1. Any number divided by itself is equal to one. Examples are:

$$\frac{24}{24} = 1 \qquad \frac{3.6}{3.6} = 1 \qquad \frac{\frac{1}{2}}{0.5} = 1$$

Focus carefully on the last example above. Even though we have expressed the same number in two different forms (½ and 0.5), we still get 1 as our answer. This is because ½ = 0.5, so if we divide ½ by 0.5, we just get 1. We can expand this idea to include any two measurements that represent the exact same quantity. For example, a container of 1 dozen eggs will have 12 eggs in the container. Since 1 dozen and 12 are really the same exact number, then we can say that:

$$\frac{1 \text{ dozen eggs}}{12 \text{ eggs}} = 1 \qquad \text{and} \qquad \frac{12 \text{ eggs}}{1 \text{ dozen eggs}} = 1$$

We can also use this concept with our base units to convert prefixes. For example, to show the relationship between grams and kilograms, recall that 1 kg = 1000 g. So:

$$\frac{1 \times 10^3 \text{ g}}{1 \text{ kg}} = 1 \qquad \text{and} \qquad \frac{1 \text{ kg}}{1 \times 10^3 \text{ g}} = 1$$

In each fraction above, the numerator is the exact same number as the denominator. So, each fraction is equal to 1.

We can use this concept to show the relationship between any two units that are related to each other. When comparing miles and feet, we would say that:

$$\frac{1 \text{ mile}}{5280 \text{ feet}} = 1 \qquad \text{and} \qquad \frac{5280 \text{ feet}}{1 \text{ mile}} = 1$$

That was our first simple math principle: any quantity divided by itself is just 1. Now let's turn to our second math principle.

2. You can multiply any number by 1 without changing the number. For example:

$$3 \times 1 = 3$$
$$6.73 \times 1 = 6.73$$

Obviously, this is not rocket science. But if we bring together the two principles we have seen so far, we get a foolproof way to convert units. The method goes like this: we just multiply by some fraction that equals 1. Let's see an example of how this works.

Imagine that you need to measure a distance, and someone tells you that the distance is 34.7 meters. You are not so familiar with meters, so you want to have the answer expressed in feet. So, you need to convert meters into feet. You look in your textbook and you find a list that shows conversions, and you see that 1 ft = 0.3048 m.

We have seen that you can always multiply any number by 1, and we have also seen that another way to write the number 1 is like this:

$$\frac{1\ \text{ft}}{0.3048\ \text{m}} \qquad \text{or} \qquad \frac{0.3048\ \text{m}}{1\ \text{ft}}$$

So, we can multiply our starting number (34.7 meters) by either one of the fractions above without changing the number. Since we want to convert meters into feet, we will need to choose the fraction that has meters in the denominator—that is the only way that units of meters will be crossed off completely:

$$34.7\ \text{m} \times \frac{1\ \text{ft}}{0.3048\ \text{m}}$$

We have not changed our number at all, because we have just multiplied by 1. Now, we can cross off units, because we have meters in the numerator and meters in the denominator:

$$34.7\ \cancel{\text{m}} \times \frac{1\ \text{ft}}{0.3048\ \cancel{\text{m}}} = \frac{34.7 \times 1\ \text{ft}}{0.3048}$$

Now we have our answer in feet. All you have to do is divide 34.7 by 0.3048, and then report your answer with the appropriate significant figures. In this case, we should report three significant figures, so our answer is 144 ft. Notice that our answer is expressed with the appropriate significant figures ***and*** with the appropriate units.

You can use this method to convert any two units for which you know the conversion factor (like we did above, when we knew the conversion factor 1 ft = 0.3048 m). Anytime you do a conversion using this method, you will have *two* choices for which multiplier to use. In the example above, we could have used

$$\frac{1\ \text{ft}}{0.3048\ \text{m}} \qquad \textbf{\textit{or}} \qquad \frac{0.3048\ \text{m}}{1\ \text{ft}}$$

but only the first fraction gave us a meaningful answer. If we had chosen the second fraction, we would have gotten an answer with too many units:

$$34.7\ \text{m} \times \frac{0.3048\ \text{m}}{1\ \text{ft}} = 34.7 \times 0.3048 \times \text{m}^2 \times \text{ft}^{-1}$$

If you look at the units you would get in the above equation, you immediately realize that you chose the wrong multiplier. You only use the second fraction if you want to change from feet to meters instead of meters to feet. After you do a few practice problems, you should get comfortable with quickly choosing which of the two multipliers to use.

Let's start off with some simple problems, and then we'll build up the complexity as we go along. As you start out, you will probably wonder why you need to use this method at all. You will probably say that you could have just gotten the answer by doing the calculation in your head. But as we build the complexity of the problems, you will soon see that it is not always so easy to do the calculations in your head. By using the method given above, you will see that even these more complex problems should still be straightforward to do.

Let's begin with some simple conversions:

EXERCISE 1.100. An object weighs 24.7 lb. What is its mass in kg? Use the following information:

$$1 \text{ lb} = 0.4536 \text{ kg}$$

Answer: You are given the conversion, so you must use one of the following two fractions:

$$\frac{1 \text{ lb}}{0.4536 \text{ kg}} = 1 \qquad \text{and} \qquad \frac{0.4536 \text{ kg}}{1 \text{ lb}} = 1$$

Since you are converting pounds to kilograms, use the second fraction, so that pounds will cross off and you will be left with units of kilograms:

$$24.7 \text{ lb} \times \frac{0.4536 \text{ kg}}{1 \text{ lb}} = 11.20392 \text{ kg}$$

Now, round your answer so that you display the appropriate significant figures. 24.7 has three significant figures, so your answer must also have three significant figures: **11.2 kg**

PROBLEM 1.101. An object weighs 153.2 lb. What is its mass in kg?

Answer: ______________________________

PROBLEM 1.102. An object has a mass of 2.0 kg. What is the weight of the object in pounds?

Answer: ______________________________

PROBLEM 1.103. An object is 54.6 inches tall. Convert the height into centimeters, using the following conversion: 1 in. = 2.54 cm.

Answer: ______________________________

1.5 USING MORE THAN ONE CONVERSION FACTOR

Sometimes, we need to use two or more conversion factors in order to convert our answer into the correct units. Let's see an example:

EXERCISE 1.104. The distance between two points is 1.07 miles. Convert this distance into inches using the following information:

1 mile = 5280 feet 1 foot = 12 inches

Answer: In order to convert miles into inches, you will have to convert miles into feet, and then feet into inches. *But you don't have to do this in two steps*. You can do it all in one step:

$$1.07\ \cancel{\text{miles}} \times \frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mile}}} \times \frac{12\ \text{in.}}{1\ \cancel{\text{ft}}} = 6779.52\ \text{in.}$$

Finally, round your answer to three significant figures, so your answer is: **6780 in.**

The trick here is that you don't have to do the problem in two steps. It is quicker and easier to do everything in one step, using both conversion factors. If you do it right, everything will cross off to give you the units you want.

This becomes important as we move on to problems with *more* than two conversion factors. Sometimes, you will do problems with five or six conversion factors. If you try to do one at a time, it takes too long. So, you need to get accustomed to using multiple conversion factors at the same time. Let's try another problem like Exercise 1.104. Make sure to do your conversions all in one step.

PROBLEM 1.105. The distance between two points is 1.07 miles. Convert this distance into yards using the following information: 1 mile = 1.609 km, and 1 km = 1094 yards.

Answer: ____________________

Sometimes, we have to convert units when exponents are involved. Let's see an example:

EXERCISE 1.106. The volume of an object is 34.0 ft^3 (cubic feet). Convert this into m^3 (cubic meters) using the following information: 1 ft = 0.3048 m.

Answer: Think about what it means when you have an exponent on a unit. ft^3 means ft × ft × ft. This should make sense, because the volume of an object is just (width) × (height) × (length). In order for all of the units of feet to cross off so that you are left with units of meters, you will need to use your conversion factor *three times*:

$$34.0\ \cancel{\text{ft}} \times \cancel{\text{ft}} \times \cancel{\text{ft}} \times \frac{0.3048\ \text{m}}{1\ \cancel{\text{ft}}} \times \frac{0.3048\ \text{m}}{1\ \cancel{\text{ft}}} \times \frac{0.3048\ \text{m}}{1\ \cancel{\text{ft}}}$$

When you calculate your answer and round to three significant figures, you get 0.963 m^3.

PROBLEM 1.107. The volume of an object is 12 cubic meters. Convert this into cubic centimeters.

Answer: ______________________________

In the previous problems, you had to change only one set of units (such as meters to feet). Now, let's consider some problems where you have to change units in the numerator AND in the denominator. Let's see an example:

EXERCISE 1.108. A molecule is traveling at 520 m/s. Convert this speed into miles/hour.

Answer: In this example, you will need to convert the numerator from meters into miles:

$$520\ \frac{\text{m}}{\text{s}} \times \frac{1\ \text{km}}{1000\ \text{m}} \times \frac{1\ \text{mile}}{1.609\ \text{km}}$$

And you will also need to convert the denominator from seconds into hours, using the multipliers:

$$\frac{60\ \text{s}}{1\ \text{min}} \times \frac{60\ \text{min}}{1\ \text{h}}$$

As we said before, always try to do everything in one step. There are four conversion factors here, so you should multiply everything at the same time. When you do this, all of the units cross off except miles/hour:

$$520\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1\ \cancel{\text{km}}}{1000\ \cancel{\text{m}}} \times \frac{1\ \text{mile}}{1.609\ \cancel{\text{km}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}}$$

Round your answer to two significant figures, and you get: 1200 miles/h. Here is a situation where you should use scientific notation in order to be clear about how many significant figures you are talking about. When you made the calculation above, your calculator said 1163.45556, but you had to round up to two significant figures, to 1200. To make it clear that we are talking about two significant figures (and not the exact number 1200, with four significant figures), use scientific notation: 1.2×10^3 miles/h. This makes it clear that there are two significant figures.

1 short

PROBLEM 1.109. An object is traveling at 342.3 meters per second. Convert this speed into miles per hour.

Answer: ______________________________

PROBLEM 1.110. An object is traveling at 22 miles per hour. Convert this speed into kilometers per minute (1 mile = 1.609 km).

Answer: ______________________________

1.6 SPECIAL CONVERSION FACTORS

In the previous section, we saw how to convert units. In all of our examples, we were converting either length into length, or time into time, etc. It makes sense to convert miles into inches, or to convert seconds into years. But does it make any sense to convert *miles* into *years*? The units of miles and the units of years are measuring two completely different properties: length and time. It turns out that physicists do in fact convert these units. Let's see how.

Length and time are actually intimately related to each other through the special theory of relativity. Now, we don't want to get into relativity here, because this is not a physics course, but on a superficial level, we can see the relationship between length and time if we just consider measurements that we regularly make. When we measure velocity, we are measuring the *distance* traveled per unit of *time*. Now, here comes the trick: if you have something that travels at constant velocity, such as light, then you can measure *how far* the light travels *in terms of the time* it takes to travel that far. When physicists talk about a "light year", they are not referring to a unit of time. Rather, they are referring to a distance—the distance that light travels in one year. When they say that a galaxy is one million *light years* away from us, they are talking about its *distance* from us.

In chemistry we also have terms that relate one set of units to another set of units. There are a few of these, and if you understand them, then you will have unlocked the keys to solving many problems. Since they are so important for solving problems, we will focus on them throughout this book. In this section, we will focus on one such special conversion factor: *density*.

The density of a substance measures the *mass* that is contained in a certain *volume*. The units are usually in g/cm^3. Another way of saying cm^3 is milliliter (mL). So, the units of density can also be expressed as g/mL. Before we can see how to use density as a special conversion factor, let's first get some practice calculating densities.

EXERCISE 1.111. A substance has a mass of 1.62×10^3 mg, and is contained in a cube with the following dimensions: 21.3 mm × 10.7 mm, × 12.0 mm. Calculate the density in g/cm^3.

1 short

Answer: You are asked to calculate the density, and you are given the mass and the volume. So, you just need to divide the mass by the volume. BUT, you were asked to give your answer in units of g/cm^3. So, first you must convert mg into g:

$$1.62 \times 10^3 \text{ mg} \times \frac{1 \text{ g}}{1000 \text{ mg}} = 1.62 \text{ g}$$

and then you must convert mm into cm:

$$21.3 \text{ mm} \times \frac{1 \text{ cm}}{10 \text{ mm}} = 2.13 \text{ cm}$$

$$10.7 \text{ mm} \times \frac{1 \text{ cm}}{10 \text{ mm}} = 1.07 \text{ cm}$$

$$12.0 \text{ mm} \times \frac{1 \text{ cm}}{10 \text{ mm}} = 1.20 \text{ cm}$$

Now, you can solve for your answer:

$$\text{Density} = \frac{1.62 \text{ g}}{2.13 \text{ cm} \times 1.07 \text{ cm} \times 1.20 \text{ cm}} = 0.592 \text{ g/cm}^3$$

Notice that the answer was recorded with three significant figures.

PROBLEM 1.112. A substance has a mass of 0.012 kg, and is contained in a volume with the following dimensions: 25.4 mm × 12.3 mm, × 13.4 mm. Calculate the density in g/cm^3 (and don't forget about significant figures).

Answer: ____

Now that we have seen how to calculate the density of a substance, we are ready to see how it can be used as a special conversion factor. Density tells us how much mass can be found in a particular volume. So, we can use this to convert from mass to volume, and vice versa. Let's say we know the density of a substance at a certain temperature, then we should be able to calculate the volume if we are given the mass. Similarly, we should be able to calculate the mass if we are given the volume. We do this in exactly the same way that we converted units in the previous section of this chapter. Let's see an example of how this works:

EXERCISE 1.113. At 25° C, the density of water is 1.00 g/mL. What is the mass in grams of 1.78 gallons of water? (1 gallon = 3785 mL).

Answer: Before you get started, note that you will need to convert gallons into milliliters, because your final answer must be expressed in terms of milliliters. So, you need to use the conversion given above, which is:

$$1.78 \text{ gallons} \times \frac{3785 \text{ mL}}{1 \text{ gallon}}$$

Now you are ready to do your calculation. You are given the density of water at room temperature. So, you know the relationship between mass and volume. You can treat this in exactly the same way that you did when you were converting units in the previous section. The density of water tells you that 1.00 g = 1 mL. Therefore, you can set up two fractions that are both equal to 1, just as you did before:

$$\frac{1.00\text{ g}}{1\text{ mL}} = 1 \qquad \text{and} \qquad \frac{1\text{ mL}}{1.00\text{ g}} = 1$$

In the fractions above, 1 mL is treated as an exact number rather than a number with one significant figure because we define density as the amount of grams in exactly 1 mL of volume. So, you can treat the number 1 mL as having an infinite amount of significant figures.

Now you just need to multiply the volume (that was converted to milliliters) by one of the fractions above. Since you want the units of volume to cancel off, use the first fraction:

$$1.78\text{ gallons} \times \frac{3785\text{ mL}}{1\text{ gallon}} \times \frac{1.00\text{ g}}{1\text{ mL}} = 6740\text{ g}$$

Notice that you record your answer with three significant figures, which makes the answer 6740 g. In this example, you have to use scientific notation to eliminate any ambiguity regarding how many significant figures you have. The answer is expressed as: **6.74×10^3 g**

Before we conclude the chapter, there are two subtle points that should be mentioned. When we use density as a conversion factor, we need to be careful of two things. There are two major differences between the conversions we saw in the previous section and the conversion we are doing here with density:

1. We need to consider the role of significant figures. Unlike the regular conversion factors, this special conversion factor (density) must be reported with the correct number of significant figures. To see what we mean by this, let's consider a regular conversion factor. When we say that 1 kg = 1000 g, we are really saying that

   $$1.00000000\ldots\text{ kg} = 1000.00000\ldots\text{ g}$$

   In other words, when we talk about this conversion (1 kg = 1000 g), we do not mean to imply that the term 1 kg has only one significant figure. It actually has an infinite number of significant figures. But, when we say that the density of an object is 0.592 g/cm^3, we are saying that we only know the value of this conversion factor to three significant figures. Whenever you deal with density (whether you are calculating a density or using it as a special conversion factor), you should take special care to follow the rules of significant figures.
2. When we deal with regular conversion factors, we are dealing with fundamental relationships that are always true. For example, consider this regular

conversion factor: 1 kg = 1000 g. This relationship is not dependent on anything else. It is true under any conditions. But when we say that 1.00 g = 1 mL, we are talking about a relationship that is dependent on many factors, such as temperature and pressure. Therefore, this relationship only holds at a specific temperature and pressure. If we change the temperature and pressure, then the density will change as well.

Let's do two final problems that incorporate everything we have seen in this chapter.

PROBLEM 1.114. At a certain temperature, the density of water is 0.9978 g/mL. What is the mass in grams of 1.782 gallons of water at this temperature? (1 gallon = 3785 mL). Report your answer using scientific notation, and make sure to use appropriate significant figures.

Answer: ______________________________

PROBLEM 1.115. At 20° C, the density of water is 0.9978 g/mL. What volume will be occupied by 1.75 kg of water at this temperature? Report your answer using scientific notation, and make sure to use appropriate significant figures.

Answer: ______________________________

QU1
Number 1.32 repeated. OK?