Chapter 1

Trouncing Trig Technicalities

In This Chapter

- Understanding what trigonometry is
- Speaking the language of trig
- Putting it all into equations
- Graphing for understanding

How did Columbus find his way across the Atlantic Ocean? How did the Egyptians build the pyramids? How did early astronomers measure the distance to the moon? No, Columbus didn't follow a yellow brick road. No, the Egyptians didn't have Legos instructions. And, no, there isn't a tape measure long enough to get to the moon. The common answer here is trigonometry.

Trigonometry is the study of angles and triangles and the wonderful things that you can do with them. For centuries, humans have been able to measure distances that they can't reach because of the power of this mathematical subject.

Taking Trig for a Ride: What Trig Is

"What's your angle?" That question isn't a come-on such as "What's your astrological sign?" In trigonometry, you measure angles in both degrees and radians. You can shove them into triangles and circles and make them do special things. Actually, angles drive trigonometry. Sure, you have to consider algebra and other math. But you can't have trigonometry without angles. Put an angle into a trig function, and out pops a special, unique number. What do you do with that number? Read on, because that's what trig is all about.

Sizing up the basic figures

Segments, rays, and lines are some of the basic forms in geometry, and they're almost as important in trigonometry. As I explain in the following sections, you use those segments, rays, and lines to form angles.

Drawing segments, rays, and lines

A *segment* is a straight figure drawn between two endpoints. You usually name it by its endpoints, which you indicate by capital letters. Sometimes, a single letter names a segment, but a lowercase letter usually refers to an angle opposite that segment.

A *ray* is another straight figure that has an endpoint on one end, and then it just keeps going forever in some specified direction. You name rays by their endpoint first and then by any other point that lies on the ray.

A *line* is a straight figure that goes forever and ever in either direction. You only need two points to determine a particular line — and only one line can go through both of those points. You can name a line by any two points that lie on it.

Figure 1-1 shows a segment, ray, and line.

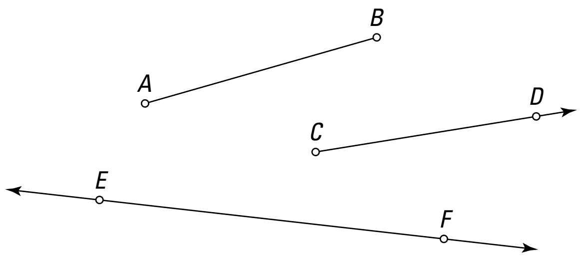

Figure 1-1: Segment *AB,* ray *CD,* and line *EF.*

Intersecting lines

When two lines intersect — if they do intersect — they can only do so at one point. They can't double back and cross one another again. Some curious things happen when two lines intersect. The angles that form between those two lines are related to one another. Any two angles that are next to one another and share a side are called *adjacent* angles. In Figure 1-2, the lines *AB* and *CD* intersect at point *E.* The two angles below the line *CD* (numbered 1 and 2) are adjacent to one another. So are the two angles to the right of line *AB* (numbered 2 and 3), the angles to the left of line *AB* (numbered 1 and 4), and the angles above line *CD* (numbered 3 and 4). So this intersection has four different pairs of adjacent angles.

The angles that are opposite one another when two lines intersect also have a special name. Mathematicians call these angles *vertical* angles. They don't have a side in common. You can find two pairs of vertical angles in Figure 1-2, the pair of angles to the left and right (numbered 1 and 3), and the pair above and below (numbered 2 and 4).

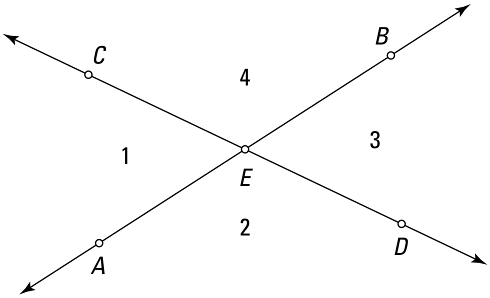

Figure 1-2: Intersecting lines form adjacent and supplementary angles.

Why are these different angles so special? They're different because of how they interact with one another. The adjacent angles here are called *supplementary* angles. The sides that they don't share form a straight line, which has a measure of 180 degrees. The vertical angles are always equal in measure.

Angling for position

When two lines, segments, or rays touch or cross one another, they form an angle. In the case of two intersecting lines, the result is four different angles. When two segments intersect, they can form one, two, or four angles, depending on how they touch, as you can see in Figure 1-3. The same goes for two rays.

These examples are just some of the ways that you can form angles. Geometry, for example, describes an angle as being created when two rays have a common endpoint. In practical terms, you can form an angle in many ways, from many figures. The business with the two rays means that you can extend the two sides of an angle out farther to help with measurements, calculations, and practical problems.

You refer to the parts of all angles in the same way. The place where the lines, segments, or rays cross is called the *vertex* of the angle. From the vertex, two sides extend.

Naming angles by size

You can name or categorize angles based on their size or measurement in degrees (see Figure 1-4):

- **Acute:** An angle measuring between 0 and 90 degrees
- **Obtuse:** An angle measuring between 90 and 180 degrees
- **Right:** An angle measuring exactly 90 degrees
- **Straight:** An angle measuring exactly 180 degrees (a straight line)

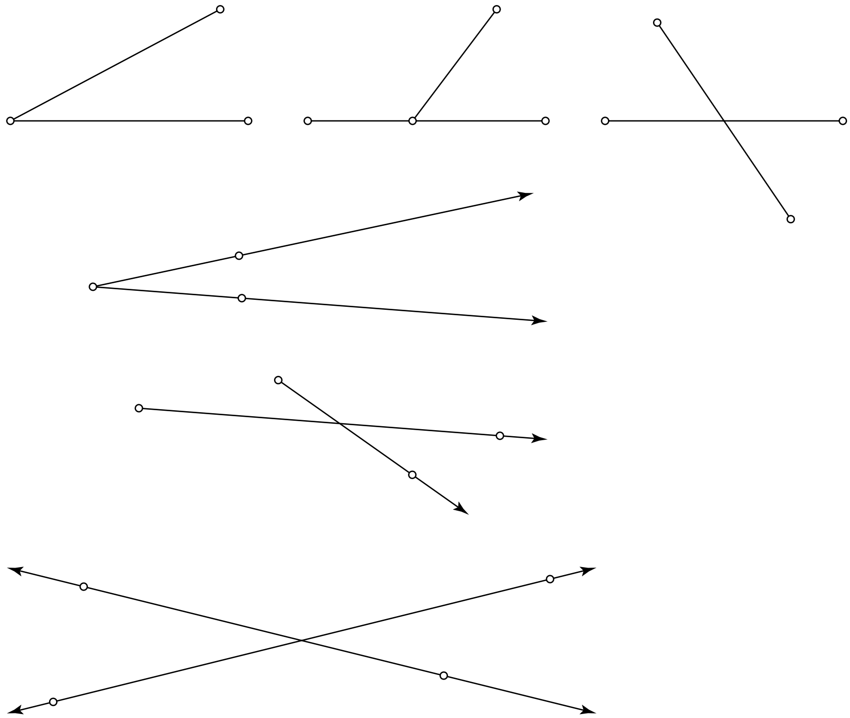

Figure 1-3: Different ways of creating angles.

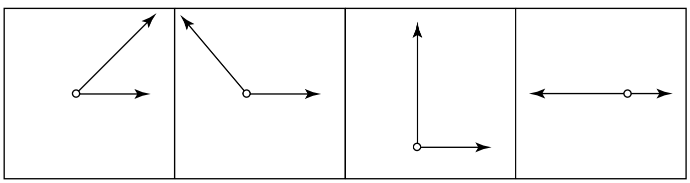

Figure 1-4: Types of angles — acute, obtuse, right, and straight.

Naming angles by letters

How do you name an angle? Why does it even need a name? In most cases, you want to be able to distinguish a particular angle from all the others in a picture. When you look at a photo in a newspaper, you want to know the names of the different people and be able to point them out. With angles, you should feel the same way.

You can name an angle in one of three different ways:

- **By its vertex alone:** Often, you name an angle by its vertex alone because such a label is efficient, neat, and easy to read. In Figure 1-5, you can call the angle *A*.
- **By a point on one side, followed by the vertex, and then a point on the other side:** For example, you can call the angle in Figure 1-5 angle *BAC* or angle *CAB.* This naming method is helpful if someone may be confused as to which angle you're referring to in a picture. ***Remember:*** Make sure you always name the vertex in the middle.
- **By a letter or number written inside the angle:** Usually, that letter is Greek; in Figure 1-5, however, the angle has the letter *w.* Often, you use a number for simplicity if you're not into Greek letters or if you're going to compare different angles later.

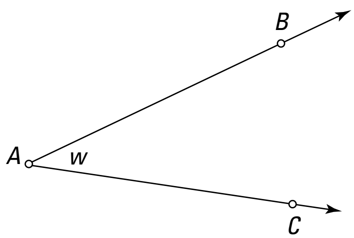

Figure 1-5: Naming an angle.

Triangulating your position

All on their own, angles are certainly very exciting. But put them into a triangle, and you've got icing on the cake. Triangles are one of the most frequently studied geometric figures. The angles that make up the triangle give them many of their characteristics.

Angles in triangles

A triangle always has three angles. The angles in a triangle have measures that always add up to 180 degrees — no more, no less. A triangle named *ABC* has angles *A, B,* and *C,* and you can name the sides *AB, BC,* and *AC,* depending on which two angles the side is between. The angles themselves can be acute, obtuse, or right. If the triangle has either an obtuse or right angle, then the other two angles have to be acute.

Naming triangles by their shape

Triangles have special names based on their angles and sides. They can also have more than one name — a triangle can be both acute and isosceles, for example. Here are their descriptions, and check out Figure 1-6 for the pictures:

- **Acute triangle:** A triangle where all three angles are acute
- **Right triangle:** A triangle with a right angle (the other two must be acute)
- **Obtuse triangle:** A triangle with an obtuse angle (the other two must be acute)
- **Isosceles triangle:** A triangle with two angles that have the same measure; the lengths of the sides opposite those angles are equal, too
- **Equilateral triangle:** A triangle where all three angles measure 60 degrees; all the lengths of the sides are equal, too
- **Scalene triangle:** A triangle with no angles or sides of the same measure

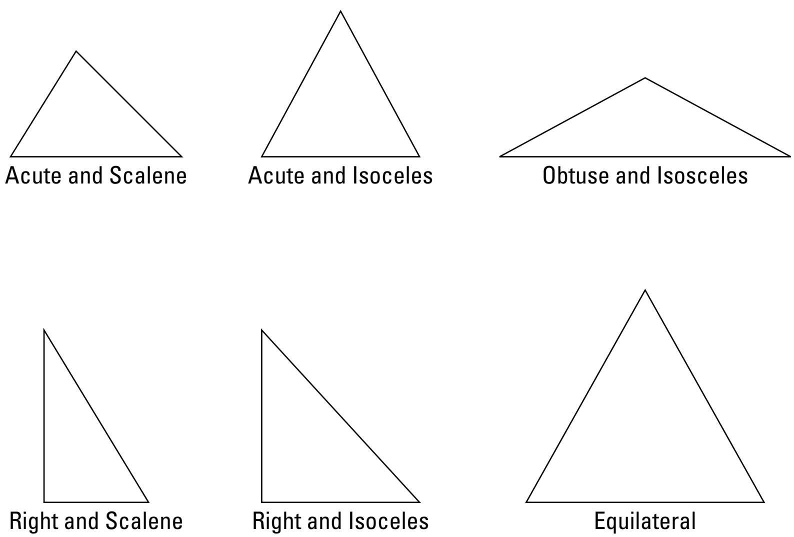

Figure 1-6: Triangles can have more than one name, based on their characteristics.

Circling the wagons

A *circle* is a geometric figure that needs only two parts to identify it and classify it by size: its *center* (or middle) and its *radius* (the distance from the center to any point on the circle). Technically, the center isn't a part of the circle, it's just a sort of anchor or reference point. The circle consists only of all those points that are the same distance from the center.

Radius, diameter, circumference, and area

After you've chosen a point to be the center of a circle and know how far that point is from all the points that lie on the circle, you can draw a fairly decent picture. With the measure of the radius, you can tell a lot about the circle: its *diameter* (the distance from one side to the other, passing through the center), its *circumference* (how far around it is), and its *area* (how many square inches, feet, yards, meters — what have you — fit into it). Figure 1-7 shows these features.

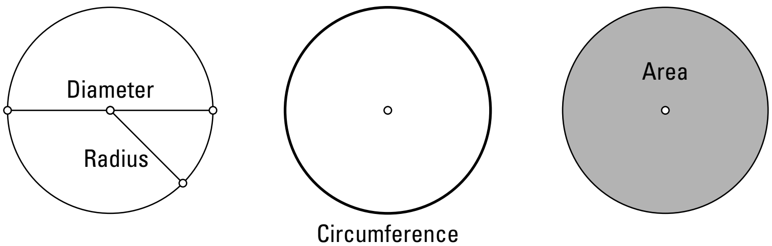

Figure 1-7: The different features of a circle.

Ancient mathematicians figured out that the circumference of a circle is always a little more than three times the diameter of a circle. Since then, they narrowed that "little more than three times" to a value called *pi* (pronounced "pie"), designated by the Greek letter π. The decimal value of *pi* isn't exact — it goes on forever and ever, but most of the time, people refer to it as being approximately 3.14 or $\frac{22}{7}$, whichever form works best in specific computations.

The formula for figuring out the circumference of a circle is tied to π and the diameter.

Circumference of a circle: $C = \pi d = 2\pi r$

The *d* represents the measure of the diameter, and *r* represents the measure of the radius. The diameter is always twice the radius, so either form of the equation works.

Similarly, the formula for the area of a circle is tied to π and the radius.

Area of a circle: $A = \pi r^2$

This formula reads, "Area equals *pi* are squared." And all this time, I thought that pies are round.

Don't give me that *jiva*

The ancient Greek mathematician Ptolemy was born some time at the end of the first century. Ptolemy based his version of trigonometry on the relationships between the chords of circles and the corresponding central angles of those chords. Ptolemy came up with a theorem involving four-sided figures that you can construct with the chords. In the meantime, mathematicians in India decided to use the measure of *half* a chord and *half* the angle to try to figure out these relationships. Drawing a radius from the center of a circle through the middle of a chord (halving it) forms a right angle, which is important in the definitions of the trig functions. These half-measures were the beginning of the sine function in trigonometry. In fact, the word *sine* actually comes from the Hindu name *jiva.*

Example: Find the radius, circumference, and area of a circle if its diameter is equal to 10 feet in length.

If the diameter *(d)* is equal to 10, you write this value as $d = 10$. The radius is half the diameter, so the radius is 5 feet, or $r = 5$. You can find the circumference by using the formula $C = \pi d = \pi \cdot 10 \approx 3.14 \cdot 10 = 31.4$. So the circumference is about 31½ feet around. You find the area by using the formula $A = \pi r^2 = \pi \cdot 5^2 = \pi \cdot 25 \approx 3.14 \cdot 25 = 78.50$, so the area is about 78½ square feet.

Chord versus tangent

You show the diameter and radius of a circle by drawing segments from a point on the circle either to or through the center of the circle. But two other straight figures have a place on a circle. One of these figures is called a chord, and the other figure is a tangent.

Chords of a circle

A *chord* of a circle is a segment that you draw from one point on the circle to another point on the circle (see Figure 1-8). This segment always stays inside the circle. The largest chord possible is the diameter — you can't get any longer than that segment.

Tangents to a circle

A *tangent* to a circle is a line, ray, or segment that touches the outside of the circle in exactly one point, as in Figure 1-9. It never crosses into the circle. A tangent can't be a chord, because a chord touches a circle in two points, crossing through the inside of the circle.

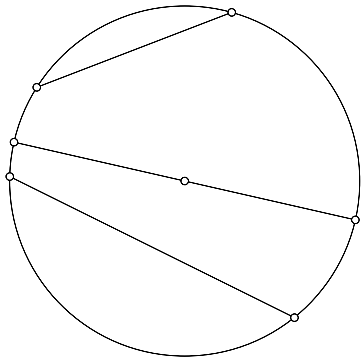

Figure 1-8: Chords of a circle join two points on the circle.

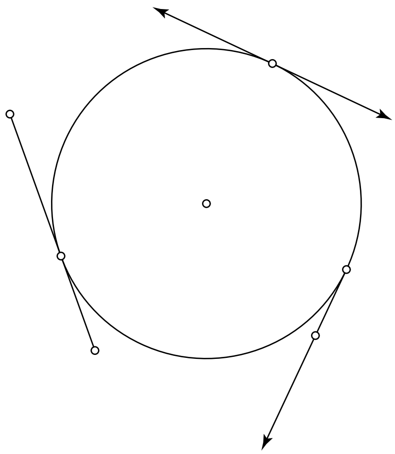

Figure 1-9: Tangents drawn to a circle.

Sectioning sectors

A *sector* of a circle is a section of the circle between two *radii* (plural for radius). You can consider this part like a piece of pie cut from a circular pie plate (see Figure 1-10).

You can find the area of a sector of a circle if you know the angle between the two radii. A circle has a total of 360 degrees all the way around the center, so if a sector has an angle measure of 60 degrees between the two radii, the sector takes up 60/360, or $\frac{1}{6}$, of the degrees all the way around. In that case, the sector has $\frac{1}{6}$ the area of the whole circle.

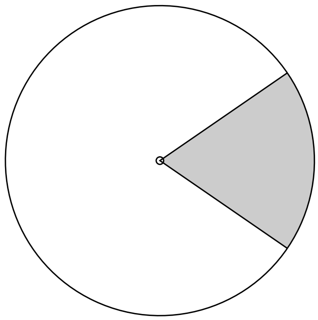

Figure 1-10: A sector of a circle.

Example: Find the area of a sector of a circle if the angle between the two radii forming the sector is 80 degrees and the radius of the circle is 9 inches.

1. **Find the area of the circle.**

   The area of the whole circle is $A = \pi r^2 = \pi \cdot (4.5)^2 \approx 3.14 \cdot 20.25 = 63.585$, or about 63½ square inches.

2. **Find the portion of the circle that the sector represents.**

   The sector takes up only 80 degrees of the circle. Divide 80 by 360 to get

   $$\frac{80}{360} = \frac{2}{9} \approx 0.222$$

3. **Calculate the area of the sector.**

   Multiply the fraction or decimal from Step 2 by the total area to get the area of the sector: $0.222 \cdot 63.585 \approx 14.116$. The whole circle has an area of almost 64 square inches, but the sector has an area of just over 14 square inches.

Understanding Trig Speak

Any math or science topic has its own unique vocabulary. Some very nice everyday words have special meanings when used in the context of that subject. Trigonometry is no exception.

Defining trig functions

Every triangle has six parts: three sides and three angles. If you measure the sides and then pair up those measurements (taking two at a time), you have three different pairings. Do division problems with the pairings — changing the order in each pair — and you have six different answers. These six different answers represent the six trig functions. For example, if your triangle has sides measuring 3, 4, and 5, then the six divisions are 3/4, 4/3, 3/5, 5/3, 4/5, and 5/4. In Chapter 7, you find out how all these fractions work in the world of trig functions by using the different sides of a right triangle. And then, in Chapter 8, you take a whole different approach as you discover how to define the trig functions with a circle.

The six trig functions are named *sine, cosine, tangent, cotangent, secant,* and *cosecant.* Many people confuse the spoken word *sine* with *sign* — you can't really tell the difference when you hear it unless you're careful with the context. You can "go off on a tangent" in some personal dealings, but that phrase has a whole different meaning in trig. Cosigning a loan isn't what trig has in mind, either. The other three ratios are special to trig speak. You can't confuse them with anything else.

Interpreting trig abbreviations

REMEMBER

Even though the word *sine* isn't all that long, you have a three-letter abbreviation for this trig function and all the others. Mathematicians find using abbreviations easier, and those versions fit better on calculator keys. The functions and their abbreviations are

- sine → sin
- cosine → cos
- tangent → tan
- cotangent → cot
- secant → sec
- cosecant → csc

As you can see, the first three letters in the full name make up the abbreviations, except for cosecant's.

Noting notation

Angles are the main focus in trigonometry, and you often don't know their measure. Many angles and their angle measures have general rules that apply to them. You can name angles by one letter, three letters, or a number, but to do trig problems and computations, mathematicians commonly refer to the angle measures with Greek letters.

The most commonly used letters for angle measures are α (alpha), β (beta), γ (gamma), and θ (theta). Also, many equations use the variable x to represent an angle measure.

Algebra has conventional notation involving superscripts, such as the 2 in x^2. In trigonometry, superscripts have the same rules and characteristics as in other mathematics. But trig superscripts often look very different. Table 1-1 presents a listing of many of the ways that trig uses superscripts.

Table 1-1 How You Use Superscripts in Trig

How to Write in Trig Notation	*What the Superscript Means*
$\sin^2 \theta$	$(\sin \theta)^2$
$\sin^{-1} \theta$	$\arcsin \theta$

The first entry in Table 1-1 shows how you can save having to write parentheses every time you want to raise a trig function to a power. This notation is neat and efficient, but it can be confusing if you don't know the "code." The second entry shows you how to write the reciprocal of a trig function. It means you should take the value of the function and divide it into the number 1. The last entry in Table 1-1 shows how you write the *inverse sine* function. Using the –1 superscript right after sine means that you're talking about inverse sine (or *arcsin*), not the reciprocal of the function. In Chapter 13, I cover the inverse trig functions in great detail, making this business about the notation for an inverse trig function even more clear.

Functioning with angles

The functions in algebra use many operations and symbols that are different from the add, subtract, multiply, and divide signs in arithmetic. For example, take a look at the square-root operation, $\sqrt{25} = 5$. Putting 25 under the *radical* (square-root symbol) produces an answer of 5. Other operations in algebra, such as absolute value, factorial, and step-function, are used in trigonometry, too. But the world of trig expands the horizon, introducing even more different processes. When working with trig functions, you have a whole new set of values to learn or find. For instance, putting 25 into the sine function looks like this: sin 25. The answer that pops out is either 0.423 or –0.132, depending on whether you're using degrees or radians (for more on those two important trig concepts, head on over to Chapters 4 and 5). You can't usually determine or memorize the values that you get by putting angle measures into trig functions very easily. So you need trig tables of values or scientific calculators to study trigonometry.

In general, when you apply a trig function to an angle measure, you get some real number. Some angles and trig functions have nice values, but most don't. Table 1-2 shows the trig functions for a 30-degree angle.

Table 1-2 The Trig Functions for a 30-Degree Angle

Trig Function	*Value Rounded to Three Decimal Places*
sin 30°	0.500
cos 30°	0.866
tan 30°	0.577
cot 30°	1.732
sec 30°	1.155
csc 30°	2.000

Some characteristics that the entries in Table 1-2 confirm are that the sine and cosine functions always have values that are between and including –1 and 1. Also, the secant and cosecant functions always have values that are equal to or greater than 1 or equal to or less than –1. I discuss these properties in more detail in Chapter 7.

Using the table in the Appendix, you can find more values of trig functions for particular angle measures (in degrees):

$$\tan 45^\circ = 1$$

$$\csc 90^\circ = 1$$

$$\sec 60^\circ = 2$$

I chose these sample values so the answers look nice and whole. Remember that most angles and most functions look much messier than these examples.

Taming the radicals

A *radical* is a mathematical symbol that means, "Find the number that multiplies itself by itself one or more times to give you the number under the radical." You can see why you use a symbol such as $\sqrt{\ }$ rather than all those words. Radicals represent functions that are used a lot in trigonometry. In Chapter 7, I define the trig functions by using a right triangle. To solve for the lengths of a right triangle's sides by using the Pythagorean theorem, you have to compute some square roots, which use radicals. Some basic answers to radical expressions are $\sqrt{16} = 4, \sqrt{121} = 11, \sqrt[3]{8} = 2, \sqrt[4]{729} = 9$.

These examples are all *perfect squares, perfect cubes,* or *perfect fourth roots,* which means that the answer is a number that ends — the decimal doesn't go on forever. The following section discusses a way to simplify radicals that aren't perfect roots.

Simplifying radical forms

Simplifying a radical form means to rewrite it with a smaller number under the radical — if possible. You can simplify this form only if the number under the radical has a perfect square or perfect cube (or perfect whatever factor) that you can factor out.

Example: Simplify $\sqrt{80}$.

The number 80 isn't a perfect square, but one of its factors, 16, is. You can write the number 80 as the product of 16 and 5, write the two radicals separately, and then evaluate each radical. The resulting product is the simplified form:

$$\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$$

Example: Simplify $\sqrt[3]{250}$.

The number 250 isn't a perfect cube, but one of its factors, 125, is. Write 250 as the product of 125 and 2; separate, evaluate, and write the simplified product:

$$\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = \sqrt[3]{125}\sqrt[3]{2} = 5\sqrt[3]{2}$$

Approximating answers

As wonderful as a simplified radical is, and as useful as it is when you're doing further computations in math, sometimes you just need to know about how much the value's worth.

Approximating an answer means to shorten the actual value in terms of the number of decimal places. You may find approximating especially useful when the decimal value of a number goes on forever without ending or repeating. When you approximate an answer, you *round* it to a certain number of decimal places, letting the rest of the decimal values drop off. Before doing that, though, you need to consider how big a value you're dropping off. If the numbers that you're dropping off start with a 5 or greater, then bump up the last digit that you leave on by 1. If what you're dropping off begins with a 4 or less, then just leave the last remaining digit alone.

They called this simpler?

Some ancient mathematicians didn't like to write fractions unless they had a numerator of 1. They only liked the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and so on. So what did they do when they needed to write the fraction $\frac{5}{6}$? They wrote $\frac{1}{2} + \frac{1}{3}$ instead (because $\frac{1}{2} + \frac{1}{3}$ is equal to $\frac{5}{6}$). What a pain to have to write $\frac{1}{2} + \frac{1}{4} + \frac{1}{10}$ rather than $\frac{17}{20}$. Or maybe you prefer this approach, too?

Example: Round the number 3.141592654 to four decimal places, three decimal places, and two decimal places.

- **Four decimal places:** This rounding value means that the 3.1415 stays. Because you get to drop off the 92654, and 9 is the first digit of those dropped numbers, bump up the last digit that you're keeping (the 5) to 6. Rounded to four places, 3.141592654 rounds to 3.1416.
- **Three decimal places:** The 3.141 stays. Because you drop off the 592654, and 5 is the first digit of those numbers, bump up the last digit that you're keeping (the 1) to 2. Rounded to three places, 3.141592654 rounds to 3.142.
- **Two decimal places:** The 3.14 stays. Because the 1592654 drops off, and 1 is the first digit of those numbers, then the last digit that you're keeping (the 4) stays the same. Rounded to two places, 3.141592654 rounds to 3.14.

Use this technique when approximating radical values. Using a calculator, the decimal value of $\sqrt{80}$ is about 8.94427191. Depending on what you're using this value for, you may want to round it to two, three, four, or more decimal places. Rounded to three decimal places, this number is 8.944.

Equating and Identifying

Trigonometry has the answers to so many questions in engineering, navigation, and science. The ancient astronomers, engineers, farmers, and sailors didn't have the current systems of symbolic algebra and trigonometry to solve their problems, but they did well and set the scene for later mathematical developments. People today benefit big-time by having ways to solve equations in trigonometry that are quick and efficient, including special techniques and trig identities to fool around with that mathematicians in days of old have already worked out.

The methods that you use for solving equations in algebra take a completely different turn when you use trig *identities* (in short, equivalences that you can substitute into equations in order to simplify them). To make matters easier (or, some say, to *complicate* them), the different trig functions can have many different identities. They almost have split personalities. When you're solving trig equations and trig identities, you're sort of like a detective working your way through to substitute, simplify, and solve. What answer should you expect when solving the equations? Why, angles, of course!

Take, for example, one trigonometric equation: $\sin\theta + \cos^2\theta = 1$.

The point of the problem is to figure out what angle or angles should replace the θ to make the equation true. In this case, if θ is 0 degrees, 90 degrees, or 180 degrees, for example, the equation is true.

If you replace θ by 0 degrees in the equation, then you get

$$\begin{aligned} \sin 0^\circ + \left(\cos 0^\circ\right)^2 &= 1 \\ 0 + (1)^2 &= 1 \\ 1 &= 1 \end{aligned}$$

If you replace θ by 90 degrees in the equation, then you get

$$\begin{aligned} \sin 90^\circ + \left(\cos 90^\circ\right)^2 &= 1 \\ 1 + (0)^2 &= 1 \\ 1 &= 1 \end{aligned}$$

Something similar happens with 180 degrees and all the other angle measures that work in this equation. But remember that not just any angle will work here. I carefully chose the angles that are *solutions,* which are the angles that make the equation true. In order to solve trig equations like this one, you have to use inverse trig functions, trig identities, and algebra techniques. You can find all the details on how to use these processes in Chapters 10 through 13. And when you've got those parts figured out, dive into Chapter 14, where the equation-solving comes in.

In this particular case, you use an *identity* to solve the equation for all its answers. You replace the $cos^2\theta$ by $1 - \sin^2\theta$ so that all the terms have a sine in them — or just a number. You actually have other ways to change the identity of $cos^2\theta$, too. I chose $1 - \sin^2\theta$, but other choices include $\frac{1}{\sec^2\theta}$ and $\frac{1+\cos 2\theta}{2}$. Discover how to actually solve equations like this one in Chapter 14.

This example just shows you that the identity of the trig functions can change an expression significantly — according to some very strict rules.

Graphing for Gold

The trig functions have distinctive graphs that you can use to help understand their values over certain intervals and in particular applications. In this section, I describe the axes and show you six basic graphs.

Describing graphing scales

You use the *coordinate plane* for graphing in algebra, geometry, and other mathematical topics. The *x*-axis goes left and right, and the *y*-axis goes up and down. You can also use the coordinate plane in trigonometry, with a little added twist.

The *x*-axis in a trig sketch has tick marks that can represent both numbers (either positive or negative) and angle measures (either in degrees or radians). You usually want the horizontal and vertical tick marks to have the same distance between them. To make equivalent marks on the *x*-axis in degrees, figure that every 90 degrees is about 1.6 units (the same units that you're using on the vertical axis). These *units* represent numbers in the real-number system. This conversion method works because of the relationship between degree measure and radian measure. Check out the method used to do the computation for this conversion in Chapter 5.

Recognizing basic graphs

The graphs of the trig functions have many similarities and many differences. The graphs of the sine and cosine look very much alike, as do the tangent and cotangent, and then the secant and cosecant. But those three groupings look different from one another. The one characteristic that ties them all together is the fact that they're *periodic,* meaning they repeat the same curve or pattern over and over again, in either direction along the *x*-axis. Check out Figures 1-11 through 1-16 to see for yourself.

I say a lot more in this book about the trig-function graphs, and you can find that discussion in Chapters 16, 17, 18, and 19.

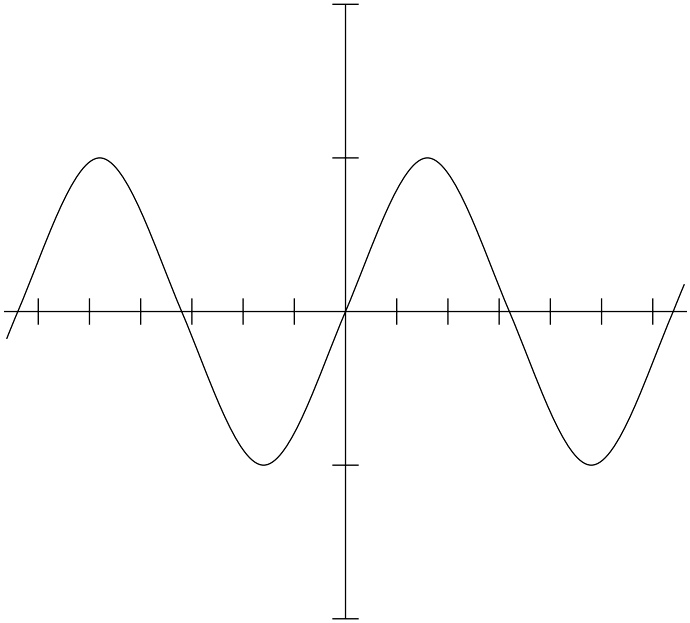

Figure 1-11: The graph of $y = \sin x$.

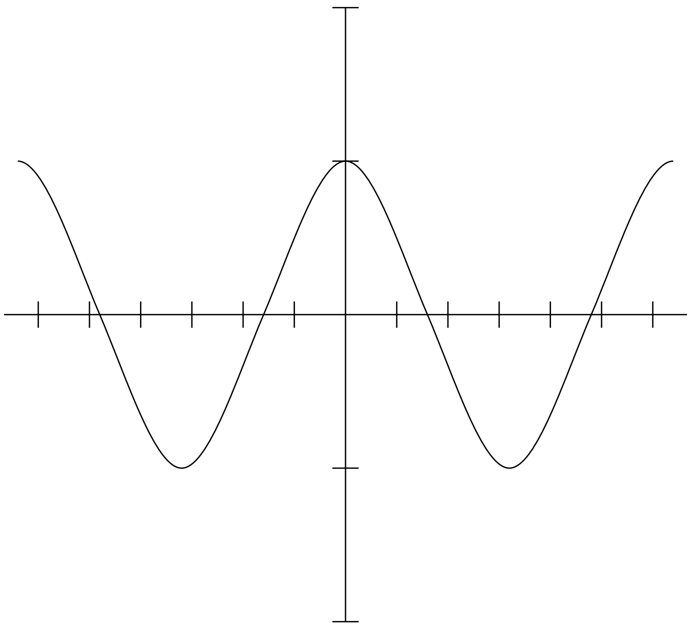

Figure 1-12: The graph of $y = \cos x$.

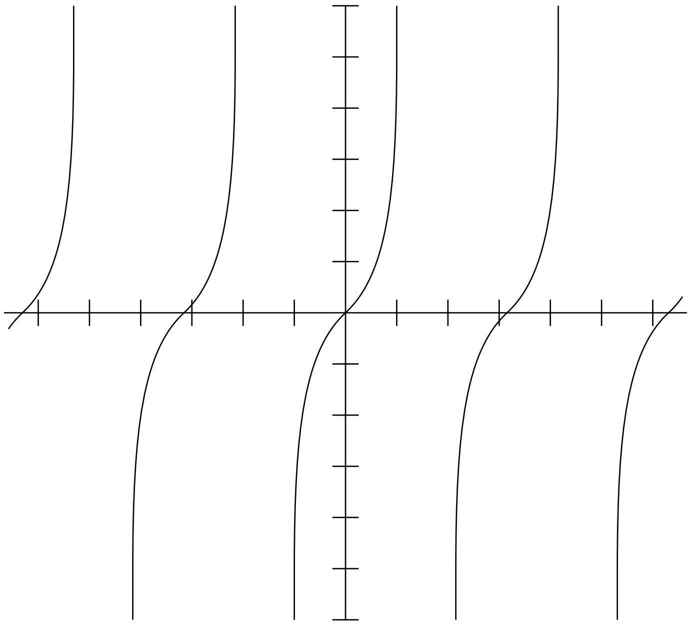

Figure 1-13: The graph of *y* = tan *x*.

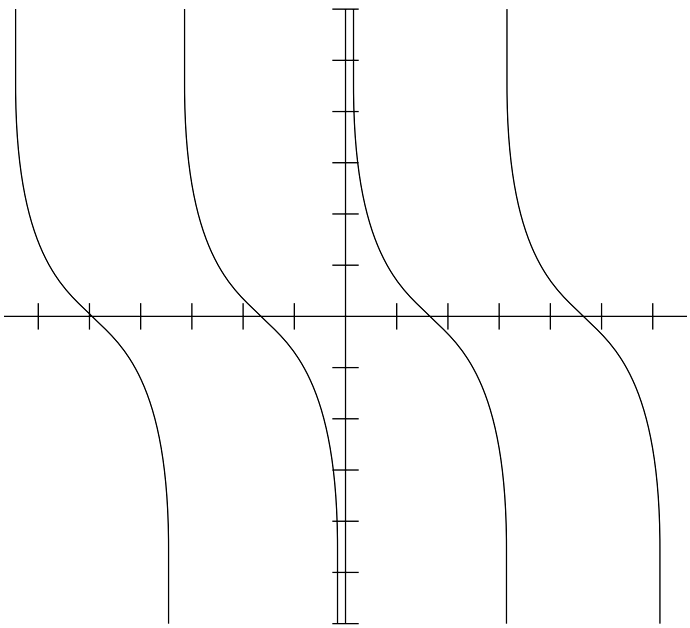

Figure 1-14: The graph of *y* = cot *x*.

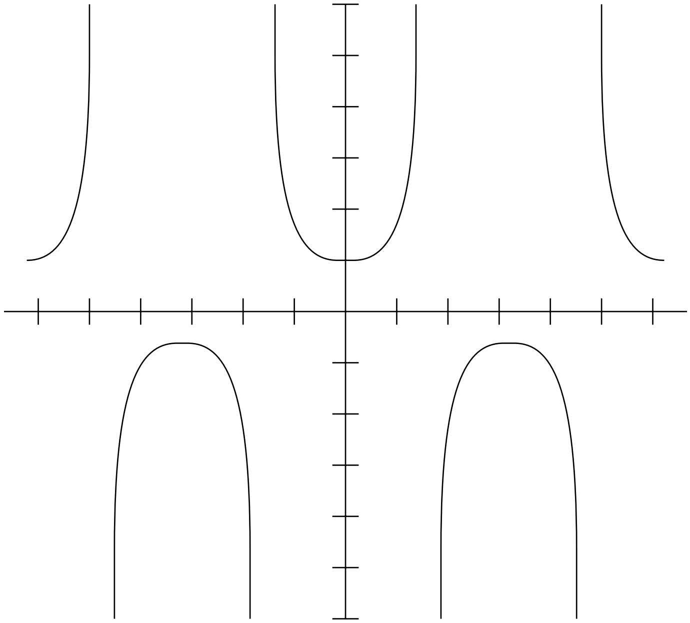

Figure 1-15: The graph of $y = \sec x$.

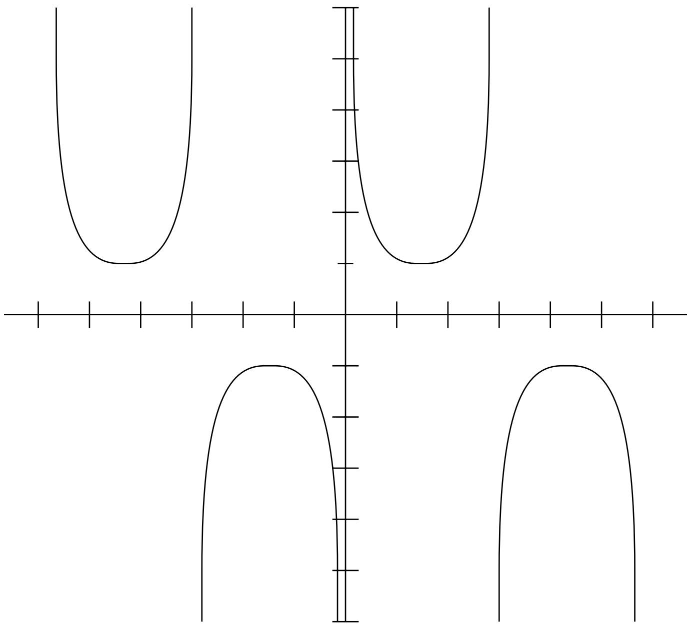

Figure 1-16: The graph of $y = \csc x$.