1

Overhead Transmission Lines and Their Circuit Constants

In order to understand fully the nature of power systems, we need to study the nature of transmission lines as the first step. In this chapter we examine the characteristics and basic equations of three-phase overhead transmission lines. However, the actual quantities of the constants are described in Chapter 2.

1.1 Overhead Transmission Lines with *LR* Constants

1.1.1 Three-phase single circuit line without overhead grounding wire

1.1.1.1 Voltage and current equations, and equivalent circuits

A three-phase single circuit line between a point m and a point n with only L and R and without an overhead grounding wire (OGW) can be written as shown in Figure 1.1a. In the figure, r_g and L_g are the equivalent resistance and inductance of the earth, respectively. The outer circuits I and II connected at points m and n can theoretically be three-phase circuits of any kind.

All the voltages V_a, V_b, V_c and currents I_a, I_b, I_c are vector quantities and the symbolic arrows show the measuring directions of the three-phase voltages and currents which have to be written in the same direction for the three phases as a basic rule to describe the electrical quantities of three-phase circuits.

In Figure 1.1, the currents I_a, I_b, I_c in each phase conductor flow from left to right (from point m to point n). Accordingly, the composite current $I_a + I_b + I_c$ has to return from right to left (from point n to m) through the earth–ground pass. In other words, the three-phase circuit has to be treated as the set of 'three phase conductors + one earth circuit' pass.

In Figure 1.1a, the equations of the transmission line between m and n can be easily described as follows. Here, voltages V and currents I are complex-number vector values:

$$\left.\begin{aligned}
{}_mV_a - {}_nV_a &= (r_a + j\omega L_{aag})I_a + j\omega L_{abg}I_b + j\omega L_{acg}I_c - {}_{mn}V_g \quad ① \\
{}_mV_b - {}_nV_b &= j\omega L_{bag}I_a + (r_b + j\omega L_{bbg})I_b + j\omega L_{bcg}I_c - {}_{mn}V_g \quad ② \\
{}_mV_c - {}_nV_c &= j\omega L_{cag}I_a + j\omega L_{cbg}I_b + (r_c + j\omega L_{ccg})I_c - {}_{mn}V_g \quad ③ \\
\text{where } {}_{mn}V_g &= (r_g + j\omega L_g)I_g = -(r_g + j\omega L_g)(I_a + I_b + I_c) \quad ④
\end{aligned}\right\} \qquad (1.1)$$

Handbook of Power System Engineering Yoshihide Hase

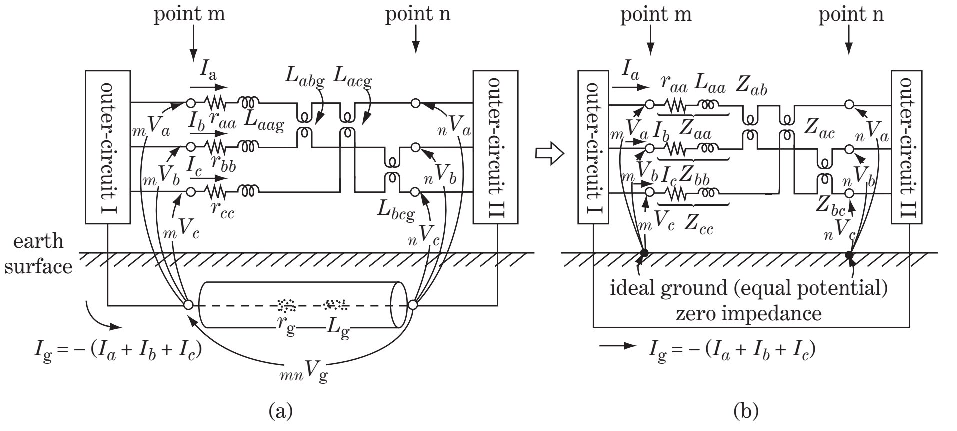


Figure 1.1 Single circuit line with *LR* constants

Substituting ④ into ①, and then eliminating ${}_{mn}V_g$, I_g,

$$ {}_mV_a - {}_nV_a = (r_a + r_g + j\omega\overline{L_{aag} + L_g})I_a + (r_g + j\omega\overline{L_{abg} + L_g})I_b + (r_g + jw\overline{L_{acg} + L_g})I_c \quad ⑤ $$

Substituting ④ into ② and ③ in the same way,

$$ {}_mV_b - {}_nV_b = (r_g + j\omega\overline{L_{bag} + L_g})I_a + (r_b + r_g + j\omega\overline{L_{bbg} + L_g})I_b + (r_g + j\omega\overline{L_{bcg} + L_g})I_c \quad ⑥ $$

$$ {}_mV_c - {}_nV_c = (r_g + j\omega\overline{L_{cag} + L_g})I_a + (r_g + j\omega\overline{L_{cbg} + L_g})I_b + (r_c + r_g + j\omega\overline{L_{ccg} + L_g})I_c \quad ⑦ $$

(1.2)

Now, the original Equation 1.1 and the derived Equation 1.2 are the equivalent of each other, so Figure 1.1b, showing Equation 1.2, is also the equivalent of Figure 1.1a.

Equation 1.2 can be expressed in the form of a matrix equation and the following equations are derived accordingly (refer to Appendix B for the matrix equation notation):

$$ \begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \end{bmatrix} $$

$$ = \begin{bmatrix} r_a + r_g + j\omega\overline{L_{aag} + L_g} & r_g + j\omega\overline{L_{abg} + L_g} & r_g + j\omega\overline{L_{acg} + L_g} \\ r_g + j\omega\overline{L_{bag} + L_g} & r_b + r_g + j\omega\overline{L_{bbg} + L_g} & r_g + j\omega\overline{L_{bcg} + L_g} \\ r_g + j\omega\overline{L_{cag} + L_g} & r_g + j\omega\overline{L_{cbg} + L_g} & r_c + r_g + j\omega\overline{L_{ccg} + L_g} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (1.3) $$

$$ \equiv \begin{bmatrix} r_{aa} + j\omega L_{aa} & r_{ab} + j\omega L_{ab} & r_{ac} + j\omega L_{ac} \\ r_{ba} + j\omega L_{ba} & r_{bb} + j\omega L_{bb} & r_{bc} + j\omega L_{bc} \\ r_{ca} + j\omega L_{ca} & r_{cb} + j\omega L_{cb} & r_{cc} + j\omega L_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} $$

$$ \equiv \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} $$

$$\left.\begin{array}{l}\text{where}\quad Z_{aa} = r_{aa} + j\omega L_{aa} = (r_a + r_g) + j\omega(L_{aag} + L_g) \\ Z_{bb}, Z_{cc} \text{ are written in similar equation forms} \\ \text{and } Z_{ac},\ Z_{bc} \text{ are also written in similar forms}\end{array}\right\} \tag{1.4}$$

Now, we can apply symbolic expressions for the above matrix equation as follows:

$$_m\boldsymbol{V}_{abc} - {}_n\boldsymbol{V}_{abc} = \boldsymbol{Z}_{abc} \cdot \boldsymbol{I}_{abc} \tag{1.5}$$

where

$$_m\boldsymbol{V}_{abc} = \begin{bmatrix} _mV_a \\ _mV_b \\ _mV_c \end{bmatrix},\quad {}_n\boldsymbol{V}_{abc} = \begin{bmatrix} _nV_a \\ _nV_b \\ _nV_c \end{bmatrix},\quad \boldsymbol{Z}_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix},\quad \boldsymbol{I}_{abc} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{1.6}$$

Summarizing the above equations, Figure 1.1a can be described as Equations 1.3 and 1.6 or Equations 1.5 and 1.6, in which the resistance r_g and inductance L_g of the earth return pass are already reflected in all these four equations, although I_g and $_{mn}V_g$ are eliminated in Equations 1.5 and 1.6. We can consider Figure 1.1b as the equivalent circuit of Equations 1.3 and 1.4 or Equations 1.5 and 1.6. In Figure 1.1b, earth resistance r_g and earth inductance L_g are already included in the line constants Z_{aa}, Z_{ab}, etc., so the earth in the equivalent circuit of Figure 1.1b is 'the ideal earth' with zero impedance. Therefore the earth can be expressed in the figure as the equal-potential (zero-potential) earth plane at any point. It is clear that the mutual relation between the constants of Figure 1.1a and Figure 1.1b is defined by Equation 1.4. It should be noted that the self-impedance Z_{aa} and mutual impedance Z_{ab} of phase a, for example, involve the earth resistance r_g and earth inductance L_g.

Generally, in actual engineering tasks, Figure 1.1b and Equations 1.3 and 1.4 or Equations 1.5 and 1.6 are applied instead of Figure 1.1a and Equations 1.1 and 1.2; in other words, the line impedances are given as Z_{aa}, Z_{ab}, etc., instead of Z_{aag}, Z_{abg}. The line impedances Z_{aa}, Z_{bb}, Z_{cc} are named '**the self-impedances of the line including the earth–ground effect**', and Z_{ab}, Z_{ac}, Z_{bc}, etc., are named '**the mutual impedances of the line including the earth–ground effect**'.

1.1.1.2 Measurement of line impedances Z_{aa}, Z_{ab}, Z_{ac}

Let us consider how to measure the line impedances taking the earth effect into account.

As we know from Figure 1.1b and Equations 1.3 and 1.4, the impedances Z_{aa}, Z_{ab}, Z_{ac}, etc., can be measured by the circuit connection shown in Figure 1.2a.

The conductors of the three phases are grounded to earth at point n, and the phase b and c conductors are opened at point m. Accordingly, the boundary conditions $_nV_a = {}_nV_b = {}_nV_c = 0, I_b = I_c = 0$ can be adopted for Equation 1.3:

$$\left.\begin{array}{ll}\begin{bmatrix} _mV_a \\ _mV_b \\ _mV_c \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} & ① \\ \therefore\quad _mV_a/I_a = Z_{aa},\qquad _mV_b/I_a = Z_{ba},\qquad _mV_c/I_a = Z_{ca} & ② \end{array}\right\} \tag{1.7}$$

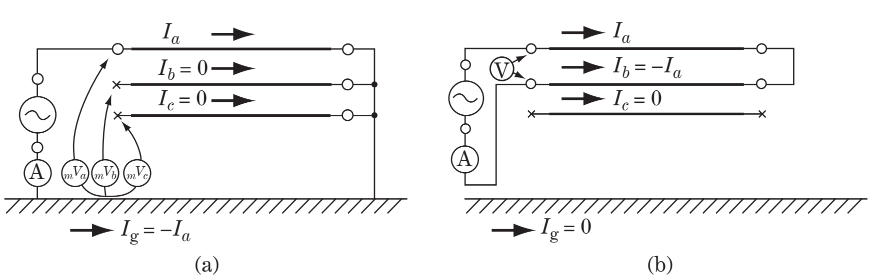


Figure 1.2 Measuring circuit of line impedance

Therefore the impedances Z_{aa}, Z_{ab}, Z_{ac} can be calculated from the measurement results of ${}_mV_a$, ${}_mV_b$, ${}_mV_c$ and I_a.

All the impedance elements in the impedance matrix $\boldsymbol{Z_{abc}}$ of Equation 1.7 can be measured in the same way.

1.1.1.3 Working inductance ($L_{aa} - L_{ab}$)

Figure 1.2b shows the case where the current I flows along the phase a conductor from point m to n and comes back from n to m only through the phase b conductor as the return pass. The equation is with boundary conditions $I_a = -I_b = I, \quad I_c = 0, \quad {}_nV_a = {}_nV_b$:

$$\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_a \\ {}_nV_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I \\ -I \\ 0 \end{bmatrix} \qquad (1.8a)$$

Therefore

$$\left.\begin{array}{ll} {}_mV_a - {}_nV_a = (Z_{aa} - Z_{ab})I: & \text{voltage drop of the phase a conductor between points m and n} \\ {}_mV_b - {}_nV_b = -(Z_{bb} - Z_{ba})I: & \text{voltage drop of the phase b conductor between points m and n} \end{array}\right\} ①$$
$$\left.\begin{array}{l} V = {}_mV_a - {}_mV_b = \{(Z_{aa} - Z_{ab}) + (Z_{bb} - Z_{ba})\}I \\ V/I = ({}_mV_a - {}_mV_b)/I = (Z_{aa} - Z_{ab}) + (Z_{bb} - Z_{ba}) = \{\text{twice values of working impedance}\} \end{array}\right\} ② \qquad (1.8b)$$

Equation 1.8① indicates the voltage drop of the parallel circuit wires a, b under the condition of the ‘go-and-return-current’ connection. The current I flows out at point m on the phase a conductor and returns to m only through the phase b conductor, so any other current flowing does not exist on the phase c conductor or earth–ground pass. In other words, Equation 1.8b① is satisfied regardless of the existence of the third wire or earth–ground pass. Therefore the impedance $(Z_{aa} - Z_{ab})$ as well as $(Z_{bb} - Z_{ba})$ should be specific values which are determined only by the relative condition of the phase a and b conductors, and they are not affected by the existence or absence of the third wire or earth–ground pass. $(Z_{aa} - Z_{ab})$ is called the **working impedance** and the corresponding $(L_{aa} - L_{ab})$ is called the **working inductance** of the phase a conductor with the phase b conductor.

Furthermore, as the conductors a and b are generally of the same specification (the same dimension, same resistivity, etc.), the ipedance drop between m and n of the phase a and b conductors should be the same. Accordingly, the working inductances of both conductors are clearly the same, namely $(L_{aa} - L_{ab}) = (L_{bb} - L_{ba})$.

The value of the working inductance can be calculated from the well-known equation below, which is derived by an electromagnetic analytical approach as a function only of the conductor radius r and the parallel distance s_{ab} between the two conductors:

$$L_{aa} - L_{ab} = L_{bb} - L_{ba} = 0.4605 \log_{10} \frac{s_{ab}}{r} + 0.05 \qquad [\text{mH/km}] \qquad (1.9)$$

This is the equation for the working inductance of the parallel conductors a and b, which can be quoted from analytical books on electromagnetism. The equation shows that the working inductance $L_{aa} - L_{ab}$ for the two parallel conductors is determined only by the relative distance between the two conductors s_{ab} and the radius r, so it is not affected by any other conditions such as other conductors or the distance from the earth surface.

The working inductance can also be measured as the value (1/2)V/I by using Equation 1.8b②.

1.1.1.4 Self- and mutual impedances including the earth–ground effect L_{aa}, L_{ab}

Now we evaluate the actual numerical values for the line inductances contained in the impedance matrix of Equation 1.3.

The currents I_a, I_b, I_c flow through each conductor from point m to n and $I_a + I_b + I_c$ returns from n to m through the ideal earth return pass. All the impedances of this circuit can be measured by the method of Figure 1.2a. However, these measured impedances are experimentally a little larger than those obtained by pure analytical calculation based on the electromagnetic equations with the assumption of an ideal, conductive, earth plane surface.

In order to compensate for these differences between the analytical result and the measured values, we can use an imaginary ideal conductive earth plane at some deep level from the ground surface as shown in Figure 1.3.

In this figure, the imaginary perfect conductive earth plane is shown at the depth H_g, and the three imaginary conductors α, β, γ are located at symmetrical positions to conductors a, b, c, respectively, based on this datum plane.

The inductances can be calculated by adopting the equations of the electromagnetic analytical approach to Figure 1.3.

1.1.1.4.1 Self-inductances L_{aa}, L_{bb}, L_{cc} In Figure 1.3, the conductor a (radius r) and the imaginary returning conductor α are symmetrically located on the datum plane, and the distance between a and α is $h_a + H_a$. Thus the inductance of conductor a can be calculated by the following equation which is a special case of Equation 1.9 under the condition $s_{ab} \to h_a + H_a$:

$$L_{aag} = 0.4605 \log_{10} \frac{h_a + H_a}{r} + 0.05 \qquad [\text{mH/km}] \tag{1.10a}$$

Conversely, the inductance of the imaginary conductor α (the radius is H_a, because the actual grounding current reaches up to the ground surface), namely the inductance of earth, is

$$L_g = 0.4605 \log_{10} \frac{h_a + H_a}{H_a} + 0.05\,[\text{mH/km}] \fallingdotseq 0.05 \quad [\text{mH/km}] \tag{1.10b}$$

Therefore,

$$L_{aa} = L_{aag} + L_g = 0.4605 \log_{10} \frac{h_a + H_a}{r} + 0.1 \quad [\text{mH/km}] \tag{1.11}$$

L_{bb}, L_{cc} can be derived in the same way.

Incidentally, the depth of the imaginary datum plane can be checked experimentally and is mostly within the range of $H_g = 300 - 1000$ m. On the whole H_g is rather shallow, say $300 - 600$ m in the

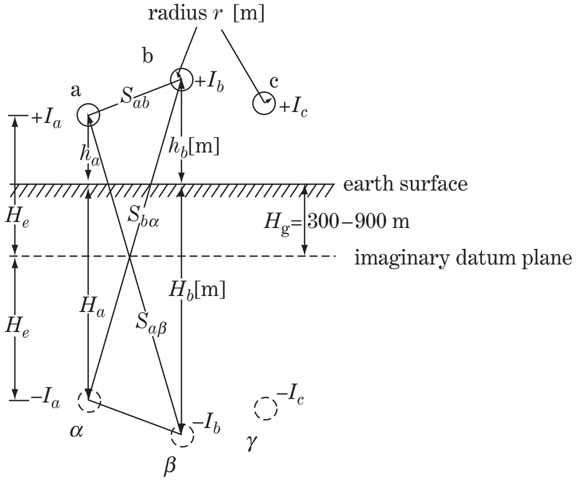


Figure 1.3 Earth–ground as conductor pass

geological younger strata after the Quaternary period, but is generally deep, say 800 − 1000 m, in the older strata of the Tertiary period or earlier.

1.1.1.4.2 Mutual inductances L_{ab}, L_{bc}, L_{ca}

The mutual inductance L_{ab} can be derived by subtracting L_{aa} from Equation 1.11 and the working inductance $(L_{aa} - L_{ab})$ from Equation 1.9:

$$\begin{aligned} L_{ab} &= L_{aa} - (L_{aa} - L_{ab}) = 0.4605\log_{10}\frac{h_a + H_a}{s_{ab}} + 0.05 \quad [\mathrm{mH/km}] \\ &\fallingdotseq 0.4605\log_{10}\frac{s_{a\beta}}{s_{ab}} + 0.05 \quad [\mathrm{mH/km}] \end{aligned} \tag{1.12a}$$

Similarly

$$\begin{aligned} L_{ba} &= 0.4605\log_{10}\frac{h_b + H_b}{s_{ab}} + 0.05 \quad [\mathrm{mH/km}] \\ &\fallingdotseq 0.4605\log_{10}\frac{s_{b\alpha}}{s_{ab}} + 0.05 \quad [\mathrm{mH/km}] \end{aligned} \tag{1.12b}$$

where $h_a + H_a = 2H_e \fallingdotseq 2H_g$, and so on.

Incidentally, the depth of the imaginary datum plane $H_g \fallingdotseq H_e = (h_a + H_a)/2$ would be between 300 and 1000 m, while the height of the transmission tower h_a is within the range of 10–100 m (UHV towers of 800–1000 kV would be approximately 100 m). Furthermore, the phase-to-phase distance S_{ab} is of order 10 m, while the radius of conductor r is a few centimetres (the equivalent radius r_{eff} of EHV/UHV multi-bundled conductor lines may be of the order of 10–50 cm).

Accordingly,

$$\left.\begin{aligned} &H_a \fallingdotseq H_b \fallingdotseq H_c \fallingdotseq 2H_e \gg h_a \fallingdotseq h_b \fallingdotseq h_c \gg s_{ab} \fallingdotseq s_{bc} \fallingdotseq s_{ca} \gg r, r_{\mathrm{eff}} \\ &s_{a\beta} \fallingdotseq s_{b\alpha} \fallingdotseq h_a + H_a = 2H_e \fallingdotseq h_b + H_b \end{aligned}\right\} \tag{1.13a}$$

Then, from Equations 1.9, 1.11 and 1.12,

$$L_{aa} \fallingdotseq L_{bb} \fallingdotseq L_{cc}, \quad L_{ab} \fallingdotseq L_{bc} \fallingdotseq L_{ca} \tag{1.13b}$$

1.1.1.4.3 Numerical check

Let us assume conditions $s_{ab} = 10\,\mathrm{m}$, $r = 0.05\,\mathrm{m}$, $H_e = (h_a + H_a)/2 \fallingdotseq H_g = 900\,\mathrm{m}$.

Then calculating the result by Equation 1.11 and 1.12,

$$L_{aa} = 2.20\,\mathrm{mH/km}, \quad L_{ab} = 1.09\,\mathrm{mH/km}$$

If $H_e = (h_a + H_a)/2 = 300\,\mathrm{m}$, then $L_{aa} = 1.98\,\mathrm{mH/km}$, $L_{ab} = 0.87\,\mathrm{mH/km}$. As $h_a + H_a$ is contained in the logarithmic term of the equations, constant values L_{aa}, L_{ab} and so on are not largely affected by $h_a + H_a$, neither is radius r nor r_{eff} as well as the phase-to-phase distance s_{ab}. Besides, 0.1 and 0.05 in the second term on the right of Equations 1.9–1.12 do not make a lot of sense.

Further, if transmission lines are reasonably transpositioned, $Z_{aa} \fallingdotseq Z_{bb} \fallingdotseq Z_{cc}$, $Z_{ab} \fallingdotseq Z_{bc} \fallingdotseq Z_{ca}$ can be justified so that Equation 1.3 is simplified into Equation 2.13 of Chapter 2.

1.1.1.5 Reactance of multi-bundled conductors

For most of the recent large-capacity transmission lines, multi-bundled conductor lines ($n = 2 - 8$ per phase) are utilized as shown in Figure 1.4. In the case of n conductors (the radius of

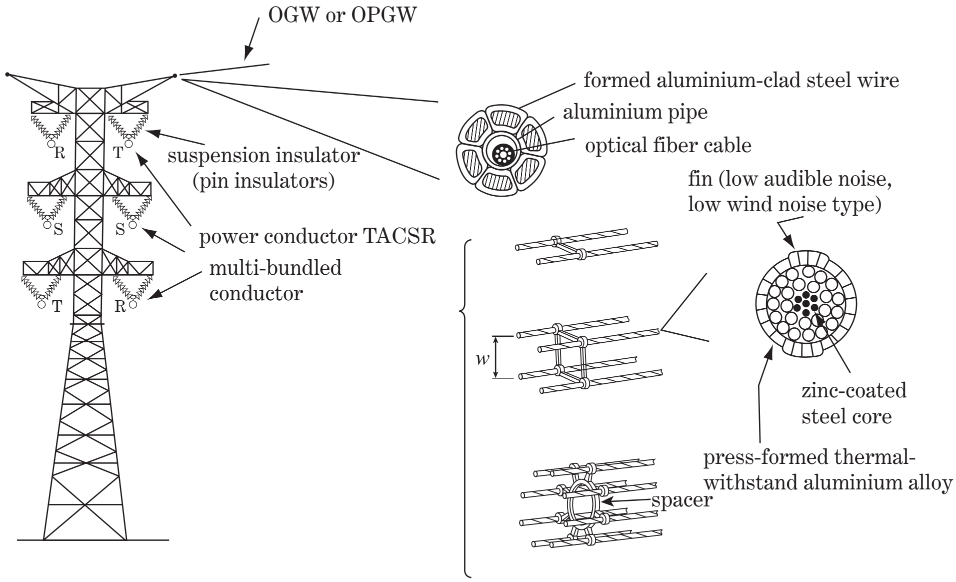


Figure 1.4 Overhead double circuit transmission line

each conductor is r), L_{aag} of Equation 1.10a can be calculated from the following modified equation:

$$\left.\begin{aligned} L_{aag} &= 0.4605\log_{10}\frac{h_a+H_a}{r^{1/n}\times w^{(n-1)/n}}+\frac{0.05}{n}\quad[\text{mH/km}]\\ &\equiv 0.4605\log_{10}\frac{h_a+H_a}{r_{\text{eff}}}+\frac{0.05}{n}\quad[\text{mH/km}]\\ &\text{where}\quad r_{\text{eff}}=r^{1/n}\times w^{(n-1)/n}\ \text{is the equivalent radius and}\\ &w\,[\text{m}]\ \text{is the geometrical averaged distance of bundled conductors} \end{aligned}\right\}\quad(1.14\text{a})$$

Since the self-inductance L_g of the virtual conductor α given by Equation 1.10b is not affected by the adoption of multi-bundled phase a conductors, accordingly

$$L_{aa}=L_{aag}+L_g=0.4605\log_{10}\frac{h_a+H_a}{r_{\text{eff}}}+0.05\left(1+\frac{1}{n}\right)\quad[\text{mH/km}]\quad(1.14\text{b})$$

1.1.1.5.1 *Numerical check*

Using TACSR = 810 mm^2 (see Chapter 2), $2r = 40$ mm and four bundled conductors ($n = 4$), with the square allocation $w = 50$ cm averaged distance

$$\left.\begin{aligned} w &= (w_{12}\cdot w_{13}\cdot w_{14}\cdot w_{23}\cdot w_{24}\cdot w_{34})^{1/6}\\ &= (50\cdot 50\sqrt{2}\cdot 50\cdot 50\cdot 50\sqrt{2}\cdot 50)^{1/6}=57.24\,\text{cm}\\ r_{\text{eff}} &= r^{1/n}\cdot w^{(n-1)/n}=20^{1/4}\cdot 57.25^{3/4}=44.0\,\text{mm} \end{aligned}\right\}\quad(1.14\text{c})$$

The equivalent radius $r_{\text{eff}} = 44$ mm is 2.2 times $r = 20$ mm, so that the line self-inductance L_{aa} can also be reduced by the application of bundled conductors. The mutual inductance L_{ab} of Equation 1.12a is not affected by the adoption of multi-bundled conductor lines.

1.1.1.6 Line resistance

Earth resistance r_g in Figure 1.1a and Equation 1.2 can be regarded as negligibly small. Accordingly, the so-called mutual resistances r_{ab}, r_{bc}, r_{ca} in Equation 1.4 become zero. Therefore, the specific resistances of the conductors r_a, r_b, r_c are actually equal to the resistances r_{aa}, r_{bb}, r_{cc} in the impedance matrix of Equation 1.3.

In addition to the power loss caused by the linear resistance of conductors, non-linear losses called the **skin-effect loss** and **corona loss** occur on the conductors. These losses would become progressionally larger in higher frequency zones, so they must be major influential factors for the attenuation of travelling waves in surge phenomena. However, they can usually be neglected for power frequency phenomena because they are smaller than the linear resistive loss and, further, very much smaller than the reactance value of the line, at least for power frequency.

In regard to the bundled conductors, due to the result of the enlarged equivalent radius, the **dielectric strength around the bundled conductors** is somewhat relaxed, so that corona losses can also be relatively reduced. Skin-effect losses of bundled conductors are obviously far smaller than that of a single conductor whose aluminium cross-section is the same as the total sections of the bundled conductors.

1.1.2 Three-phase single circuit line with OGW, OPGW

Most high-voltage transmission lines are equipped with **OGW** (**overhead grounding wires**) and/or **OPGW** (**OGW with optical fibres** for communication use).

In the case of a single circuit line with single OGW, the circuit includes four conductors and the fourth conductor (x in Figure 1.5) is earth grounded at all the transmission towers. Therefore, using the figure for the circuit, Equation 1.3 has to be replaced by the following equation:

$$\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \\ {}_mV_x=0 \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \\ {}_nV_x=0 \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{ax} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bx} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cx} \\ Z_{xa} & Z_{xb} & Z_{xc} & Z_{xx} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_x \end{bmatrix} \tag{1.15a}$$

Extracting the fourth row,

$$I_x = -\frac{1}{Z_{xx}}(Z_{xa}I_a + Z_{xb}I_b + Z_{xc}I_c) \tag{1.15b}$$

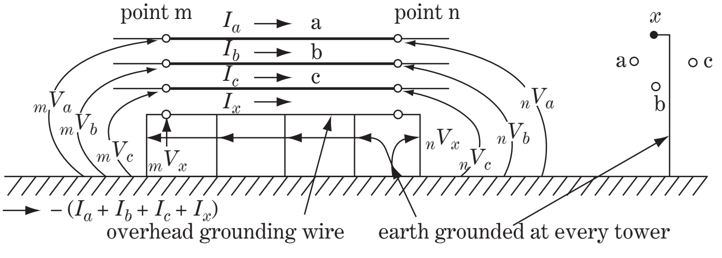


Figure 1.5 Single circuit line with OGW

Substituting I_x into the first, second and third rows of Equation 1.15a,

$$
\left.
\begin{array}{l}
\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} Z_{ax}I_x \\ Z_{bc}I_x \\ Z_{cx}I_x \end{bmatrix} \\
= \begin{bmatrix} Z_{aa} - \dfrac{Z_{ax}Z_{xa}}{Z_{xx}} & Z_{ab} - \dfrac{Z_{ax}Z_{xb}}{Z_{xx}} & Z_{ac} - \dfrac{Z_{ax}Z_{xc}}{Z_{xx}} \\ Z_{ba} - \dfrac{Z_{bx}Z_{xa}}{Z_{xx}} & Z_{bb} - \dfrac{Z_{bx}Z_{xb}}{Z_{xx}} & Z_{bc} - \dfrac{Z_{bx}Z_{xc}}{Z_{xx}} \\ Z_{ca} - \dfrac{Z_{cx}Z_{xa}}{Z_{xx}} & Z_{cb} - \dfrac{Z_{cx}Z_{xb}}{Z_{xx}} & Z_{cc} - \dfrac{Z_{cx}Z_{xc}}{Z_{xx}} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\
\equiv \begin{bmatrix} Z'_{aa} & Z'_{ab} & Z'_{ac} \\ Z'_{ba} & Z'_{bb} & Z'_{bc} \\ Z'_{ca} & Z'_{cb} & Z'_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\
\text{where } Z_{ax} = Z_{xa}, \quad Z_{bx} = Z_{xb}, \quad Z_{cx} = Z_{xc} \\
Z'_{aa} = Z_{aa} - \delta_{aa}, \quad Z'_{ab} = Z_{ab} - \delta_{ab} \\
\delta_{aa} = -\dfrac{Z_{ax}Z_{xa}}{Z_{xx}}, \quad \delta_{ab} = -\dfrac{Z_{ax}Z_{xb}}{Z_{xx}}
\end{array}
\right\} \tag{1.16}
$$

This is the fundamental equation of the three-phase single circuit line with OGW in which I_x has already been eliminated and the impedance elements of the grounding wire are slotted into the three-phase impedance matrix. Equation 1.16 is obviously of the same form as Equation 1.3, while all the elements of the rows and columns in the impedance matrix have been revised to smaller values with corrective terms $\delta_{ax} = -Z_{ax}Z_{xa}/Z_{xx}$ etc.

The above equations indicate that the three-phase single circuit line with OGW can be expressed as a 3×3 impedance matrix equation in the form of Equation 1.16 regardless of the existence of OGW, as was the case with Equation 1.3. Also, we can comprehend that OGW has roles not only to shield lines against lightning but also to reduce the self- and mutual reactances of transmission lines.

1.1.3 Three-phase double circuit line with *LR* constants

The three-phase double circuit line can be written as in Figure 1.6 and Equation 1.17 regardless of the existence or absence of OGW:

$$
\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \\ {}_mV_A \\ {}_mV_B \\ {}_mV_C \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \\ {}_nV_A \\ {}_nV_B \\ {}_nV_C \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{aA} & Z_{aB} & Z_{aC} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bA} & Z_{bB} & Z_{bC} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cA} & Z_{cB} & Z_{cC} \\ Z_{Aa} & Z_{Ab} & Z_{Ac} & Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{Ba} & Z_{Bb} & Z_{Bc} & Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{Ca} & Z_{Cb} & Z_{Cc} & Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_A \\ I_B \\ I_C \end{bmatrix} \tag{1.17}
$$

In addition, if the line is appropriately phase balanced, the equation can be expressed by Equation 2.17 of Chapter 2.

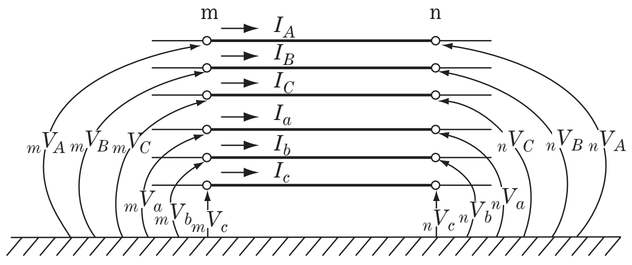


Figure 1.6 Three-phase double circuit line with *LR* constants

1.2 Stray Capacitance of Overhead Transmission Lines

1.2.1 Stray capacitance of three-phase single circuit line

1.2.1.1 Equation for electric charges and voltages on conductors

Figure 1.7a shows a single circuit line, where electric charges q_a, q_b, q_c [C/m] are applied to phase a, b, c conductors and cause voltages v_a, v_b, v_c [V], respectively. The equation of this circuit is given by

$$\left.\begin{array}{l} \underbrace{\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}}_{\boldsymbol{v}_{abc}} = \underbrace{\begin{bmatrix} p_{aa} & p_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix}}_{\boldsymbol{p}_{abc}} \cdot \underbrace{\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}}_{\boldsymbol{q}_{abc}} \quad \therefore \boldsymbol{v}_{abc} = \boldsymbol{p}_{abc} \cdot \boldsymbol{q}_{abc} \\ \text{where } q\,[\mathrm{C/m}],\ v\,[\mathrm{V}] \text{ are instantaneous real numbers} \end{array}\right\} \tag{1.18}$$

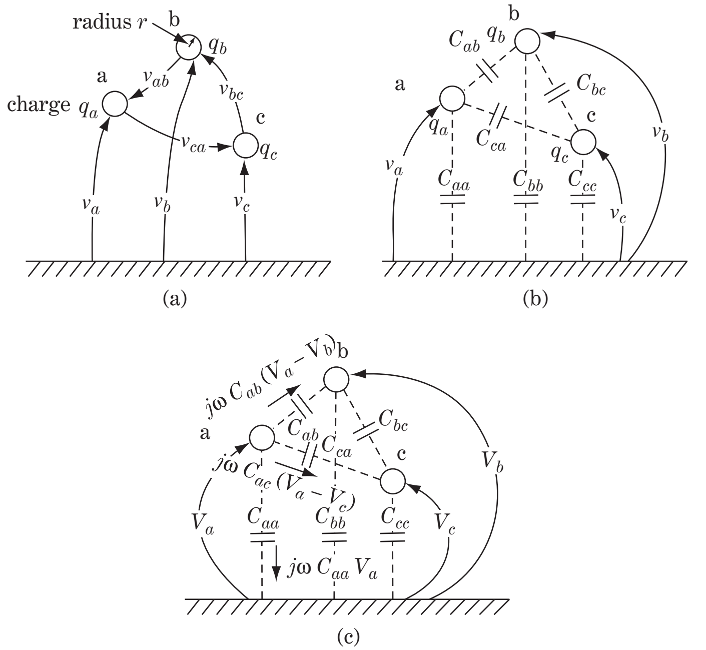


Figure 1.7 Stray capacitance of single circuit line

The inverse matrix equation can be derived from the above equation as

$$\underbrace{\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}}_{\boldsymbol{q}_{abc}} = \underbrace{\begin{bmatrix} k_{aa} & k_{ab} & k_{ac} \\ k_{ba} & k_{bb} & k_{bc} \\ k_{ca} & k_{cb} & k_{cc} \end{bmatrix}}_{\boldsymbol{k}_{abc}} \cdot \underbrace{\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}}_{\boldsymbol{v}_{abc}} \quad \therefore \boldsymbol{q}_{abc} = \boldsymbol{k}_{abc} \cdot \boldsymbol{v}_{abc} \tag{1.19}$$

Here, $\boldsymbol{p}_{abc}$ and $\boldsymbol{k}_{abc}$ are inverse 3×3 matrices of each other, so that $\boldsymbol{p}_{abc} \cdot \boldsymbol{k}_{abc} = \mathbf{1}$ (**1** is the 3×3 unit matrix; refer to Appendix B).

Accordingly,

$$\left.\begin{aligned} k_{aa} &= (p_{bb}p_{cc} - p_{bc}^2)/\Delta & &[\mathrm{F/m}] \\ k_{bb} &= (p_{cc}p_{aa} - p_{ca}^2)/\Delta & &[\mathrm{F/m}] \\ k_{cc} &= (p_{aa}p_{bb} - p_{ab}^2)/\Delta & &[\mathrm{F/m}] \\ k_{ab} &= k_{ba} = -(p_{ab}p_{cc} - p_{ac}p_{bc})/\Delta & &[\mathrm{F/m}] \\ k_{bc} &= k_{cb} = -(p_{bc}p_{aa} - p_{ba}p_{ca})/\Delta & &[\mathrm{F/m}] \\ k_{ca} &= k_{ac} = -(p_{ca}p_{bb} - p_{cb}p_{ab})/\Delta & &[\mathrm{F/m}] \\ \Delta &= p_{aa}p_{bb}p_{cc} + 2p_{ab}p_{bc}p_{ac} - (p_{aa}p_{bc}^2 + p_{bb}p_{ca}^2 + p_{cc}p_{ab}^2) & &[\mathrm{m/F}]^3 \end{aligned}\right\} \tag{1.20}$$

where p [m/F] are the **coefficients of the potential** and k [F/m] are the electrostatic **coefficients of static capacity**.

Modifying Equation 1.19 a little,

$$\left.\begin{aligned} q_a &= k_{aa}v_a + k_{ab}v_b + k_{ac}v_c \\ &= (k_{aa} + k_{ab} + k_{ac})v_a + (-k_{ab})(v_a - v_b) + (-k_{ac})(v_a - v_c) & &[\mathrm{C/m}] \\ q_b &= (k_{ba} + k_{bb} + k_{bc})v_b + (-k_{bc})(v_b - v_c) + (-k_{ba})(v_b - v_a) & &[\mathrm{C/m}] \\ q_c &= (k_{ca} + k_{cb} + k_{cc})v_c + (-k_{ca})(v_c - v_a) + (-k_{ca}) + (v_c - v_b) & &[\mathrm{C/m}] \end{aligned}\right\} \tag{1.21}$$

then

$$\left.\begin{aligned} q_a &= C_{aa}v_a + C_{ab}(v_a - v_b) + C_{ac}(v_a - v_c) & &[\mathrm{C/m}] \\ q_b &= C_{bb}v_b + C_{bc}(v_b - v_c) + C_{ba}(v_b - v_a) & &[\mathrm{C/m}] \\ q_c &= C_{cc}v_c + C_{ca}(v_c - v_a) + C_{cb}(v_c - v_b) & &[\mathrm{C/m}] \end{aligned}\right\} \tag{1.22}$$

with q_a, q_b, q_c [C/m], v_b, v_b, v_c [V] and

$$\left.\begin{aligned} C_{aa} &= k_{aa} + k_{ab} + k_{ac} & &[\mathrm{F/m}] & C_{ab} &= -k_{ab} & &[\mathrm{F/m}] \\ C_{bb} &= k_{ba} + k_{bb} + k_{bc} & &[\mathrm{F/m}] & C_{bc} &= -k_{bc} & &[\mathrm{F/m}] \\ C_{cc} &= k_{ca} + k_{cb} + k_{cc} & &[\mathrm{F/m}] & C_{ca} &= -k_{ca} & &[\mathrm{F/m}] \\ C_{ac} &= -k_{ac} & &[\mathrm{F/m}] \\ C_{ba} &= -k_{ba} & &[\mathrm{F/m}] \\ C_{cb} &= -k_{cb} & &[\mathrm{F/m}], \end{aligned}\right\} \tag{1.23}$$

Equations 1.22 and 1.23 are the fundamental equations of stray capacitances of a three-phase single circuit overhead line. Noting the form of Equation 1.22, Figure 1.7b can be used for another expression of Figure 1.7a: C_{aa}, C_{bb}, C_{cc} are the phase-to-ground capacitances and $C_{ab} = C_{ba}$, $C_{bc} = C_{cb}$, $C_{ca} = C_{ac}$ are the phase-to-phase capacitances between two conductors.

1.2.1.2 Fundamental voltage and current equations

It is usually convenient in actual engineering to adopt current $i(= dq/dt)$ [A] instead of charging value q [C], and furthermore to adopt effective (rms: root mean square) voltage and current of complex-number V, I instead of instantaneous value $v(t)$, $i(t)$.

As electric charge q is the integration over time of current i, the following relations can be derived:

$$
\left.\begin{array}{l}
q = \int i dt, \quad i = \dfrac{dq}{dt} \quad ① \\
i(t) = \mathrm{Re}(\sqrt{2}I) = \mathrm{Re}(\sqrt{2}\,|I| \cdot \mathrm{e}^{j(\omega t+\theta_1)}) = \sqrt{2}\,|I|\cos(\omega t+\theta_1) \quad ② \\
\text{Re() shows the real part of the complex number}(\mathrm{Re}(a+jb)=a). \\
v(t) = \mathrm{Re}(\sqrt{2}V) = \mathrm{Re}(\sqrt{2}\,|V| \cdot \mathrm{e}^{j(\omega t+\theta_2)}) \\
\quad = \sqrt{2}\,|V|\cos(\omega t+\theta_2) \quad ③ \\
\therefore \\
q(t) = \int i dt = \int \mathrm{Re}(\sqrt{2}\,|I| \cdot \mathrm{e}^{j(\omega t+\theta_1)})dt \\
\quad = \mathrm{Re}(\sqrt{2}\,|I| \cdot \int \mathrm{e}^{j(\omega t+\theta_1)}dt) \\
\quad = \mathrm{Re}\left(\sqrt{2}\,|I| \cdot \dfrac{\mathrm{e}^{j(\omega t+\theta_1)}}{j\omega}\right) = \mathrm{Re}\left(\dfrac{\sqrt{2}I}{j\omega}\right) \quad ④
\end{array}\right\} \quad (1.24)
$$

(note that, in this book, the exponential function will be denoted by 'e').

Equation 1.22 can be modified to the following form by adopting Equation 1.24④ and by replacement of $v_a \to \sqrt{2}V_a$ etc.:

$$
\left.\begin{array}{l}
\mathrm{Re}\left(\dfrac{\sqrt{2}I_a}{j\omega}\right) = \mathrm{Re}\{C_{aa}\cdot\sqrt{2}V_a + C_{ab}\cdot\sqrt{2}(V_a-V_b) + C_{ac}\cdot\sqrt{2}(V_a-V_c)\} \\
\mathrm{Re}\left(\dfrac{\sqrt{2}I_b}{j\omega}\right) = \mathrm{Re}\{C_{bb}\cdot\sqrt{2}V_b + C_{bc}\cdot\sqrt{2}(V_b-V_c) + C_{ba}\cdot\sqrt{2}(V_b-V_a)\} \\
\mathrm{Re}\left(\dfrac{\sqrt{2}I_c}{j\omega}\right) = \mathrm{Re}\{C_{cc}\cdot\sqrt{2}V_c + C_{ca}\cdot\sqrt{2}(V_c-V_a) + C_{cb}\cdot\sqrt{2}(V_c-V_b)\}
\end{array}\right\} \quad (1.25)
$$

Therefore

$$
\left.\begin{array}{l}
I_a = j\omega C_{aa}V_a + j\omega C_{ab}(V_a-V_b) + j\omega C_{ac}(V_a-V_c) \\
I_b = j\omega C_{bb}V_b + j\omega C_{bc}(V_b-V_c) + j\omega C_{ba}(V_b-V_a) \\
I_c = j\omega C_{cc}V_c + j\omega C_{ca}(V_c-V_a) + j\omega C_{cb}(V_c-V_b)
\end{array}\right\} \quad (1.26a)
$$

or, with a small modification,

$$
\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = j\omega \begin{bmatrix} C_{aa}+C_{ab}+C_{ac} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{ba}+C_{bb}+C_{bc} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{ca}+C_{cb}+C_{cc} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (1.26b)
$$

This is the fundamental equation for stray capacitances of a three-phase single circuit transmission line. Also Figure 1.7c is derived from one-to-one correspondence with Equation 1.26.

1.2.1.3 Coefficients of potential (p_{aa}, p_{ab}), coefficients of static capacity (k_{aa}, k_{ab}) and capacitances (C_{aa}, C_{ab})

The earth surface can be taken as a perfect equal-potential plane, so that we can use Figure 1.8, in which the three imaginary conductors α, β, γ are located at symmetrical positions of conductors a, b, c, respectively, based on the earth surface plane. By assuming electric charges $+q_a$, $+q_b$, $+q_c$ and $-q_a$, $-q_b$, $-q_c$ per unit length on conductors a, b, c, and α, β, γ respectively, the following voltage equation can be derived:

$$
\begin{aligned}
v_a = &\left(\text{voltage of conductor a due to} \pm q_a \text{ of conductor a, } \alpha : 2q_a \log_\mathrm{e}\frac{2h_a}{r} \times 9\times 10^9\ [\mathrm{V}]\right) \\
&+\left(\text{voltage of conductor a due to} \pm q_b \text{ of conductor b, } \beta : 2q_b \log_\mathrm{e}\frac{s_{a\beta}}{s_{ab}} \times 9\times 10^9\ [\mathrm{V}]\right) \\
&+\left(\text{voltage of conductor a due to} \pm q_c \text{ of conductor c, } \gamma : 2q_c \log_\mathrm{e}\frac{s_{a\gamma}}{s_{ac}} \times 9\times 10^9\ [\mathrm{V}]\right) \quad ①
\end{aligned}
$$

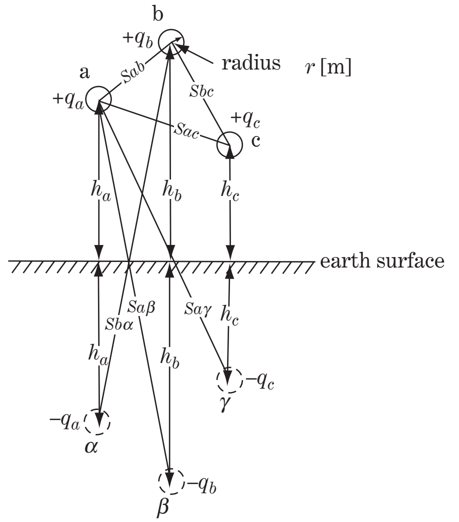


Figure 1.8 Three parallel overhead conductors

Equations for v_b, v_c can be derived in the same way. Then

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} p_{aa} & p_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix} \cdot \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

$$= 2 \times 9 \times 10^9 \times \begin{bmatrix} \log_e \dfrac{2h_a}{r} & \log \dfrac{s_{a\beta}}{s_{ab}} & \log_e \dfrac{s_{a\gamma}}{s_{ac}} \\ \log_e \dfrac{s_{b\alpha}}{s_{ba}} & \log_e \dfrac{2h_b}{r} & \log_e \dfrac{s_{b\gamma}}{s_{bc}} \\ \log_e \dfrac{s_{c\alpha}}{s_{ca}} & \log_e \dfrac{s_{c\beta}}{s_{cb}} & \log_e \dfrac{2h_c}{r} \end{bmatrix} \cdot \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} ② \qquad (1.27)$$

where $s_{a\beta} = s_{b\alpha} = \sqrt{\{s_{ab}^2 - (h_a - h_b)^2\} + (h_a + h_b)^2} = \sqrt{s_{ab}^2 + 4h_a h_b}$.
Refer to the supplement at the end of this chapter for the derivation of Equation 1.27①.

The equation indicates that the coefficients of potential (p_{aa}, p_{ab}, etc.) are calculated as a function of the conductor's radius r, height (h_a, h_b, h_c) from the earth surface, and phase-to-phase distances (s_{ab}, s_{ac}, etc.) of the conductors. p_{aa}, p_{ab}, etc., are determined only by physical allocations of each phase conductor (in other words, by the structure of towers), and relations like $p_{ab} = p_{ba}$ are obvious.

In conclusion, the coefficients of potential (p_{aa}, p_{ab}, etc.), the coefficients of static capacity (k_{aa}, k_{ab}, etc.) and the capacitance (C_{aa}, C_{ab}, etc.) are calculated from Equations 1.27, 1.20 and 1.23, respectively. Again, all these values are determined only by the physical allocation of conductors and are not affected by the applied voltage.

1.2.1.4 Stray capacitances of phase-balanced transmission lines

Referring to Figure 1.8, a well-phase-balanced transmission line, probably by transposition, can be assumed. Then

$$\left.\begin{aligned} &h \equiv h_a \fallingdotseq h_b \fallingdotseq h_c, \qquad s_{ll} \equiv s_{ab} = s_{ba} \fallingdotseq s_{bc} = s_{cb} \fallingdotseq s_{ca} = s_{ac} \\ &s_{a\beta} \fallingdotseq s_{b\alpha} \fallingdotseq s_{a\gamma} \fallingdotseq s_{c\alpha} \fallingdotseq s_{b\gamma} \fallingdotseq s_{c\beta} \end{aligned}\right\} \tag{1.28}$$

$$\left.\begin{aligned} &p_s \equiv p_{aa} \fallingdotseq p_{bb} \fallingdotseq p_{cc} \\ &p_m \equiv p_{ab} = p_{ba} \fallingdotseq p_{ac} = p_{ca} \fallingdotseq p_{bc} = p_{cb} \end{aligned}\right\} \tag{1.29}$$

Accordingly, Equation 1.20 can be simplified as follows:

$$\left.\begin{aligned} \Delta &= p_s^3 + 2p_m^3 - 3p_s p_m^2 \\ &= (p_s - p_m)^2 (p_s + 2p_m) \\ k_s &\equiv k_{aa} \fallingdotseq k_{bb} \fallingdotseq k_{cc} \fallingdotseq (p_s^2 - p_m^2)/\Delta = \frac{p_s + p_m}{(p_s - p_m)(p_s + 2p_m)} \\ k_m &\equiv k_{ab} = k_{ba} \fallingdotseq k_{ac} = k_{ca} \fallingdotseq k_{bc} = k_{cb} = -(p_m p_s - p_m^2)/\Delta \\ &= \frac{-p_m}{(p_s - p_m)(p_s + 2p_m)} \\ k_s + 2k_m &= \frac{1}{p_s + 2p_m} \end{aligned}\right\} \tag{1.30}$$

and from Equation 1.23

$$\left.\begin{aligned} C_s &\equiv C_{aa} \fallingdotseq C_{bb} \fallingdotseq C_{cc} = k_s + 2k_m = \frac{1}{p_s + 2p_m} \\ C_m &\equiv C_{ab} = C_{ba} \fallingdotseq C_{ac} = C_{ca} \fallingdotseq C_{bc} = C_{cb} = -k_m \\ &= \frac{p_m}{(p_s - p_m)(p_s + 2p_m)} = \frac{p_m}{p_s - p_m} \cdot C_s \end{aligned}\right\} \tag{1.31}$$

and from Equation 1.27

$$p_s \equiv p_{aa} \fallingdotseq p_{bb} \fallingdotseq p_{cc} = 2 \times 9 \times 10^9 \log_e \frac{2h}{r} \qquad [\mathrm{m/K}] \qquad ①$$

$$\begin{aligned} p_m &\equiv p_{ab} \fallingdotseq p_{bc} \fallingdotseq p_{ca} = 2 \times 9 \times 10^9 \log_e \frac{s_{b\alpha}}{s_{ll}} \qquad [\mathrm{m/F}] \\ &\fallingdotseq 2 \times 9 \times 10^9 \log_e \frac{\sqrt{s_{ll}^2 + (2h)^2}}{s_{ll}} \\ &= 2 \times 9 \times 10^9 \log_e \left\{1 + \left(\frac{2h}{s_{ll}}\right)^2\right\}^{1/2} \qquad [\mathrm{m/F}] \qquad ② \end{aligned} \tag{1.32}$$

where generally

$$h > s_{ll}, \quad \left(\frac{2h}{s_{ll}}\right)^2 \gg 1$$

and

$$\therefore \; p_m \fallingdotseq 2 \times 9 \times 10^9 \log_e \frac{2h}{s_{ll}} \quad [\mathrm{m/F}] \qquad ②'$$

Substituting p_s, p_m from Equation 1.32 into Equation 1.31,

$$C_s = \frac{1}{p_s + 2p_m} \doteqdot \frac{1}{2\times 9\times 10^9\left(\log_e \frac{2h}{r} + 2\log_e \frac{2h}{s_{ll}}\right)} = \frac{1}{2\times 9\times 10^9 \log_e \frac{8h^3}{rs_{ll}^2}}$$

$$= \frac{0.02413}{\log_{10}\frac{8h^3}{rs_{ll}^2}}\times 10^{-9}[\text{F/m}] = \frac{0.02413}{\log_{10}\frac{8h^3}{rs_{ll}^2}}[\mu\text{F/km}] \quad ①$$

(zero-sequence capacitance)

while

$$\frac{p_m}{p_s - p_m} \doteqdot \frac{\log_e \frac{2h}{s_{ll}}}{\log_e \frac{2h}{r} - \log_e \frac{2h}{s_{ll}}} = \frac{\log_{10}\frac{2h}{s_{ll}}}{\log_{10}\frac{s_{ll}}{r}}$$

$$\therefore C_m = C_s \cdot \frac{p_m}{p_s - p_m} \doteqdot C_s \cdot \frac{\log_{10}\frac{2h}{s_{ll}}}{\log_{10}\frac{s_{ll}}{r}} = \frac{0.02413}{\log_{10}\frac{8h^3}{rs_{ll}^2}} \cdot \frac{\log_{10}\frac{2h}{s_{ll}}}{\log_{10}\frac{s_{ll}}{r}}[\mu\text{F/km}] \quad ② \qquad (1.33)$$

In conclusion, a well-phase-balanced transmission line can be expressed by Figure 1.9a1 and Equation 1.26b is simplified into Equation 1.34, where the stray capacitances C_s, C_m can be calculated from Equation 1.33:

$$\underbrace{\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}}_{\boldsymbol{I}_{abc}} = j\omega \underbrace{\begin{bmatrix} C_s + 2C_m & -C_m & -C_m \\ -C_m & C_s + 2C_m & -C_m \\ -C_m & -C_m & C_s + 2C_m \end{bmatrix}}_{\boldsymbol{C}_{abc}} \cdot \underbrace{\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}}_{\boldsymbol{V}_{abc}} \qquad (1.34)$$

$$\therefore \boldsymbol{I}_{abc} = j\omega \boldsymbol{C}_{abc} \cdot \boldsymbol{V}_{abc}$$

Incidentally, Figure 1.9a1 can be modified to Figure 1.9a2, where the total capacitance of one phase $C \equiv C_s + 3C_m$ is called the **working capacitance** of single circuit transmission lines, and can be calculated by the following equation:

$$C \equiv C_s + 3C_m = (k_s + 2k_m) + 3(-k_m) = k_s - k_m = \frac{1}{p_s - p_m}$$

$$= \frac{1}{2\times 9\times 10^9\left(\log_e \frac{2h}{r} - \log_e \frac{2h}{s_{ll}}\right)} = \frac{1}{2\times 9\times 10^9 \log_e \frac{s_{ll}}{r}} \quad [\text{F/m}]$$

$$= \frac{0.02413}{\log_{10}\frac{s_{ll}}{r}} \quad [\mu\text{F/km}] \quad \text{(positive sequence capacitance)} \quad ①$$

In case of multi-bundled (n) conductor lines, the radius r is replaced by the equivalent radius r_{eff},

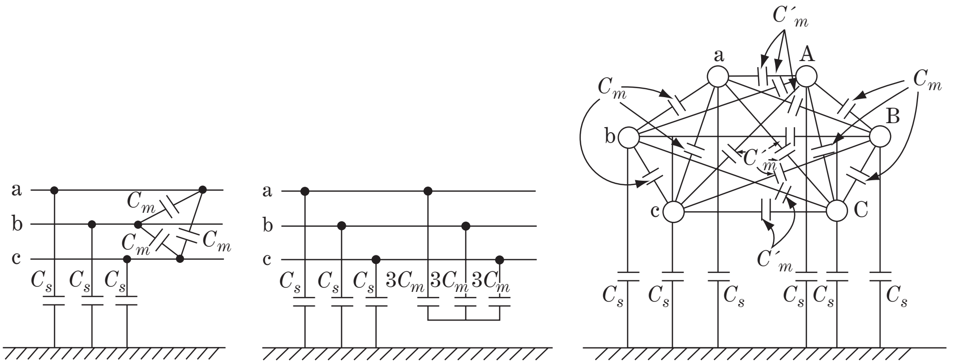
$$r_{\text{eff}} = r^{1/n} \times w^{(n-1)/n} \quad [\text{m}] \quad ② \qquad (1.35)$$

where w is the geometrical averaged distance between bundled conductors.

1.2.1.4.1 Numerical check

Taking the conditions conductor radius $r = 0.05$ m, averaged phase-to-phase distance $s_{ll} = 10$ m and average height $h = 60$ m, then by Equations 1.33 and 1.35, we have

$$C_s = 0.00436\,\mu\text{F/km},\ C_m = 0.00204\,\mu\text{F/km} \text{ and } C = C_s + 3C_m = 0.01048\,\mu\text{F/km}$$



(a1) single circuit line (a2) single circuit line (b) double circuit line

Figure 1.9 Stray capacitances of overhead line (well balanced)

1.2.2 Three-phase single circuit line with OGW

Four conductors of phase names a, b, c, x exist in this case, so the following equation can be derived as an extended form of Equation 1.26a:

$$I_a = j\omega C_{aa}V_a + j\omega C_{ab}(V_a - V_b) + j\omega C_{ac}(V_a - V_c) + j\omega C_{ax}(V_a - V_x) \tag{1.36a}$$

where $V_x = 0$, because OGW is earth grounded at every tower. Accordingly,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = j\omega \begin{bmatrix} C_{aa}+C_{ab}+C_{ac}+C_{ax} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{ba}+C_{bb}+C_{bc}+C_{bx} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{ca}+C_{cb}+C_{cc}+C_{cx} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{1.36b}$$

This matrix equation is again in the same form as Equation 1.26b. However, the phase-to-ground capacitance values (diagonal elements of the matrix $\boldsymbol{C}$) are increased (the value of C_{ax} is increased for the phase a conductor, from $C_{aa} + C_{ab} + C_{ac}$ *to* $C_{aa} + C_{ab} + C_{ac} + C_{ax}$).

1.2.3 Three-phase double circuit line

Six conductors of phase names a, b, c, A, B, C exist in this case, so the following equation can be derived as an extended form of Equation 1.26a:

$$\begin{aligned} I_a = j\omega[&C_{aa}V_a + C_{ab}(V_a - V_b) + C_{ac}(V_a - V_c) + C_{aA}(V_a - V_A) + C_{aB}(V_a - V_B) \\ &+ C_{aC}(V_a - V_C)] \end{aligned} \tag{1.37a}$$

Then

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_A \\ I_B \\ I_C \end{bmatrix} = j\omega \begin{bmatrix} \begin{matrix} C_{aa}+C_{ab}+ \\ C_{ac}+C_{aA}+ \\ C_{aB}+C_{aC} \end{matrix} & -C_{ab} & -C_{ac} & -C_{aA} & -C_{aB} & -C_{aC} \\ -C_{ba} & \begin{matrix} C_{ba}+C_{bb}+ \\ C_{bc}+C_{bA}+ \\ C_{bB}+C_{bC} \end{matrix} & -C_{bc} & -C_{bA} & -C_{bB} & -C_{bC} \\ -C_{ca} & -C_{cb} & \begin{matrix} C_{ca}+C_{cb}+ \\ C_{cc}+C_{cA}+ \\ C_{cB}+C_{cC} \end{matrix} & -C_{cA} & -C_{cB} & -C_{cC} \\ -C_{Aa} & -C_{Ab} & -C_{Ac} & \begin{matrix} C_{AA}+C_{AB}+ \\ C_{AC}+C_{Aa}+ \\ C_{Ab}+C_{Ac} \end{matrix} & -C_{AB} & -C_{AC} \\ -C_{Ba} & -C_{Bb} & -C_{Bc} & -C_{BA} & \begin{matrix} C_{BA}+C_{BB}+ \\ C_{BC}+C_{Ba}+ \\ C_{Bb}+C_{Bc} \end{matrix} & -C_{BC} \\ -C_{Ca} & -C_{Cb} & -C_{Cc} & -C_{CA} & -C_{CB} & \begin{matrix} C_{CA}+C_{CB}+ \\ C_{CC}+C_{Ca}+ \\ C_{Cb}+C_{Cc} \end{matrix} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_A \\ V_B \\ V_C \end{bmatrix} \quad (1.37b)$$

It is obvious that the double circuit line with OGW can be expressed in the same form.

The case of a well-transposed double circuit line is as shown in Figure 1.9b:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_A \\ I_B \\ I_C \end{bmatrix} = j\omega \begin{bmatrix} \begin{matrix} C_s+2C_m \\ +3C'_m \end{matrix} & -C_m & -C_m & -C'_m & -C'_m & -C'_m \\ -C_m & \begin{matrix} C_s+2C_m \\ +3C'_m \end{matrix} & -C_m & -C'_m & -C'_m & -C'_m \\ -C_m & -C_m & \begin{matrix} C_s+2C_m \\ +3C'_m \end{matrix} & -C'_m & -C'_m & -C'_m \\ -C'_m & -C'_m & -C'_m & \begin{matrix} C_s+2C_m \\ +3C'_m \end{matrix} & -C_m & -C_m \\ -C'_m & -C'_m & -C'_m & -C_m & \begin{matrix} C_s+2C_m \\ +3C'_m \end{matrix} & -C_m \\ -C'_m & -C'_m & -C'_m & -C_m & -C_m & \begin{matrix} C_s+2C_m \\ +3C'_m \end{matrix} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_A \\ V_B \\ V_C \end{bmatrix}$$

$$\begin{aligned} &C_s \equiv C_{aa} \fallingdotseq C_{bb} \fallingdotseq C_{cc} \fallingdotseq C_{AA} \fallingdotseq C_{BB} \fallingdotseq C_{CC} && \text{: one phase-to-ground capacitance} \\ &C_m \equiv C_{ab} \fallingdotseq C_{bc} \fallingdotseq \cdots \fallingdotseq C_{AB} \fallingdotseq C_{BC} \fallingdotseq \cdots && \text{: capacitance between two conductors of the same circuit} \\ &C'_m \equiv C_{aA} \fallingdotseq C_{bC} \fallingdotseq \cdots \fallingdotseq C_{Aa} \fallingdotseq C_{Bb} \fallingdotseq \cdots && \text{: capacitance between two conductors of a different circuit} \end{aligned} \quad (1.38)$$

Above, we have studied the fundamental equations and circuit models of transmission lines and the actual calculation method for the L, C, R constants. Concrete values of the constants are investigated in Chapter 2.

1.3 Supplement: Additional Explanation for Equation 1.27

Equation 1.27① can be explained by the following steps.

Step-1: The induced voltage v at arbitrary point y in Figure a

Figure a shows two parallel conductors x, x' (radius r) whose mutual distance is s [m] and the conductor length is l [m], where $l \gg s$.

When charges $+q, -q$ are applied to per unit length of the conductors x, x' respectively, the voltage potential v at the arbitrary single point y is given by the following equation (expressed in MKS units):

$$v = \frac{2q}{4\pi\varepsilon_0} \cdot \log_e \frac{s_2}{s_1} = 2q \cdot 9 \times 10^9 \log_e \frac{s_2}{s_1} \quad (1)$$

where

$$4\pi\varepsilon_0 = \frac{1}{9\times10^9} \quad \text{(refer the Equation (4) in the next page.)}$$

The voltage of the centre line g is obviously zero.

Step-2: The induced voltage v at arbitrary point y when $+q_a$ is applied to the overhead conductor a in Figure b

This is a special case shown in Figure b in which the names of the conductors have been changed ($x \to$ a, $x' \to \alpha$). The upper half zones of Figures a and b (the open space above the earth surface) are completely the same. Accordingly, under the state of a single overhead conductor a with an existing charge $+q$, the voltage v at the arbitrary point y in the open space can be calculated from Equation 1.

Step-3: The induced voltage v_a on the conductor a when $+q_a$ is applied to conductor a

This case corresponds to choosing the arbitrary point y on the conductor surface in Figure b. Therefore the voltage v_a can be derived by replacing $s_1 \to r$, $s_2 \to 2h$ in Equation 1:

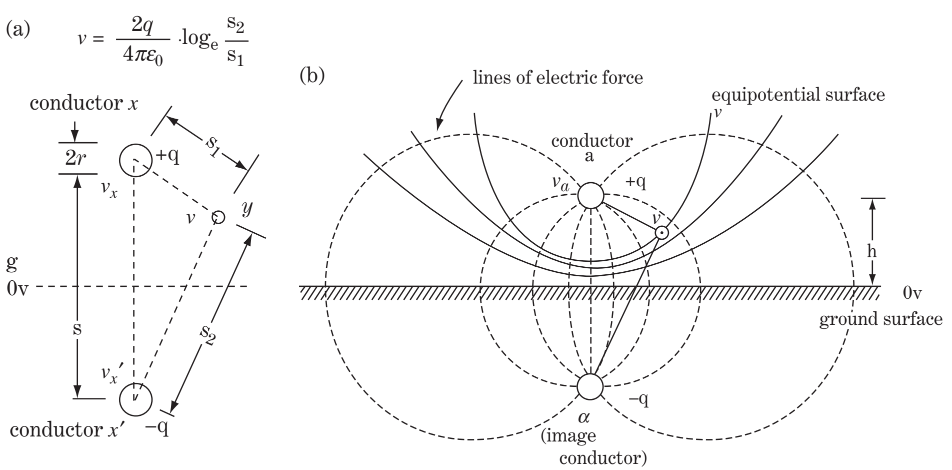
$$\therefore v_a = 2q_a \cdot 9\times10^9 \log_e \frac{2h}{r} \tag{2}$$

Step-4: The induced voltage v_a on the conductor a when $+q_b$ is applied to conductor b

This case corresponds to replacement of $x \to$ b, $x' \to \beta$, $y \to$ a in Figure a. Accordingly, $s_1 \to s_{ab}$, $s_2 \to_{a\beta}$ in Equation 1,

$$\therefore v_a = 2q_b \cdot 9\times10^9 \log_e \frac{s_{a\beta}}{s_{ab}} \tag{3}$$

Equations 2 and 3 are the first and second terms on the right-hand side of Equation 1.27①. The equation is the expanded case for three conductors by applying the theorem of superposition. Clearly, the equation can be expanded to the cases of parallel multi-conductors of arbitrary number n. (Note that $4\pi\varepsilon_0 = 1/(9\times10^9)$.)



In the rationalized MKS system of units, $4\pi\varepsilon_0$ is given by the equation

$$4\pi\varepsilon_0 = \frac{1}{c_0^2} \cdot 10^7 = \frac{1}{(3 \times 10^8)^2} \cdot 10^7 = \frac{1}{9 \times 10^9} \tag{4}$$

where
c_0 is the velocity of electromagnetic waves (light), $c_0 = 3 \times 10^8$ [m/s]
10^7 is the coefficient of translation from CGS to MKS units, namely

$$\text{energy} = (\text{force}) \cdot (\text{distance}) = ((\text{kg} \cdot \text{m/s}^2) \cdot \text{m}) = ((\text{g} \cdot \text{cm/s}^2) \cdot \text{cm}) \times 10^7$$

Coffee break 1: Electricity, its substance and methodology

The new steam engine of **James Watt** (1736–1819) ushered in the great dawn of the Industrial Revolution in the 1770s. Applications of the steam engine began to appear quickly in factories, mines, railways, and so on, and the curtain of modern mechanical engineering was raised. The first steam locomotive, designed by **George Stephenson** (1781–1848), appeared in 1830.

Conversely, electrical engineering had to wait until Volta began to provide ‘stable electricity’ from his voltaic pile to other electrical scientists in the 1800s. Since then, scientific investigations of the unseen electricity on one hand and practical applications for **telegraphic communication** on the other hand have been conducted by scientists or electricians simultaneously, often the same people. In the first half of the nineteenth century, the worth of electricity was recognized for telegraphic applications, but its commercial application was actually realized in the 1840s. Commercial telegraphic communication through wires between New York and Boston took place in 1846, followed at Dover through a submarine cable in 1851. However, it took another 40 years for the realization of commercial applications of electricity as the replacement energy for steam power or in lighting.