Chapter 1
TRANSLATING EXPRESSIONS

Chapter Check-In

- ❑ Addition and subtraction keywords
- ❑ Multiplication and division keywords
- ❑ Turnaround words

The first step in solving a word problem is always to read the problem. You need to be able to **translate** words into mathematical symbols, focusing on **keywords** that indicate the mathematical procedures required to solve the problem—both the operation and the order of the expression. In much the same way that you can translate Spanish into English, you can translate English words into symbols, the language of mathematics. Many (if not all) keywords that indicate mathematical operations are familiar words.

Keywords of Basic Mathematical Operations

To begin, you translate English phrases into algebraic **expressions.** An algebraic expression is a collection of numbers, **variables,** operations, and **grouping symbols.** You will translate an unknown number as the variable *x* or *n.* The grouping symbols are usually a set of parentheses, but they can also be sets of brackets or braces.

In translating expressions, you want to be well acquainted with basic keywords that translate into mathematical operations: addition keywords, subtraction keywords, multiplication keywords, and division keywords, which are covered in the four following sections.

Addition keywords

Some common examples of addition keywords are as follows:

SUM OF_____ AND _____
TOTAL OF _____ AND _____
_____ PLUS _____
_____ INCREASED BY _____
GAIN
RAISE
MORE
INCREASE OF

The first two keywords (SUM and TOTAL) are called **leading keywords** because they lead the expression. The second two keywords (PLUS and INCREASED BY) are keywords that indicate the exact placement of the plus sign. The last four keywords can be found in word problems and may indicate addition.

When an expression begins with the leading keywords SUM or TOTAL, the leading keyword defines the corresponding AND. The plus sign then physically replaces the AND in the expression.

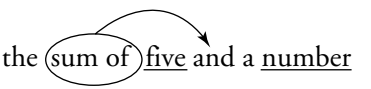
Example 1: Translate the following: the sum of five and a number

The following steps help you translate this problem:

1. **Underline the words before and after AND when it corresponds to the leading keyword SUM OF.**

   the sum of five and a number

2. **Circle the leading keyword and indicate the corresponding AND that it defines.**

3. **Translate each underlined expression and replace AND with a plus sign.**

   The expression translates to $5 + x$.

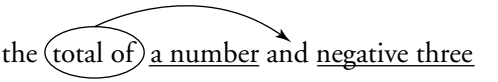
Example 2: Translate the following: the total of a number and negative three

Use the following steps to translate this problem:

1. **The keyword TOTAL OF is a leading keyword that defines AND, so underline the words before and after AND: "a number" and "negative three."**

   the total of a number and negative three

2. **Circle the leading keyword and indicate the corresponding AND that it defines.**

3. **Translate each underlined expression and replace AND with a plus sign.**

   The expression translates to $x + -3$.

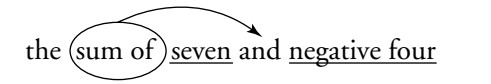
Example 3: Translate the following: the sum of seven and negative four

Translate this example in the following way:

1. **The word SUM OF is a leading keyword that defines AND , so underline the words before and after AND: "seven" and "negative four."**

   the sum of seven and negative four

2. **Circle the leading keyword and indicate the corresponding AND that it defines.**

3. **Translate each underlined expression and replace AND with a plus sign.**

   The expression translates to $7 + -4$.

Reminder: The AND keyword translates to mean "plus" because the leading keyword is SUM OF. With other leading keywords (discussed in the following sections), AND can mean other things. Also notice that you do not simplify the expression and get "3" for the answer because you are just translating words into symbols and not performing the math.

Two other keywords on the addition keyword list, PLUS and INCREASED BY, can be correctly translated by the **direct translation strategy.** In the direct translation strategy, you translate each word into its corresponding algebraic symbol, one at a time, in the same order as written, as shown in Example 4.

Example 4: Translate the following: a number increased by twenty-four

The expression translates to $x + 24$.

Some additional keywords, such as GAIN, MORE, INCREASE OF, and RAISE, are commonly found in story problems, as in Example 5.

Example 5: Translate the following story problem into a mathematical expression about the weight of the linebacker: The defensive linebacker weighed two hundred twenty-two pounds at the beginning of spring training. He had a gain of seventeen pounds after working out with the team for four weeks.

The expression translates to 222 + 17.

Note: Not all numbers mentioned in a word problem should be included in the mathematical expression. The number "four" is just interesting fact, but it is not information you need in order to write an expression about the linebacker's weight.

You may also be wondering why the answer isn't 239 pounds. That's because the question asks you to translate the story problem into a mathematical expression, not to evaluate the expression.

Example 6: Translate the following word problem into a mathematical expression about the cashier's current hourly wage: A cashier at the corner grocery was earning $6.25 an hour. He received a raise of 25 cents an hour.

The expression translates to 6.25 + 0.25.

Note: The hourly wage is stated in dollars, and the raise is stated in cents. Any time you are adding two numbers that have **units,** make sure both numbers are measured with the same units; if they aren't, convert one of the numbers to the same units as the other. Having both numbers measured with the same units is called **homogeneous units.** In this example, you convert his raise, the 25 cents, to $0.25 because his hourly wage is measured in dollars, not cents, so the raise must also be in dollars.

Subtraction keywords

Subtraction keywords also include leading keywords, keywords that can be translated one word at a time, and keywords that are found in story problems. Look at the following list of subtraction keywords:

DIFFERENCE BETWEEN _____ AND _____
_____ MINUS _____

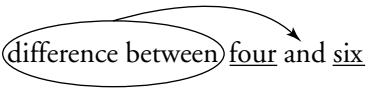
_____ DECREASED BY _____
LOSS
LESS
FEWER
TAKE AWAY

One subtraction keyword (DIFFERENCE BETWEEN) is a two-part expression that begins with a leading keyword that defines the corresponding AND. You can use the same methods of underlining and circling the keywords shown in the preceding section to translate these expressions.

Example 7: Translate the following: the difference between four and six

Here is how you translate Example 7:

1. **Because the keyword DIFFERENCE BETWEEN is a leading keyword that defines the corresponding AND, underline the words before and after AND: "four" and "six."**
   the difference between four and six
2. **Circle the leading keyword and indicate the corresponding AND that it defines.**



3. **Translate each underlined expression and replace AND with a minus sign.**
   The expression translates to 4 – 6.

Note: AND is not always translated to mean addition. Here, the DIFFERENCE BETWEEN is the leading keyword that defines the AND to mean subtraction**.**

Other subtraction keywords, such as MINUS and DECREASED BY, use the direct translation strategy. Example 8 is a subtraction word problem that is translated one keyword at a time, in the exact order of the expression.

Example 8: Translate the following: twenty-four decreased by a number

The expression translates to $24 - x$.

In a story problem, you may find the subtraction keywords LOSS, LESS, FEWER, and TAKE AWAY, as shown in Example 9.

Example 9: Translate the following word problem into a mathematical expression about the current value of materials at the job site: A construction company stored $1,253 worth of materials at the job site. The company suffered a loss of $300 due to storm damage.

The expression translates to 1,253 – 300.

Multiplication keywords

Some common examples of multiplication keywords are as follows:

MULTIPLY _____ BY _____
PRODUCT OF _____ AND _____
_____ TIMES _____
DOUBLE _____
TWICE _____
TRIPLE _____
PERCENT OF _____
FRACTION OF _____

For two of the multiplication keywords, MULTIPLY and PRODUCT OF, a leading keyword defines the corresponding BY or AND, as shown in Example 10.

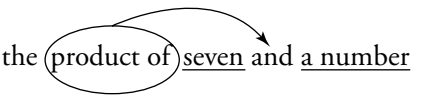
Example 10: Translate the following: the product of seven and a number

Translate this example in the following way:

1. **Because PRODUCT OF is a leading keyword that corresponds to AND, underline the words before and after AND: "seven" and "a number."**

   the product of seven and a number

2. **Circle the leading keyword and indicate the corresponding AND that it defines.**

3. **Translate each underlined expression and replace AND with a times sign.**

   The expression translates to $7 \times x$.

Note: Keep in mind that AND does not always indicate addition. The keyword PRODUCT OF defines the AND in this expression to mean multiplication.

A multiplication expression that is translated by the direct translation method is shown in Example 11.

Example 11: Translate the following: a number times fifteen

The expression translates to $x \times 15$.

Some multiplication keywords, such as DOUBLE, TWICE, and TRIPLE, translate into a number and the operation of multiplication, as shown in Examples 12 and 13.

Example 12: Translate the following: twice a number

The expression translates to $2 \times x$.

Example 13: Translate the following word problem into a mathematical expression: Jennifer had $15 dollars in the bank. Over the next two weeks she doubled her money.

The expression translates to 2×15.

One of the keywords that indicates multiplication is OF. In word problems, however, you may see more than one use of the word "of." The only OF that indicates multiplication is the one that follows the keyword PERCENT, the percent sign, the keyword FRACTION, or a fraction. See Examples 14 and 15.

Example 14: Translate the following: twenty five percent of four hundred dollars

The expression translates to 0.25×400.

Note: Remember that a percent is changed to a decimal before multiplying.

Example 15: Translate the following: one-third of twenty-seven

The expression translates to $\frac{1}{3} \times 27$.

Division keywords

Some common examples of division keywords are as follows:

QUOTIENT OF _____ AND _____
DIVIDE _____ BY _____

_____ DIVIDED BY _____
DIVIDED EQUALLY
PER

The keywords PRODUCT OF and QUOTIENT OF are difficult for some people to differentiate. Here is a hint to help you remember which one indicates division and which one indicates multiplication: Quotient is a "harder" word than "product," and division is a "harder" operation than multiplication.

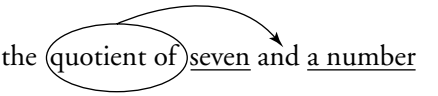
Remember: Leading keywords define the corresponding AND or BY to mean divide, usually designated with the symbol ÷.

Example 16: Translate the following: the quotient of seven and a number

1. **Because the keyword QUOTIENT OF is a leading keyword that defines AND, underline the words before and after AND: "seven" and "a number."**

   the quotient of seven and a number

2. **Circle the leading keyword and indicate the corresponding AND that it defines.**



3. **Translate each underlined expression and replace AND with a division sign.**

   The expression translates to $7 \div n$.

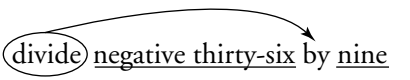
Note: Here, the keyword QUOTIENT OF defines AND to mean division.

Example 17: Translate the following: divide negative thirty-six by nine

1. **Because the word DIVIDE is a leading keyword that defines the BY, underline the words before and after BY: "negative thirty-six" and "nine."**

   divide negative thirty-six by nine

2. **Circle the leading keyword and indicate the corresponding BY that it defines.**



3. **Translate each underlined expression and replace AND with a division sign.**
   The expression translates to $-36 \div 9$, $\frac{-36}{9}$, or $9\overline{)-36}$.

Note: The first number goes in the numerator when using a fraction bar to indicate division. The number in the numerator (the –36) goes inside the "house" when using the long division symbol.

Some division keywords can be translated one word at a time. Instead, you just follow the sentence and replace with algebraic notations as you go along.

Example 18: Translate the following: a number divided by 16

The expression translates to $x \div 16$ or $\frac{x}{16}$.

Often, in story problems, the keyword that indicates division is PER. When a story problem asks for the speed of a vehicle in miles per hour, set up the expression to divide the number of miles by the number of hours. You not only directly translate "miles" ÷ "hours," but also identify the number of miles and number of hours by finding them elsewhere in the problem. See Example 19.

Example 19: Translate the following word problem into a mathematical expression about speed: It takes three hours to travel 150 miles to grandmother's house. How do you find your average speed in miles per hour?

You find "miles" ÷ "hours" in the question. In the first part of the word problem, you find the number of miles, 150 miles, and the number of hours, three hours.

The expression translates to $150 \div 3$.

Keywords That Indicate a Change in Order

Some keywords are not included in other lists in this chapter because they are a bit different from other types of keywords. This section gives you additional keywords, called **turnaround words,** and these indicate a change in order from the original English phrase.

Throughout this chapter, you use keywords in one of two ways:

- Translate the words directly, in the order they are given.
- Recognize leading keywords and find the corresponding AND, TO, BY, or FROM that tell you how to translate the equation.

To help you understand turnaround words, think about the directions on a box of cake mix. If the directions say "three eggs added to the mix," which do you put into the bowl first? You put the mix into the bowl first, and then add the eggs. The word TO is one of the basic turnaround words discussed in the following section, and to help you remember to turn the expression around, you box the word.

three eggs added [to] the mix

Basic turnaround words

Certain keywords indicate a turn around in the order of the translation. All of the keywords indicating a change in order contain the following words:

TO
FROM
THAN

Addition turnaround words

Addition keywords that indicate a turnaround are

ADD _____ TO _____
_____ ADDED TO _____
_____ MORE THAN _____

Example 20: Translate the following: twelve added to negative four

To help you translate this problem, box the turnaround word.

twelve added [to] negative four

Replace each word with algebraic symbols and turn the expression around.

The expression translates to $-4 + 12$.

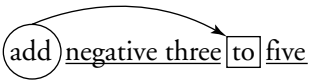
Example 21: Translate the following: add negative three to five

Example 21 uses both a turnaround word and a leading keyword, and it is translated as follows:

1. **Because the first word in the expression, ADD, indicates an operation, ADD is a leading keyword. ADD defines the TO, so underline the words before and after TO: "negative three" and "five."**

   add <u>negative three</u> to <u>five</u>

2. **Circle the leading keyword and indicate the corresponding TO that it defines; box the turnaround word, TO.**



3. **Translate each underlined expression, replace TO with a plus sign, and turn the expression around.**

   The expression translates to 5 + –3.

People sometimes argue that a turnaround word is not necessary with addition because of the **Commutative Property of Addition;** that is, both –3 + 5 and 5 + –3 result in the same answer for Example 21 (+2) when **simplified.** However, the latter (5 + –3) shows an understanding of the order indicated by the keyword. You have two reasons to learn to translate expressions in the correct order:

- Good habits are formed for correct translation of subtraction and division expressions, which are not commutative.
- Just as the makers of a cake mix intend for you to put the cake mix in the bowl first and add the eggs later, the author of the problem intends to have you perform the addition in the prescribed order.

Subtraction turnaround words

Subtraction keywords that indicate a turnaround are

SUBTRACT _____ FROM _____
_____ SUBTRACTED FROM _____
_____ LESS THAN _____

Example 22: Translate the following: a number less than seven

To help you solve this problem, box the turnaround word, THAN.

a number less than seven

Replace each word with algebraic symbols and turn the expression around.

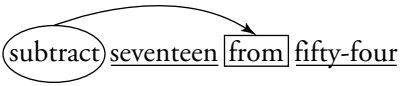
The expression translates to $7 - x$.

Example 23: Translate the following: subtract seventeen from fifty-four

1. **Because the word SUBTRACT is a leading keyword that defines FROM, underline the words before and after FROM: "seventeen" and "fifty-four."**

   subtract seventeen from fifty-four

2. **Circle the leading keyword and indicate the corresponding FROM that it defines; box the turnaround word, FROM.**



3. **Translate each underlined expression, replace FROM with a minus sign, and turn the expression around.**

   The expression translates to 54 – 17.

Multiplication turnaround words

None of the multiplication keywords indicates a turnaround. All multiplication expressions can be translated using the direct translation strategy or leading keywords.

The product of a number and 8 can be translated $n \times 8$, but most often, you see the expression written as $8 \times n$ or $8n$, because mathematicians have set a standard that the **coefficient** is written before the variable. (In this example, the number 8 is the coefficient.)

Note: The expression $8n$ uses **implied multiplication.** Multiplication is implied when a number is placed next to a variable, or when a number is placed next to an expression surrounded by parentheses. Although a multiplication sign is not shown, its use is implied.

Division turnaround words

Division keywords that indicate a turnaround are:

DIVIDE _____ INTO _____
_____ DIVIDED INTO _____

Notice that TO, a basic turnaround word, is included in the word INTO and indicates a turnaround.

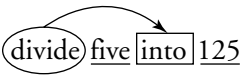
Example 24: Translate the following: divide five into 125

Solve Example 24 as follows:

1. **Because the word DIVIDE is a leading keyword that defines INTO, underline the words before and after INTO: "five" and "125."**

   divide five into 125

2. **Circle the leading keyword and indicate the corresponding INTO that it defines; box the turnaround word, INTO.**



3. **Translate each underlined expression, replace INTO with a division sign, and turn the expression around.**
   The expression translates to 125 ÷ 5.

Example 25: Translate the following: twenty-five divided into one hundred

To help you translate this problem, box the turnaround word, INTO.

twenty-five divided into 100

Replace each word with algebraic symbols and turn the expression around.

The expression translates to 100 ÷ 25.

Chapter Checkout

Q&A

Translate the following English phrases into algebraic expressions or fill in the blanks.

1. The sum of six and a number
2. Seven plus five
3. The difference between a number and eleven
4. Three minus a number
5. Multiply four by a number
6. Ten percent of twenty
7. One fifth of forty-five
8. Triple a number
9. A number divided by eight
10. The quotient of forty-nine and a number
11. The basic turnaround words are _____, _____, and _____.
12. Add four to a number
13. A number added to three
14. Seven more than five

15. Subtract three from four

16. Thirty-six less than a number

17. Divide five into twenty-five

Answers: 1. $6 + x$ **2.** $7 + 5$ **3.** $x - 11$ **4.** $3 - x$ **5.** $4x$ **6.** 0.10×20 **7.** $\frac{1}{5} \times 45$ **8.** $3x$ **9.** $x \div 8$ **10.** $49 \div x$ **11.** TO, FROM, THAN **12.** $x + 4$ **13.** $3 + x$ **14.** $5 + 7$ **15.** $4 - 3$ **16.** $x - 36$ **17.** $25 \div 5$