1

Newton's Laws

1.1 What is Mechanics?

Even those uninterested in physics seem to have an intuitive notion about why and how things move. If a ball flies through the air, it does so because we have projected it—a *force* has been applied to a *body* which impels it to *move*. Glossing deftly over any difficulties there may be in defining what the italicised words actually mean, a description of the sequence force–body–motion is, conventionally, what is meant by **mechanics**. Mechanics is about forces and motion as applied to bodies.

Some people further resolve mechanics into **kinematics** and **dynamics**. Kinematics is the description of motion in terms of its trajectory through space as time progresses, or technically as $\mathbf{r}(t)$ where $\mathbf{r}$ is the position vector of the (dimensionless) body at time t. Typically this trajectory is calculated by solving an equation of motion. Dynamics, on the other hand, relates changes in a body's motion to their causes, i.e. forces. Dynamics is therefore the 'why' of motion, a typical dynamical problem being to find the resultant force acting on a body. Personally I have never found the delineation of kinematics and dynamics as branches of mechanics particularly useful, and so these terms will be avoided in this book.

1.2 Mechanics as a Scientific Theory

So how can we quantify the relation between the forces that act on bodies and their resultant motions? Given the obvious immediacy of this problem to our everyday lives, it is not surprising that progress in this area, firstly by Galileo and then by Newton, heralded the first truly scientific theory. A fair question to ask at this stage is: 'What are the characteristics of a scientific theory?' Without becoming too philosophical about the issue, it can be said that a scientific[1] theory is a concise summary of scientific ideas describing the results of experiments. Furthermore, a good theory will have predictive power beyond the domain of experimental experience to date. In physics, the theory is usually expressed as a mathematical relationship between experimentally determinable quantities. This means that the acid test which all physical theories must pass is:

[1]The adjective 'scientific' is perhaps unnecessary as, in my view, any theory in the sense described here can only be applied in the scientific context. Thus, 'management theory', 'education theory' and similar trendy oxymorons do not constitute bodies of ideas of any meaningful predictive power, and have merely empirical status.

Classical Mechanics – Second Edition From Newton to Einstein: A Modern Introduction Martin W. McCall

A good theory must agree with experiment.

Famous physicists have apparently disagreed over this fundamental idea. Feynman, in a popular lecture, commented that it does not matter how elegant the mathematical structure of the theory; if it disagrees with experiment, then it must be rejected. Dirac, however, famously reflected that it is better to have beauty in the mathematical statement of the theory, than for it to agree with experiment. They are both right. At the crude level, any gross disagreement with experiment must signal the rejection of a theory. However, Dirac's keen insight was that the most profound physical theories invariably have a beautiful mathematical structure, and apparent disparity with experiments may be due to extraneous factors. Even in well-established theories like classical mechanics, their mathematical structure is still being explored in interesting ways.

1.3 Newtonian vs. Einsteinian Mechanics

For 300 years the bastion of mechanics were the laws enunciated by Isaac Newton. And yet, even now, when better theories are known, 99.9% of the time practising physicists can still use Newton's laws with confidence. Again, this tells us something about the nature of scientific theories, for a minimum requirement of a new theory is that it reproduces all the results of its predecessors in appropriate limiting cases. Niels Bohr codified this precisely as the so-called Correspondence Principle. Thus when, in 1905, Einstein revolutionised physics by carefully refining our notions of space, time and frames of reference, the new laws of mechanics had to reproduce those of Newton when the speeds under consideration were much less than the speed of light.

Why bother to study Newtonian mechanics then, when we have better theories? Why not start and finish with the very best theory of mechanics to date, our predecessors having carefully checked its consistency with previous theoretical descriptions? The simple answer is that we learn incrementally from our experience, and so until we have some exposure to speeds approaching those of light we simply have no intuition on which to guide our deliberations, thus making the better theory harder to learn. Once in a lecture, and to the amusement of the class, I inadvertently omitted the '$\times 10^8$' when writing down the speed of light. When my slip was pointed out, I remarked how intuitive the notions of special relativity would become if indeed the speed of light were the 'everyday' speed of 3 ms^{-1}.

However, I believe there is a deeper answer relating to what might be called 'nature's hierarchy of beauty'. Yes, Einstein's relativistic description is more symmetric and, if we take symmetry to be a measure of aesthetic appeal (this is probably what Dirac meant), more beautiful than classical mechanics, but that symmetry is simply not apparent when analysing 'everyday' situations when $v \ll c$. In fact, at low speeds Newtonian mechanics develops a completely distinct geometrical structure (in which time plays the role of a parameter rather than a coordinate), in such a way as to make Newtonian theory a simplifying, or clarifying, description of nature at this level. In this sense nature appears to have a natural hierarchy of simplicity, and a theory that is naturally suited to the description of phenomena at different levels of detail.

Whatever the reason, most students find Einstein's relativity harder to understand at first reading than a review of the Newtonian mechanics they began to learn at school and so, since

Newtonian mechanics is still a remarkably successful theory for most situations, that is where we will begin.

1.4 Newton's Laws

Let us state Newton's laws, not in the archaic language of the *Principia* (1687), but in modern terminology:[2]

1. A body remains at rest or moves with constant velocity, $\mathbf{v}$, when no external force acts on it:
$$\frac{d}{dt}\mathbf{v} = \mathbf{0} \quad \Leftrightarrow \quad \mathbf{F} = \mathbf{0}. \tag{1.1}$$

2. The rate of change of momentum of a body is proportional to the force on the body:
$$\mathbf{F} \propto \frac{d}{dt}m\mathbf{v}\ . \tag{1.2}$$

3. When two bodies interact, they exert on each other equal, but opposite, forces:
$$\mathbf{F}_{12} = -\mathbf{F}_{21}\ . \tag{1.3}$$

Now let us carefully unpack these statements seeking, as we go, their real content. Ask yourself the following: bearing in mind the brief discussion above on the nature of physical theories, are the statements laws, postulates, definitions or empirical observations?

The first law is Galileo's law of inertia and breaks away from the ideas of mechanics dating back to Aristotle that a body's natural state is at rest. The intellectual leap in formulating the first law should not be underestimated. We are mostly familiar with the notion that bodies slow down as they move, apparently striving to achieve, according to the Aristotelean view, a 'natural state of rest'. Of course we are now comfortable with the idea that bodies slow down because of the application of frictional forces, though this could hardly have been self-evident at the time of Galileo and Newton. If constant velocity is the 'natural' state of motion when a body is 'free' then interactions from external influences change this 'natural state'. The second law tells us how.

The second law (which in fact was first written down by Robert Hooke) is the kernel of Newtonian mechanics, quantifying the relationship between forces and motion, and asserting that the rate of change of a body's momentum is proportional to the force impelling a body to move. The definition that momentum is mass times the velocity is implicit. The constant of proportionality, k, is, by choice of units, defined to be equal to one, the resulting unit of force being equal to one newton (appropriately enough), usually written as $1\ \mathrm{N}$. Thus a force of $1\ \mathrm{N}$ causes a mass of $1\mathrm{kg}$ to accelerate at $1\,\mathrm{m\,s^{-2}}$. The weight of an apple is (appropriately enough) about $1\ \mathrm{N}$. We are taught at school to differentiate carefully between mass and weight, the latter being the force exerted on a mass due to gravity at the surface of

[2] Vector quantities, with which the reader is assumed to have a little familiarity, appear in bold face. However, when considering motion in one dimension vector quantities will appear in italic face, the direction being dictated by the sign. Note the distinction between the null vector $\mathbf{0}$, whose components all vanish, and the scalar 0.

a planet (usually the Earth). Most systems to which the second law is applied have constant mass (though rocket motion is an important exception – see Section 4.8), in which case

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} \,, \tag{1.4}$$

where $\mathbf{a}$ is the body's acceleration. This is the form in which the second law is most frequently stated, though it is very important to remember that it can only be written in this way for constant mass systems. Only then can we say that the first law is a special case of the second, since if $\mathbf{F} = \mathbf{0}$, then integrating Equation (1.4), gives,

$$\mathbf{v} = \mathbf{constant} \,, \tag{1.5}$$

which is the first law.

The third law codifies the relationship between forces acting between two bodies. If $\mathbf{F}_{12}$ is the force acting on body one due to body two, then Newton's third law asserts that body two exerts an equal and opposite force on body one, i.e. $\mathbf{F}_{12} = -\mathbf{F}_{21}$. Careful: never has there been an elementary law of physics so misunderstood. The third law does *not* refer to two forces acting on the *same* body. So when a book rests on a table, the force of gravity *on the book* is balanced by the electromagnetic forces which keep both bodies rigid, thus providing a reaction *on the book* (see Figure 1.1). Apply more downward force to the book and the

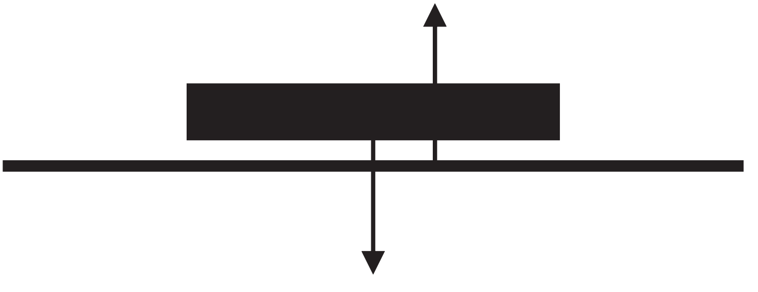

Figure 1.1 A book resting on a table: the forces acting on the book are *not* a good example of Newton's third law.

electromagnetic forces maintaining the rigidity of the system will continue to keep the book at rest until the table collapses. The third law applies to the reciprocal forces between two bodies interacting with each other. When two masses are connected by a spring the forces acting on each mass due to the other mass are equal and opposite (see Figure 1.2). This is a good example of the third law in action, which we will use shortly to measure mass. Avoid using the phrase 'Action and reaction are equal and opposite', which is at best unclear. A consequence of Newton's laws is that if $\mathbf{F}_{12} = -\mathbf{F}_{21}$ then, by the second law,

$$\frac{d}{dt}\left(m_1\mathbf{v}_1 + m_2\mathbf{v}_2\right) = \mathbf{0} \,, \tag{1.6}$$

or

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = \mathbf{constant} \,. \tag{1.7}$$

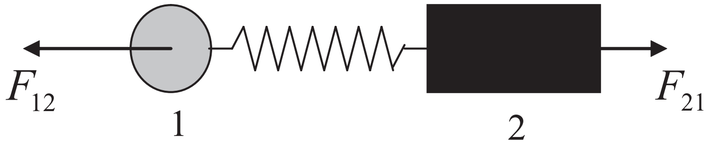

Figure 1.2 The mutual interaction of two masses connected by a spring as an example of Newton's third law.

Thus, for a two-body system in the absence of any external forces, the momentum is conserved. Clearly this idea can be extended to an arbitrary number of bodies so that we can assert that for an isolated system (i.e. one where no external forces are acting) of N bodies, the total momentum is conserved:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_N\mathbf{v}_N = \mathbf{constant}\ . \tag{1.8}$$

Beware! Whenever a lecturer or author uses the phrase 'clearly it follows that so-and-so is true', almost invariably it conceals some extra subtleties. The extension of momentum conservation to a system of N bodies is explored more rigorously in Problem 1.1 at the end of this chapter.

1.5 A Deeper Look at Newton's Laws

Let us start by examining carefully the terms used in the statement of the laws. What is a *force*? It is actually rather hard to say what a force *is*, rather than what it *does*. What it does is produce acceleration (for constant mass systems), but how can we test the proportionality between force and acceleration, without some independent quantification of force? Maybe we could use Hooke's law in relation to spring extensions to measure forces, though we might accidentally exceed the elastic limit and obtain spurious results (which actually violate the second law). So it seems that the second 'law' is almost a kind of definition of force. To some extent this is true, since mechanics provides no prescription as to how forces arise. The forces are presumed to be 'given' from other branches of physics – for gravitation and electrostatics by inverse square laws, for example. Once a force law is given, the second law allows us to make further progress by relating how that force impels a body to move. Moreover, as shown above, in the absence of external forces, the momentum (*defined* as mv) is conserved for an isolated system. Although momentum has been defined as mv, the *prediction* of its conservation in isolated systems is testable by experiment. It is then the business of experimental physics to confirm that momentum for an isolated system is indeed conserved. If it is, then we have found out something fundamental about nature and the statements are beyond mere definitions. Momentum *is* found to be conserved experimentally.

What about mass? Mass measures the amount of matter present in a body. As implied by the second law, it measures how hard we have to push a body to achieve a given acceleration. But again we encounter the potentially circular reasoning whereby the second law is, initially at any rate, being used as a kind of definition of mass. To progress further, and to have some measure of mass that is somehow independent of the second law, we must subject two masses (a test mass and the mass we wish to measure, say) to the *same* force and compare their accelerations. How can we know that the same magnitude of force is being applied? Answer: use the third law! (See Figure 1.2.) From the second law, the magnitude of the acceleration of the two masses is

$$a_1 = \frac{|\mathbf{F}_{12}|}{m_1} \text{ and } a_2 = \frac{|\mathbf{F}_{21}|}{m_2} ,$$

so that the mass ratio is given by

$$\frac{m_2}{m_1} = \frac{|\mathbf{F}_{21}|/a_2}{|\mathbf{F}_{12}|/a_1} = \frac{a_1}{a_2}, \tag{1.9}$$

since $|\mathbf{F}_{12}| = |\mathbf{F}_{21}|$, by the third law. Hence, if the accelerations of each of the masses are measured as the spring pushes them apart, then the mass m_2 is given by

$$m_2 = m_1 \left(\frac{a_1}{a_2} \right) . \tag{1.10}$$

Equation (1.10) then measures the mass of an unknown test mass, m_2, in units of a standard mass, m_1.

In this context it is worth drawing the distinction between **inertial mass** and **gravitational mass**. What we have discussed above is *inertial* mass, being the mass used in Newton's second law. However, Newton's theory of gravitation states that the attraction between two bodies separated by a distance r is given by

$$\mathbf{F} = -\frac{G m_G M_G}{r^2} \hat{\mathbf{r}} , \tag{1.11}$$

where G is a universal constant and $\hat{\mathbf{r}}$ is a unit vector directed along the separation between the two masses, the minus sign indicating attraction. The subscript G on the two masses, m and M, indicates that there is no a priori connection between the mass appearing in Equation (1.11) and the inertial mass of the second law. Mass in the gravitation law is referred to as *gravitational* mass. It is the 'charge' associated with the gravitation law (the corresponding law for electrostatic attraction in vacuum is $\mathbf{F} = \left(Q_1 Q_2 / 4\pi\epsilon_0 r^2\right) \hat{\mathbf{r}}$ where Q_1 and Q_2 are the charges and ε_0 is a constant). Now it happens that experimentally it is found that the gravitational mass of a body is proportional to its inertial mass. Moreover, by appropriate choice of units, the gravitational mass is *equal* to the inertial mass. Galileo's experiment to show the proportionality is the most famous physics experiment of the post-Renaissance era. Whether Galileo actually dropped balls from the top of the Leaning Tower of Pisa (Figure 1.3) is debatable. What matters is whether he did the experiment at all, which undoubtedly he did. Applying Newton's second law to the two masses:

$$-\frac{G M_G^{(E)} m_G^{(1)}}{R_E^2} = m_I^{(1)} a_1 , \quad -\frac{G M_G^{(E)} m_G^{(2)}}{R_E^2} = m_I^{(2)} a_2 , \tag{1.12}$$

where $M_G^{(E)}$ is the (gravitational) mass of the Earth and R_E is the Earth's radius. Dividing Equations (1.12)

$$\frac{a_1}{a_2} = \left(\frac{m_G^{(1)} \big/ m_I^{(1)}}{m_G^{(2)} \big/ m_I^{(2)}} \right) = \left(\frac{m_G^{(1)} m_I^{(2)}}{m_G^{(2)} m_I^{(1)}} \right) . \tag{1.13}$$

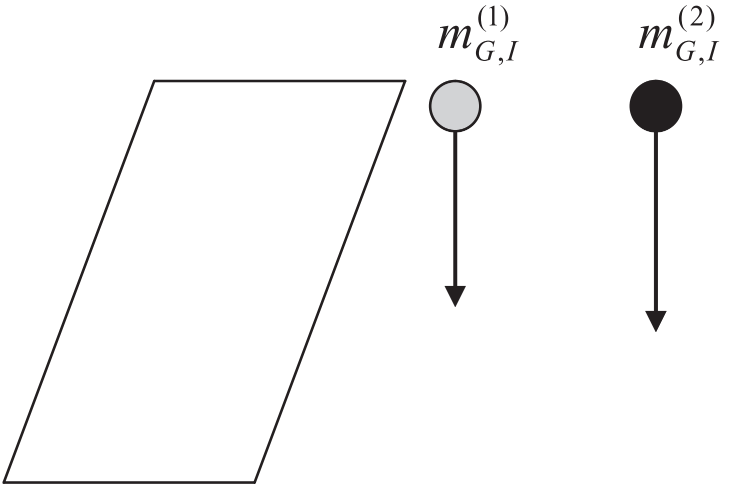

Figure 1.3 The Leaning Tower of Pisa experiment. Different masses are dropped from the same height. If they fall with the same acceleration then inertial mass is proportional to gravitational mass. Air resistance is neglected.

Thus *if* m_G/m_I is the same for all masses, then all bodies will fall with the same acceleration, and $a_1 = a_2 = -g$, the acceleration due to gravity. If not, then the bodies will separate as they fall. Modern experiments have tested the equality between gravitational and inertial mass to better than 1 part in 10^{12} and thus experimentally, $m_G/m_I =$ *constant* which, as noted above, can be set equal to one by choice of units, that is

$$m_G = m_I \ . \tag{1.14}$$

Of course it would be a remarkable coincidence if nature had simply decided that gravitational mass is to be proportional to inertial mass without any deeper significance. It suggests that gravitation can be described as a form of inertia, or, in the context of inertial frames, that gravitation is an inertial force (see below). A careful exposition of such issues is the basis of Einstein's general theory of relativity, which is beyond the scope of this book.

1.6 Inertial Frames

So far we have said nothing about the most fundamental assumptions in Newton's laws, concerning the nature of space and time. Space is presumed to follow the rules of geometry we learned at school, that is Euclidean geometry.[3] Although this may seem 'obvious', it is an underlying assumption for which we must supply an experimental test to verify its applicability. The appropriate test is to use Pythagoras' theorem. If Pythagoras' theorem holds for all right-angled triangles in three-dimensional space, then space is Euclidean, or 'flat'.

[3] An example of a *non*-Euclidean geometry is *spherical geometry* where 'lines' are great circles on the surface of a sphere. The angles of a spherical triangle sum to more than $180°$.

Newton regarded both space and time as 'absolute' entities, implying that they are the same for everyone. This immediately brings forth the notion of a *frame of reference*. For our purposes a reference frame may be considered as an imaginary latticework of rulers extending orthogonally in the three spatial directions. Via this frame the location of bodies may be determined. To complete the mechanical description, to each lattice is appended a clock that pinpoints when a particle is at a given location. Thus the trajectory through space as time evolves is determined in the frame of reference. Newton's notion of 'absolute time' is the tacit assumption that the time measured in one frame is the same as the time measured in any other. Time measured in two frames that are differently oriented, for example, is assumed to be the same[4]. This seemingly intuitive aspect of time fails dramatically when frames in relative motion are considered, but that will be the business of special relativity to be considered in Chapters 5 and 6. It is interesting to note, and it is a tribute to Newton's genius, that although he based his mechanics on absolute space and time he was actually uncomfortable with the rigidity that this imposed. However, and again in tribute to his genius, he recognised this as the only way to make practical progress at the time.

Even within the above definition of reference frames, there is still considerable latitude in choosing frames. Frames can be in motion relative to each other, and the question arises as to whether Newton's laws apply in all frames, or just some special class of frames. Newton's first law supplies the clue. If I hold a cup of coffee at my desk, or in the cabin of an aeroplane, I hardly notice any difference. Thus frames in *uniform* relative motion are apparently equivalent from the mechanical viewpoint. However, try the same experiment on a helter-skelter. Evidently, a frame that is accelerating with respect to another frame is not one in which bodies move uniformly and must be rejected. If we designate frames that pass the Newton's first law test as inertial frames, we are left with only one possible definition:

- An **inertial frame** is one in which Newton's first law is valid.

Yet again we must examine closely whether our reasoning is circular. First let us remove any extraneous external interactions by setting up our ideal inertial frame in deep space, far removed from other matter. We can decide whether or not our frame is rotating by looking at the motion of a 'free' body within the frame. If our record of the trajectory of the body shows it to be of a complex spiralling nature, then the frame is rotating. If, however, the motion is uniform, then our frame is inertial.[5] Such an ideal inertial frame may also be referred to as a **free-float frame** – see Figure 1.4.

All of this is seen to be a conceptual idealisation for which in practice successive approximations must be made. Let us consider a few examples:

1. *Deep space* – The cosmos exhibits a 'clumpiness' on a scale of about 150 million light years ($\sim 10^{25}\,\mathrm{m}$). By removing ourselves to a remote region, we may still have to think about the odd hydrogen atom, but this is the best possible approximation to the 'free-float' or 'ideal' inertial frame considered above. Such a frame is of interest only to cosmologists and astronomers.

[4]Of course the observers in different frames may set their time origins differently, so what is really meant here is the *time difference* between the same pair of events as recorded in different frames.

[5]There are additional subtleties here concerning the nature of time. If we have a 'bad' clock that measures time as a nonlinear function of the time shown by a 'good' clock, motion will appear nonuniform even in an inertial frame. But how do we know a priori whether our clock is good or bad?

2. *Frame of 'fixed stars' in our galaxy* – Over the orbital periods of the planets in our solar system, the stars of our galaxy show little motion, and furthermore we can suppose that the stars beyond the sun hardly influence the motions of the planets. Thus planetary motion may in practice be analysed in a frame tied to the 'fixed stars' of our galaxy.

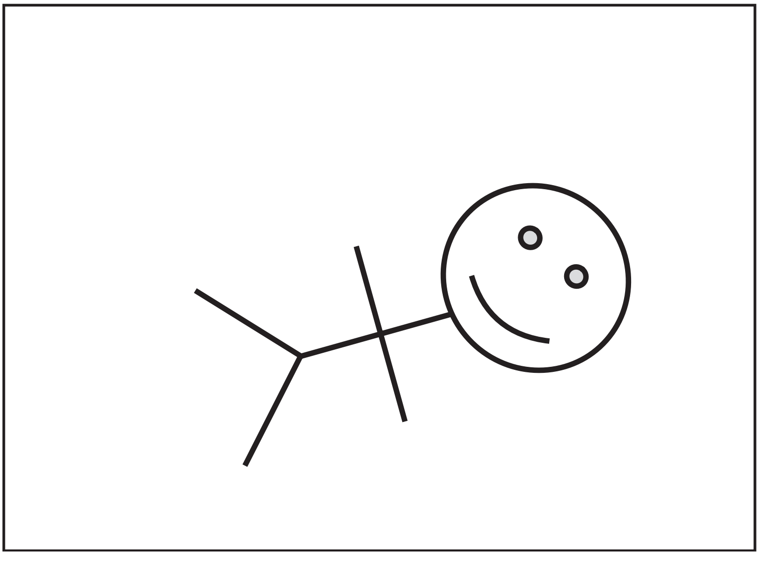

Figure 1.4 A free-float frame: a region of space removed as far as possible from other matter.

3. *Earth* – The use of the Earth as an inertial frame presents certain difficulties. First, whilst the initial motion of a bullet fired from a gun might pass the inertial frame test, most motions we want to consider are over a sufficient duration to be significantly affected by the Earth's gravity. One way to proceed is to regard Earth gravity as an external force applied to the motion of all bodies within an Earth-bound inertial frame:

$$\mathbf{F}_{\text{grav}} = m\mathbf{a} = -mg\mathbf{k} \tag{1.15}$$

where the standard unit Cartesian basis vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ have been introduced, with the unit vector $\mathbf{k}$ pointing 'up'.

However, since we cannot switch gravity off (at least not directly, see below), we have no way of applying the Newton III test. At best we can verify Newton's third law in a plane orthogonal to the direction of gravity by counteracting its effects with a vertically applied force, for example a puck on a frictionless table, with the table supplying the upwards reaction to gravity.

Moreover, the fact that the Earth rotates means that it is an accelerating frame. The rotation rate, ω, is 2π radians every 24 hrs, or $\omega = 7.3 \times 10^{-5}\,\text{rad}\,\text{s}^{-1}$. As a rough guide, we can ignore any effects due to the Earth's rotation over times that are short compared with the rotation period, though we must bear in mind that rotational effects will always be detectable with sufficiently accurate measurements. Effects due to the

Earth's rotation are the centrifugal effect and the Coriolis effect, to be discussed in Chapter 9.

All of the above approximations to inertial frames are 'valid', in that it depends entirely on the accuracy required as to whether a given approximation to an inertial frame is adequate.

Ultimately nowhere is remote enough to be classed as an inertial frame—suggesting that it should be possible to formulate mechanics without this conceptual idealisation. Indeed this is possible, but it would lead us too far afield for this book. The difficulties in defining an ideal inertial frame strike right at the heart of Newton's system for describing mechanics.

Since it appears that for terrestrial experiments we are stuck with analysing mechanics in noninertial frames, it is useful to consider briefly whether we can salvage anything in terms of attempting to apply Newton's laws in them.

1.7 Newton's Laws in Noninertial Frames

Consider the situation shown in Figure 1.5 which depicts a plumb-bob suspended from the ceiling of a railway carriage which is accelerating to the right with acceleration $\mathbf{a}$. The string tension $\mathbf{T}$, and the bob weight $m\mathbf{g}$, are the identified forces acting in the directions indicated. Now what is the effect of attempting to apply Newton's laws in the frame of the carriage? In this frame the bob is in equilibrium, so that the vector sum of the forces must cancel. What is there to oppose the horizontal component of the tension $T \sin\theta$? An observer in the inertial frame of the platform sees the train accelerate by a to the right and, applying Newton's second law, writes $T \sin\theta = ma$. In order for the train-based observer to obtain equilibrium he must invent a force $\mathbf{F}_I = -m\mathbf{a}$. The introduction of this force is necessary purely due to the attempt to use Newton's laws in an accelerating frame of reference.

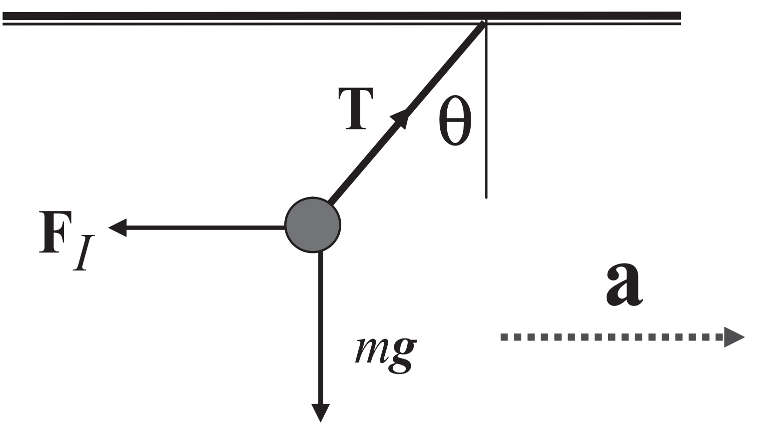

Figure 1.5 Plumb-bob in an accelerating railway carriage.

In fact this is a general result. Newton's laws may be applied in a frame accelerating with constant acceleration $\mathbf{a}$, provided that in addition to the identified forces acting, each body is subjected to a force

$$\mathbf{F}_I = -m\mathbf{a} \tag{1.16}$$

where m is the mass of the body. These forces are sometimes called **inertial forces** or, less helpfully, **fictitious forces**.

1.8 Switching Off Gravity

Now comes a beautiful trick. Can we combine the ideas of the previous two sections to manufacture a good inertial frame near the Earth? Let us analyse how mechanics looks to an individual in a lift cabin when the lift cable breaks and the cabin is in freefall – see Figure 1.6.

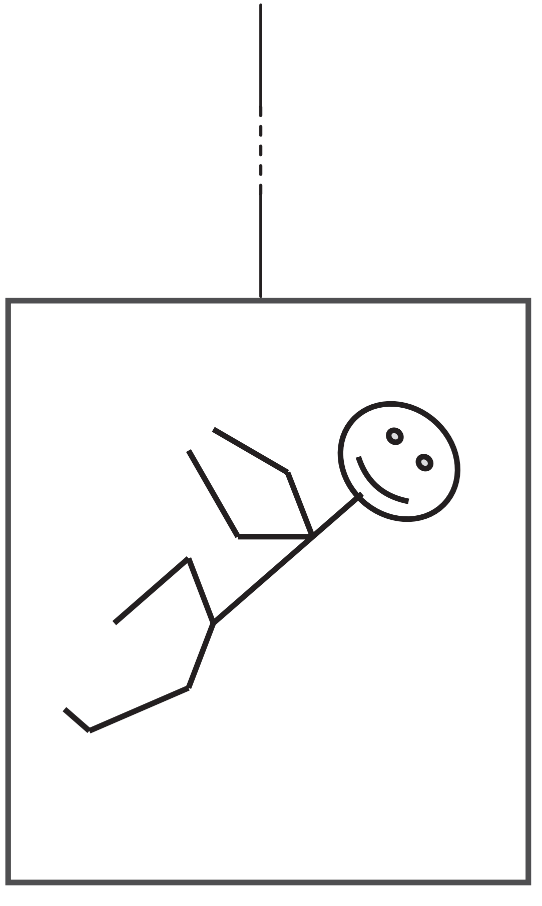

Figure 1.6 Free-fall lift cabin; despite his impending fate, the passenger is pleased that, for a while, he is in a good inertial frame.

The only force acting is gravity, which is given by $F_{\text{grav}} = mg$, taking downwards as positive. But the elevator frame is accelerating down with precisely the same acceleration, so that applying the prescription above, when analysing in this frame, we must introduce a force given by $F_I = -mg$. The net force on the individual, and any other free body in the cabin, is thus given by

$$F_{\text{grav}} + F_I = mg - mg = 0. \tag{1.17}$$

Any additional forces that may be acting between the bodies in the cabin can now safely be analysed using Newton's laws. We have thus succeeded in constructing a 'quasi'-inertial frame near a large gravitating mass (the Earth), in which Newton's laws can be applied. Gravity has been 'switched off'!

Having said that, we must recognise that the lift cabin can only occupy a small volume, and can only fall for a short time. If not, then so-called tidal effects become significant, and gravity can be detected again – see Problem 1.2 at the end of this chapter.

A profound insight from Einstein was to use the above ideas to postulate that since an inertial force can oppose gravity, then gravity itself may be described as an inertial force. Succinctly, uniform gravity is indistinguishable from acceleration. This line of reasoning also leads to a very natural explanation for the equality of inertial and gravitational mass, a complex of ideas known as the equivalence principle.

1.9 Finale – Laws, Postulates or Definitions?

Let us return to the question posed just after the statement of Newton's laws: are they laws, postulates, definitions or empirical observations? As we have seen their real nature is actually very hard to penetrate, as they contain elements of all four, though they are certainly more than any aspect individually. Perhaps the best we can do is to summarise and say that they constitute a body of ideas passing all the tests for a good physical theory: summary of observational data in mathematical statements, and prediction of results of experiments not yet performed. The crowning achievement of Newton's theory was its application beyond our everyday, terrestrial experience to describe lunar and planetary motion. Newton's theory was thus the first attempt at a universal, or *fundamental*, description of nature, being applied to the motion of *all* bodies subjected to forces of *every* possible origin.

1.10 Summary

Section 1.4. Newton's Laws:

1. A body remains at rest or moves with constant velocity, v, when no external force acts on it:

$$\frac{d}{dt}\mathbf{v} = \mathbf{0} \;\Leftrightarrow\; \mathbf{F} = \mathbf{0}\,. \tag{1.1}$$

2. The rate of change of momentum of a body is proportional to the force on the body:

$$\mathbf{F} \propto \frac{d}{dt}m\mathbf{v}\,. \tag{1.2}$$

3. When two bodies interact, they exert on each other equal, but opposite, forces:

$$\mathbf{F}_{12} = -\mathbf{F}_{21}\,. \tag{1.3}$$

The consequence of Equations (1.2) and (1.3) is that momentum is conserved for an isolated system:

$$\frac{d}{dt}(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = \mathbf{0} \qquad \Rightarrow \qquad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = \mathbf{constant}\ .$$

Equations (1.6) and (1.7).

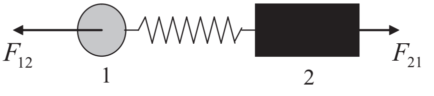

Figure 1.2

Section 1.5. Use Newton's third law to measure (inertial) mass:

$$\frac{m_2}{m_1} = \frac{|\mathbf{F}_{21}|/a_2}{|\mathbf{F}_{12}|/a_1} = \frac{a_1}{a_2}\ . \tag{1.9}$$

Distinction with gravitational mass, the mass appearing in Newton's law of gravitation: $\mathbf{F} = -\left(Gm_G M_G/r^2\right)\hat{\mathbf{r}}$ (Equation (1.11)). Tower of Pisa experiment shows, using $-GM_G^{(E)}m_G/R_E^2 = m_I a_1$ (Newton's second law, Equation (1.12)), that $m_I \propto m_G$.

Section 1.6. An inertial frame is one in which Newton's first law applies (resort to successive approximation to avoid argument being circular).

Section 1.7. Attempts to use Newton's laws in non-inertial frames with constant acceleration require the introduction of artificial 'inertial' forces, $\mathbf{F}_I = -m\mathbf{a}$ (Equation (1.16)), where $\mathbf{a}$ is the frame acceleration.

1.11 Problems

1.1 Show that for N bodies the number of equations of the form of Equation (1.7) that can be written is $N(N-1)/2$. Show that when they are added together they may be cast in the form

$$(N-1)\ \ (m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_N\mathbf{v}_N) = \ \textbf{constant}\ .$$

Hence deduce Equation (1.8).

1.2 Imagine a very large free-fall lift cabin of typical dimension $r \sim R_E$, where R_E is the radius of the Earth. How might gravity be detectable in such a frame?

The answers to the problems in all chapters can be found in the companion website at www.wiley.com/go/mccall.