Chapter 1

Basic Notions in Acoustic and Electromagnetic Diffraction Problems

1.1 FORMULATION OF THE DIFFRACTION PROBLEM

This book develops the Physical Theory of Diffraction (PTD) for both acoustic and electromagnetic waves diffracted at perfectly reflecting objects.

> In the case of two-dimensional (2-D) problems, this theory is valid for both *electromagnetic* and *acoustic* waves.

First we present the theoretical fundamentals for acoustic waves and then for electromagnetic waves. In the linear approximation, the velocity potential u of harmonic acoustic waves satisfies the wave equation (Kinsler et al., 1982; Pierce, 1994)

$$\nabla^2 u + k^2 u = I. \tag{1.1}$$

Here $k = 2\pi/\lambda = \omega/c$ is the wave number, λ is the wavelength, ω the angular frequency, c the speed of sound, and I the source strength characteristic. The time dependence is assumed to be in the form $\exp(-i\omega t)$, and is suppressed below. The acoustic pressure p and the velocity v of fluid particles, caused by sound waves, are determined through the velocity potential (Kinsler et al., 1982; Pierce, 1994)

$$p = -\rho \frac{\partial u}{\partial t} = i\omega\rho u, \qquad \vec{v} = \nabla u \tag{1.2}$$

where ρ is the mass density of a fluid. The power flux density of sound waves, which is the analog of the Poynting vector for electromagnetic waves, equals

$$\vec{P} = p\vec{v} = p\nabla u. \tag{1.3}$$

Fundamentals of the Physical Theory of Diffraction. By Pyotr Ya. Ufimtsev

Its value averaged over the period of oscillations $T = 2\pi/\omega$ equals

$$\vec{P}_{\text{av}} = \frac{1}{2}\text{Re}(p^*\vec{v}). \tag{1.4}$$

Here and everywhere below, the superscript asterisk is used for complex conjugate quantities.

Two types of boundary conditions are imposed on the surface of perfectly reflecting objects: the Dirichlet condition

$$u = 0 \qquad \text{or} \qquad p = 0 \qquad \text{(soft)} \tag{1.5}$$

for objects with a soft (pressure-release) surface, and the Neumann condition

$$\frac{\partial u}{\partial n} = \hat{n} \cdot \nabla' u = 0 \qquad \text{(hard)} \tag{1.6}$$

for objects with a hard (rigid) surface. Here u is the total field that is the *sum of the incident and scattered waves.* The symbol $\hat{n}$ stands for a unit outward vector, which is normal to the scattering surface S (Fig. 1.1). The gradient operator ∇' is applied to coordinates of the integration /source point Q.

To complete the formulation of the diffraction problem, and to ensure the uniqueness of its solution, the above wave equation and the boundary conditions are supplemented by the Sommerfeld radiation condition for the scattered field,

$$\lim r\left(\frac{\partial u}{\partial r} - iku\right) = 0 \qquad \text{with } r \to \infty, \tag{1.7}$$

where r is the distance from the scattering object to the observation point.

In the International System (SI) of units, the quantities introduced above have the following dimensions

$$\begin{gathered}[r] = \text{m}, \qquad [t] = \text{sec}, \qquad [\vec{v}] = [c] = \frac{\text{m}}{\text{sec}}, \qquad [\omega] = \frac{1}{\text{sec}}, \\ [\rho] = \frac{\text{kg}}{\text{m}^3}, \qquad [\text{p}] = \frac{\text{kg}}{\text{m} \cdot \text{sec}^2}, \qquad [u] = \frac{\text{m}^2}{\text{sec}}, \qquad [\vec{P}] = \frac{\text{kg}}{\text{sec}^3}.\end{gathered} \tag{1.8}$$

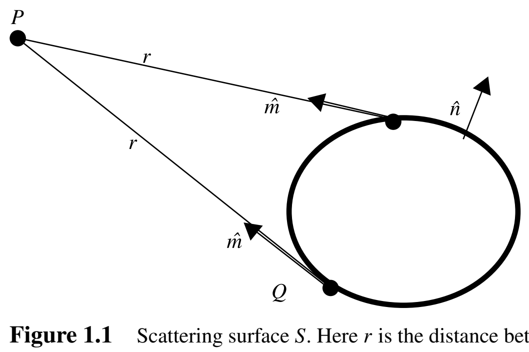

Figure 1.1 Scattering surface S. Here r is the distance between the observation point P (which can be in the far zone) and the integration point Q (on the surface of the scatterer), the unit vector $\hat{m}$ is directed from the point Q to the point P.

The standard denotations are used here: m for meter, kg for kilogram, and sec for second. The pressure unit is called Pascal (Pa) and $1\,\text{Pa} = 1\ \text{Newton}/1\,\text{m}^2$. The SI unit of the power flux density is $1\,\text{Watt}/1\,\text{m}^2 = 1\,\text{Joule}/(1\ \text{sec} \cdot 1\,\text{m}^2)$.

In the scattering problems, which admit the electromagnetic interpretation, the quantity u plays the role of the electric field intensity ($[E] = \text{Volt/m}$) or magnetic field intensity ($[H] = \text{Ampere/m}$), depending on the polarization of electromagnetic waves. Their power flux density is called the *Poynting* vector and is defined as

$$\vec{P} = \vec{E} \times \vec{H} \qquad \text{and} \qquad \vec{P}_{\text{av}} = \frac{1}{2}\text{Re}[\vec{E} \times \vec{H}^*]. \tag{1.9}$$

1.2 SCATTERED FIELD IN THE FAR ZONE

The scattered field is determined by the Helmholtz integral expressions (Bakker and Copson, 1939):

$$u_{\text{s}} = -\frac{1}{4\pi}\int_S \frac{\partial u}{\partial n}\frac{\text{e}^{ikr}}{r}\,\text{ds}, \qquad u_{\text{h}} = \frac{1}{4\pi}\int_S u\frac{\partial}{\partial n}\frac{\text{e}^{ikr}}{r}\,\text{ds}, \tag{1.10}$$

where the integrals are taken over the scattering surface S. The function u_{s} describes the field scattered by an acoustically soft object, and the function u_{h} relates to the field scattered by an acoustically hard object. The field quantities u and $\partial u/\partial n$ in the integrands belong to the total field on the object surface, that is, to the sum of the incident and scattered fields. These quantities represent the surface sources of the scattered field induced by the incident wave. We denote them by symbols

$$j_{\text{s}} = \frac{\partial u}{\partial n}, \qquad j_{\text{h}} = u, \tag{1.11}$$

similar to those used for induced sources/currents in the electromagnetic version of PTD (Ufimtsev, 2003). The quantity e^{ikr}/r in Equation (1.10) represents the Green function of a homogeneous medium, that is, the fundamental solution of the wave equation, and $\hat{n}$ is a unit outward vector normal to the surface S.

In the far field, where $r \gg kd^2$ (d is the characteristic linear dimension of the object), the field expressions (1.10) can be simplified. We choose the origin of the coordinate system somewhere inside the object, as shown in Figure 1.2. Under the conditions $R \gg r'$, $R \gg kd^2$, we have

$$r \approx R - r'\cos\Omega, \qquad \frac{\text{e}^{ikr}}{r} \approx \frac{\text{e}^{ikR}}{R}\text{e}^{-ikr'\cos\Omega} \tag{1.12}$$

with

$$\cos\Omega = \cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\varphi - \varphi'). \tag{1.13}$$

In addition,

$$\frac{\partial}{\partial n}\frac{\text{e}^{ikr}}{r} = \nabla'\frac{\text{e}^{ikr}}{r}\cdot\hat{n} = -\left(ik - \frac{1}{r}\right)\frac{\text{e}^{ikr}}{r}\nabla r\cdot\hat{n}, \qquad \text{with } \nabla r = \hat{m}, \tag{1.14}$$

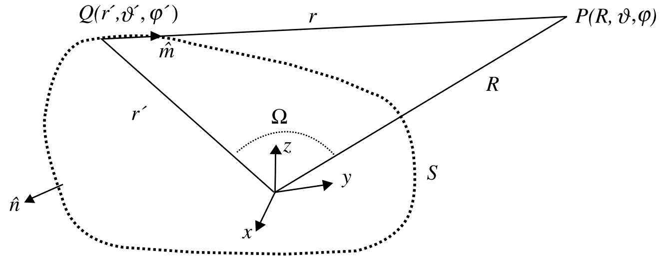

Figure 1.2 S is the surface of the scattering object; Q is the integration point (with the spherical coordinates r', ϑ', φ') on the surface S; P is the observation point (with the coordinates R, ϑ, φ) in the far zone; Ω is the angle between the directions from the origin to the integration and observation points.

or in view of Equations (1.12)

$$\frac{\partial}{\partial n}\frac{e^{ikr}}{r} \approx -ik\frac{e^{ikR}}{R}e^{-ikr'\cos\Omega}\cdot(\hat{m}\cdot\hat{n}). \tag{1.15}$$

Finally, we obtain the following approximations for the field in the far-away point P:

$$u_s = -\frac{1}{4\pi}\frac{e^{ikR}}{R}\int_S j_s e^{-ikr'\cos\Omega}\,ds, \tag{1.16}$$

$$u_h = -\frac{ik}{4\pi}\frac{e^{ikR}}{R}\int_S j_h e^{-ikr'\cos\Omega}(\hat{m}\cdot\hat{n})ds \tag{1.17}$$

where $\hat{m}$ and $\hat{n}$ are unit vectors. In this book, we develop asymptotic approximations first for the surface sources $j_{s,h}$ and then for the scattered field (1.16), (1.17).

Expressions (1.16) and (1.17) can be written in the generic form

$$u_{s,h} = u_0\Phi_{s,h}\frac{e^{ikR}}{R} \tag{1.18}$$

where the functions

$$\Phi_s = -\frac{1}{4\pi u_0}\int_S j_s e^{-ikr'\cos\Omega}\,ds, \qquad \Phi_h = -\frac{ik}{4\pi u_0}\int_S j_h e^{-ikr'\cos\Omega}(\hat{m}\cdot\hat{n})ds \tag{1.19}$$

represent the directivity patterns of the scattered field, and u_0 is the complex amplitude of the incident wave at the origin of the coordinates ($R = 0$). Notice that in the vicinity of the scattering object located in the far zone from the source Q , the incident wave can be approximated by the equivalent plane wave with the amplitude u_0.

According to Equations (1.2), (1.3) and (1.4), the power flux density of the scattered field is determined by

$$\vec{P}^{sc} = i\omega\rho u \nabla u. \tag{1.20}$$

In the far field,

$$\nabla u \approx iku \cdot \hat{R}, \qquad \text{with } \hat{R} = \nabla R. \tag{1.21}$$

Therefore, the power flux density averaged over the oscillation period $T = 2\pi/\omega$ equals

$$\vec{P}^{sc}_{av} = \frac{1}{2}\mathrm{Re}[p^* \vec{v}] = \frac{1}{2}\mathrm{Re}[(i\omega\rho u)^*(iku)] \cdot \hat{R} = \frac{1}{2}k^2 Z|u|^2 \cdot \hat{R}, \tag{1.22}$$

where

$$Z = \rho c \tag{1.23}$$

is the characteristic impedance of the medium.

Usually, the far field is characterized by the *bistatic cross-section* σ introduced through the relation

$$P^{sc}_{av} = \frac{\sigma \cdot P^{inc}_{av}}{4\pi R^2}, \tag{1.24}$$

where

$$P^{inc}_{av} = \frac{1}{2}k^2 Z|u^{inc}|^2 \tag{1.25}$$

is the power flux density of the incident wave. This definition suggests the interpretation given in the following paragraphs.

The bistatic cross-section is the area σ of a hypothetical plate perpendicular to the direction of the incident wave. This plate intercepts the incident power $P^{inc}_{av} \cdot \sigma$ and distributes it uniformly into the whole surrounding space *with the power flux density that is equal to the actual one scattered by the object in the direction of observation.* Because the scattered power depends on the direction of scattering, the scattering cross-section σ is a function of this direction. The term bistatic means that the direction of scattering can be arbitrary. In the particular case when the scattering direction coincides with the direction to the source of the incident wave, the quantity σ is called the *backscattering* cross-section or *monostatic* cross-section. Thus, according to Equations (1.22) and (1.24)

$$\sigma = 4\pi R^2 \frac{P^{sc}_{av}}{P^{inc}_{av}} = 4\pi R^2 \frac{|u^{sc}|^2}{|u^{inc}|^2}. \tag{1.26}$$

In the directions where the field scattered from a *smooth convex* surface has a ray structure, the bistatic cross-section is predicted by Geometrical Optics (Geometrical

Acoustics) and equals

$$\sigma = \pi\rho_1\rho_2. \tag{1.27}$$

Here ρ_1 and ρ_2 are principal radii of curvature of the scattering surface at the *reflection point*. It is also assumed that this surface is perfectly reflecting (soft or hard). Two interesting features of this quantity should be emphasized.

First, the expression (1.27) is universal. It is applicable both for acoustic and electromagnetic waves. The reason for this is that the ray structure does not depend on the nature of the waves, and it is totally determined by the geometry of the scattering surface. If the geometry is the same, the divergence of reflected rays will be the same for both acoustic and electromagnetic rays. Also, the modulus of reflection coefficient for any perfectly reflecting surfaces (soft or hard for acoustic waves, or perfectly conducting for electromagnetic waves) equals unity. However, just these two factors, the ray divergence and the reflection coefficient, totally determine the amplitude of reflected rays, and eventually the bistatic cross-section.

Equation (1.27) can be generalized for imperfect reflecting surfaces:

$$\sigma = |\mathcal{R}|^2\pi\rho_1\rho_2, \tag{1.28}$$

where $\mathcal{R}$ is the reflection coefficient, which can be different for acoustic and electromagnetic waves.

The second interesting and not obvious feature of Equation (1.27) is the following. This expression does not depend on the angle between the incident and reflected rays at the same reflection point (Fig. 1.3). In other words, it is constant for any bistatic angles, including zero angle related to backscattering. This property of scattering from perfectly reflecting objects follows from the theory of Fock (1965) as it was shown in Ufimtsev (1999).

The theory of Fock (1965) is more general. It is also valid for imperfect scattering surfaces characterized by the reflection coefficient $\mathcal{R}$. In this case, Fock's theory leads straight to Equation (1.28), where $\mathcal{R}$ depends on the bistatic angle as well as on the boundary conditions.

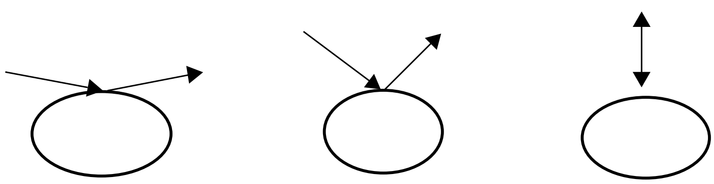

Figure 1.3 Scattering from the same reflection point (at the same reflecting object) for different bistatic angles. Bistatic cross-section σ of this perfectly reflecting object is constant for all of these angles and equals the monostatic cross-section.

1.3 PHYSICAL OPTICS

This high-frequency approach is widely used in acoustic and electromagnetic diffraction problems.

1.3.1 Definition of the Physical Optics

Physical Optics (PO) was suggested by Macdonald (1912), and since then it has been successfully applied in the theory of diffraction. In particular it is often used in the analysis of electromagnetic waves scattered from large metallic objects. Basic features of this approach in the study of electromagnetic diffraction are exposed in the article by Ufimtsev (1999). The scalar version of PO is applicable for acoustic waves and it is known in acoustics as the extended Kirchhoff approximation (Brill and Gaunaurd, 1993; Menounou et al., 2000; Moser et al., 1993). Physical Optics is a constituent part of the Physical Theory of Diffraction developed in the present book. According to this approximation, the field induced on the surface of the object is determined by Geometrical Optics (Geometrical Acoustics).

The physics behind this is as follows. Geometrical Optics describes a wave field in the limiting case when a wavelength tends to zero. With respect to such a small wavelength, the scattering surface at the reflection point can be considered approximately as a tangential plane. Therefore, the surface field induced at the tangential infinite plane is a good high-frequency approximation for true scattering sources induced on a large scattering object. Two such planes P_1 and P_2 tangent at the points Q_1 and Q_2 are shown in Figure 1.4. These points are located at the "illuminated" side of the object. Notice that according to Geometrical Optics, the field equals zero in the shadow region, including the points on the object surface.

Thus, the reflection from a tangential plane is an appropriate canonical ("fundamental") problem. Its exact solution can be easily found using Geometrical Optics, as well as by image theory. The total field generated by an external source above an infinite reflecting plane (Fig. 1.5) is the sum of the incident field and the reflected field, which can be interpreted as the field created by the image source. On the acoustically soft plane, the total field is zero as a result of the boundary condition (1.5), but its normal derivative equals

$$\frac{\partial u_{\mathrm{s}}}{\partial n} = 2\frac{\partial u_{\mathrm{s}}^{\mathrm{inc}}}{\partial n} \tag{1.29}$$

due to the exact solution of this problem. On the acoustically hard plane, the normal derivative of the total field is zero as a result of the boundary condition (1.6), and the field itself equals

$$u_{\mathrm{h}} = 2u_{\mathrm{h}}^{\mathrm{inc}}, \tag{1.30}$$

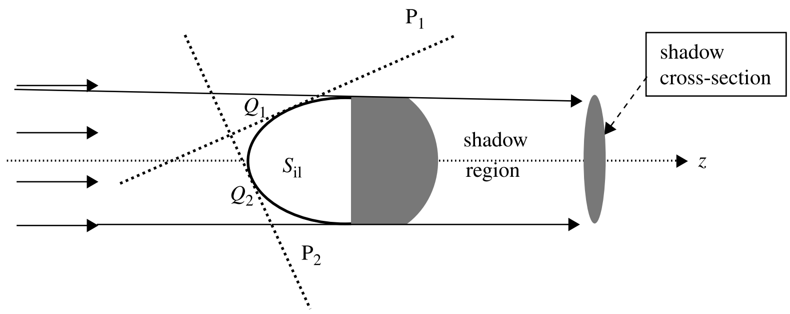

Figure 1.4 Surface fields induced on the scattering objects at the points Q_1 and Q_2 are asymptotically identical to the fields induced at the tangential planes P_1 and P_2, respectively. S_{il} is the illuminated part of the object surface. A dark plate behind the object displays the cross-section of the geometrical shadow region.

as follows from the solution of this reflection problem. In the general PTD, these quantities are interpreted as the *uniform components* of induced sources on a smooth convex scattering surface:

$$j_s^{(0)} = 2\frac{\partial u^{\text{inc}}}{\partial n}, \qquad j_h^{(0)} = 2u^{\text{inc}}. \tag{1.31}$$

These expressions define the induced sources only on the “illuminated” part of the scattering object. On the shadowed part, these components are set to zero.

By substituting Equation (1.31) into Equation (1.10) one obtains expressions for the scattered field at any distance from the scatterer (Fig. 1.4):

$$u_s^{\text{PO}} \equiv u_s^{(0)} = -\frac{1}{4\pi}\int_{S_{il}} j_s^{(0)} \frac{e^{ikr}}{r}\, ds,$$
$$u_h^{\text{PO}} \equiv u_h^{(0)} = \frac{1}{4\pi}\int_{S_{il}} j_h^{(0)} \frac{\partial}{\partial n}\frac{e^{ikr}}{r}\, ds. \tag{1.32}$$

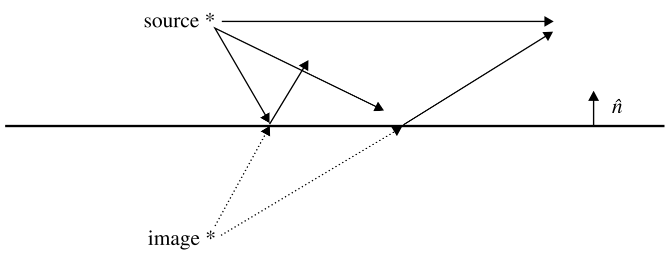

Figure 1.5 Reflection from an infinite plane.

These expressions represent the *scalar Physical Optics* approximation, also known in Acoustics as the *extended Kirchhoff Approximation* (KA). In the present book we use the term Physical Optics for both acoustic and electromagnetic waves. The symbols $u_{\mathrm{s}}^{(0)}$ and $u_{\mathrm{h}}^{(0)}$ are introduced in Equations (1.32) to emphasize that these fields are generated by the *uniform* component of the induced surface sources. Thus, Physical Optics, which deals with these uniform components, is a constituent part of the general PTD.

The Physical Optics of Equations (1.32) possesses a special property related to the field scattered in the direction to the source of the incident wave. According to Equations (1.16) and (1.17) the PO far field is determined as

$$u_{\mathrm{s}}^{(0)} = -\frac{1}{2\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\mathrm{il}}}\frac{\partial u^{\mathrm{inc}}}{\partial n}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}s \tag{1.33}$$

and

$$u_{\mathrm{h}}^{(0)} = -\frac{ik}{2\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\mathrm{il}}}u^{\mathrm{inc}}\mathrm{e}^{-ikr'\cos\Omega}(\hat{m}\cdot\hat{n})\mathrm{d}s. \tag{1.34}$$

The field incident on the scattering object (being at the large distance from the source) can be represented in the form

$$u^{\mathrm{inc}} = \mathrm{const}\,\mathrm{e}^{ik\phi^i}. \tag{1.35}$$

The unit vector $\nabla\phi^i = \hat{k}^i$ indicates the direction of the incident wave, and the unit vector $\hat{m} = \nabla r = \hat{k}^{\mathrm{s}}$ shows the direction of scattering. In the case of backscattering, the equality $\hat{k}^{\mathrm{s}} = -\hat{k}^i$ is valid. Note also that

$$\frac{\partial u^{\mathrm{inc}}}{\partial n} = \nabla u^{\mathrm{inc}}\cdot\hat{n} = iku^{\mathrm{inc}}(\nabla\phi^i\cdot\hat{n}) = iku^{\mathrm{inc}}(\hat{k}^i\cdot\hat{n}). \tag{1.36}$$

The substitution of Equation (1.36) and the quantity $(\hat{m}\cdot\hat{n}) = -(\hat{k}^i\cdot\hat{n})$ into Equations (1.33) and (1.38) leads to the equation

$$u_{\mathrm{s}}^{(0)} = -u_{\mathrm{h}}^{(0)} = -\frac{ik}{2\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\mathrm{il}}}u^{\mathrm{inc}}\mathrm{e}^{-ikr'\cos\Omega}(\hat{k}^i\cdot\hat{n})\mathrm{d}s. \tag{1.37}$$

Hence, in the frame of the Physical Optics approximation, the *backscattered fields* created by the *soft* and *hard* objects (of the same shape and size) have *equal magnitudes* and differ only in *sign*.

1.3.2 Total Scattering Cross-Section

The power flux density of the scattered waves is defined by Equation (1.22). By the integration of this quantity over the object surface, one can find the total power scattered in all directions. In the PO approximation, the total power scattered from an acoustically soft object equals

$$P^{\text{tot}} = \frac{1}{2}\text{Re}\int_{S_{\text{il}}} (p_{\text{s}}^{\text{sc}})^*(\vec{v}_{\text{s}}^{\text{tot}} \cdot \hat{n})\text{d}s, \tag{1.38}$$

where

$$\vec{v}_{\text{s}}^{\text{tot}} \cdot \hat{n} = 2\frac{\partial u^{\text{inc}}}{\partial n}, \tag{1.39}$$

and in accordance with the boundary condition (1.5), $p_{\text{s}}^{\text{sc}} = -p^{\text{inc}} = -i\omega\rho u^{\text{inc}}$. The incident wave in the vicinity of the scattering object can be approximated by the plane wave (Fig. 1.4)

$$u^{\text{inc}} = u_0 \text{e}^{ikz}. \tag{1.40}$$

Then

$$\vec{v}_{\text{s}}^{\text{tot}} \cdot \hat{n} = 2\frac{\partial u^{\text{inc}}}{\partial n} = 2iku_0\text{e}^{ikz}(\hat{z}\cdot\hat{n}), \qquad (p_{\text{s}}^{\text{sc}})^* = i\omega\rho u_0^*\,\text{e}^{-ikz} \tag{1.41}$$

and

$$P^{\text{tot}} = -k\omega\rho|u_0|^2\int_{S_{\text{il}}} (\hat{z}\cdot\hat{n})\text{d}s = k^2 ZA|u_0|^2, \tag{1.42}$$

where A is the area of the object's projection on the plane perpendicular to the direction of propagation or, in other words, the area of the shadow cross-section (Fig. 1.4). In view of Equation (1.25), the power flux density of the incident wave equals

$$P_{\text{av}}^{\text{inc}} = \frac{1}{2}k^2 Z|u_0|^2. \tag{1.43}$$

The total cross-section is defined by the ratio

$$\sigma^{\text{tot}} = P^{\text{tot}}/P_{\text{av}}^{\text{inc}} \tag{1.44}$$

and equals

$$\sigma^{\text{tot}} = 2A. \tag{1.45}$$

This result is also valid for hard objects and for perfectly conducting objects, which scatter electromagnetic waves. It can be easily verified for hard objects. Indeed in this case,

$$P^{\text{tot}} = \frac{1}{2}\text{Re}\int_{S_{\text{il}}} (p_{\text{h}}^{\text{tot}})^{*}(\vec{v}_{\text{h}}^{\text{sc}} \cdot \hat{n})\text{d}s \tag{1.46}$$

and

$$p_{\text{h}}^{\text{tot}} = 2p^{\text{inc}} = 2i\,\omega\rho u_0 \text{e}^{ikz}, \qquad (\vec{v}_{\text{h}}^{\text{sc}} \cdot \hat{n}) = -(\vec{v}^{\text{inc}} \cdot \hat{n}) = -iku_0\text{e}^{ikz}(\hat{z}\cdot\hat{n}). \tag{1.47}$$

The substitution of Equation (1.47) into (1.46) leads to Equations (1.42) and to (1.45).

1.3.3 Optical Theorem

There is a specific connection between the total scattering cross-section and the far scattered field in the shadow/forward direction. In the PO approximation, the far-field expressions (1.18) and (1.19) take the form

$$u_{\text{s}}^{\text{PO}} = u_0 \Phi_{\text{s}}^{\text{PO}} \frac{\text{e}^{ikR}}{R}, \qquad u_{\text{h}}^{\text{PO}} = u_0 \Phi_{\text{h}}^{\text{PO}} \frac{\text{e}^{ikR}}{R}, \tag{1.48}$$

where

$$\Phi_{\text{s}}^{\text{PO}} = -\frac{1}{2\pi u_0}\int_{S_{\text{il}}} \frac{\partial u^{\text{inc}}}{\partial n}\text{e}^{-ikr'\cos\Omega}\,\text{d}s, \tag{1.49}$$

$$\Phi_{\text{h}}^{\text{PO}} = -\frac{ik}{2\pi u_0}\int_{S_{\text{il}}} u^{\text{inc}}\text{e}^{-ikr'\cos\Omega}(\hat{m}\cdot\hat{n})\text{d}s \tag{1.50}$$

with $\cos\Omega$ defined in Equation (1.13). The incident wave (1.40) propagates in the z-direction. For the observation point in the forward direction, we have $\vartheta = 0$, $\cos\Omega = \cos\vartheta'$, $\hat{m} = \hat{z}$, $r'\cos\vartheta' = z'$, and

$$\frac{\partial u^{\text{inc}}}{\partial n} = \nabla u^{\text{inc}} \cdot \hat{n} = iku_0\text{e}^{ikz'}(\hat{z}\cdot\hat{n}) \tag{1.51}$$

and also

$$\Phi_{\text{s}}^{\text{PO}}(\vartheta = 0) = \Phi_{\text{h}}^{\text{PO}}(\vartheta = 0) = -\frac{ik}{2\pi}\int_{S_{\text{il}}}(\hat{z}\cdot\hat{n})\text{d}s = \frac{ik}{2\pi}A, \tag{1.52}$$

where A is the area of the shadow cross-section (Fig. 1.4). Comparison of Equation (1.52) with Equation (1.45) shows that

$$\sigma^{\text{tot}} = \frac{4\pi}{k}\text{Im}\Phi \qquad (\vartheta = 0). \tag{1.53}$$

This equation is well known as the Optical Theorem (Born and Wolf, 1980).

1.3.4 Introducing the Notion of "Shadow Radiation"

This notion was introduced for electromagnetic waves by Ufimtsev (1968). It was additionally investigated in Ufimtsev (1990) and discussed in Ufimtsev (1996). A significant part of the results relating to electromagnetic shadow radiation was included in *Theory of Edge Diffraction in Electromagnetics* (Ufimtsev, 2003). In the present section, the notion of shadow radiation is introduced in conjunction with scalar waves. Consider again the reflection from an *infinite* perfectly reflecting plane (Fig. 1.6) located in an homogeneous medium. The scattered field in the region $z > 0$ is determined by the Helmholtz integral expression (Bakker and Copson, 1939)

$$u^{\text{sc}} = \frac{1}{4\pi} \int_S \left(u^{\text{tot}} \frac{\partial}{\partial n} \frac{e^{ikr}}{r} - \frac{\partial u^{\text{tot}}}{\partial n} \frac{e^{ikr}}{r} \right) ds, \tag{1.54}$$

where $u^{\text{tot}} = u^{\text{inc}} + u^{\text{sc}}$ is the total field, $ds = dxdy$ is a differential area of the infinite plane S $(z = 0)$, and r is the distance between the integration and observation points.

Let the reflecting plane be acoustically soft. Then, on its surface,

$$u_{\text{s}}^{\text{sc}} = -u^{\text{inc}},$$

but

$$\frac{\partial u_{\text{s}}^{\text{tot}}}{\partial n} = 2\frac{\partial u^{\text{inc}}}{\partial n} = \frac{\partial u^{\text{inc}}}{\partial n} + \frac{\partial u^{\text{inc}}}{\partial n}. \tag{1.55}$$

Therefore, Equation (1.54) can be rewritten as

$$u_{\text{s}}^{\text{tot,sc}} = u_{\text{s},1}^{\text{sc}} + u_{\text{s},2}^{\text{sc}}, \tag{1.56}$$

where

$$u_{\text{s},1}^{\text{sc}} = \frac{1}{4\pi} \int_S \left(u^{\text{inc}} \frac{\partial}{\partial n} \frac{e^{ikr}}{r} - \frac{\partial u^{\text{inc}}}{\partial n} \frac{e^{ikr}}{r} \right) ds, \tag{1.57}$$

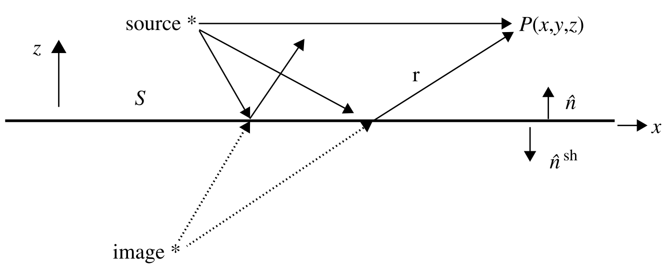

Figure 1.6 Reflection of waves from an infinite plane S in an homogeneous medium. The source is in the region $z > 0$. The total field in the shadow region $(z < 0)$ equals zero.

and

$$u_{s,2}^{sc} = \frac{1}{4\pi}\int_S \left(-u^{inc}\frac{\partial}{\partial n}\frac{e^{ikr}}{r} - \frac{\partial u^{inc}}{\partial n}\frac{e^{ikr}}{r}\right)ds. \tag{1.58}$$

To evaluate the integral in Equation (1.57), we utilize the Helmholtz equivalency theorem (Bakker and Copson, 1939). According to this theorem, the field u created by the acoustic source at the point P in an homogeneous medium (Fig. 1.7) can be represented as the radiation generated by the equivalent sources u^{inc} and $\partial u^{inc}/\partial N$ distributed over the closed imaginary surface Σ of the volume V:

$$u(P) = \frac{1}{4\pi}\oint_\Sigma \left(u^{inc}\frac{\partial}{\partial N}\frac{e^{ikr}}{r} - \frac{\partial u^{inc}}{\partial N}\frac{e^{ikr}}{r}\right)ds = \begin{cases} 0, & \text{when } P \text{ inside } V \\ u^{inc}(P), & \text{when } P \text{ outside } V. \end{cases} \tag{1.59}$$

Here it is supposed that the source of the incident wave is located inside the volume V. One should emphasize the following wonderful property of this theorem. The field at the point P inside V or outside V does not depend on the shape of its surface Σ. One can deform this surface in any way, but the result of the integration in Equation (1.59) will be the same: $u(P) = 0$ if $P \in V$, and $u(P) = u^{inc}(P)$ if $P \notin V$.

In order to evaluate integral (1.57), we first apply the equivalency theorem (1.59) to the closed surface $\Sigma = S_R + H_R$ (Fig. 1.8). Here S_R is a circular plate with a radius R, which is a part of the infinite plane S shown in Figure 1.6, and H_R is a hemisphere with the same radius R. It is supposed that the source of the incident wave is inside the volume V.

According to the equivalency theorem (1.59),

$$\frac{1}{4\pi}\oint_{S_R+H_R} \left(u^{inc}\frac{\partial}{\partial N}\frac{e^{ikr}}{r} - \frac{\partial u^{inc}}{\partial N}\frac{e^{ikr}}{r}\right)ds = \begin{cases} 0, & \text{when } P \text{ inside } V \\ u^{inc}(P), & \text{when } P \text{ outside } V \end{cases} \tag{1.60}$$

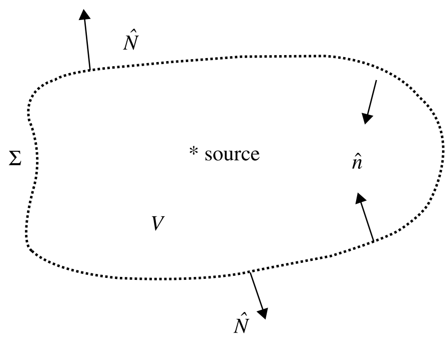

Figure 1.7 Illustration of the equivalency principle. Σ is an arbitrary imaginary surface covering a volume V of a free homogeneous medium, $\hat{n}$ and $\hat{N}$ are respectively the inward and outward unit vectors normal to Σ, and a source of the incident wave is inside V.

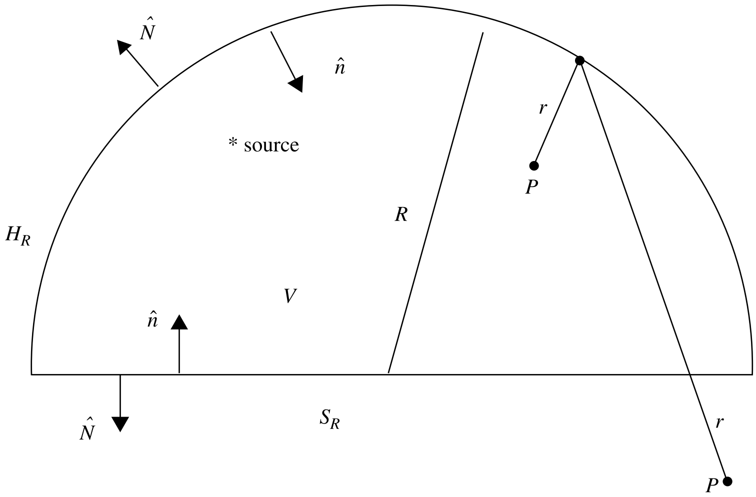

Figure 1.8 Surface of integration $\Sigma = S_R + H_R$ in Equation (1.60). A source of the incident wave is inside the volume V.

or after replacement of $\hat{N}$ by $(-\hat{n})$,

$$\frac{1}{4\pi}\oint_{S_R+H_R}\left(u^{\text{inc}}\frac{\partial}{\partial n}\frac{e^{ikr}}{r} - \frac{\partial u^{\text{inc}}}{\partial n}\frac{e^{ikr}}{r}\right)ds = \begin{cases} 0, & \text{when } P \text{ inside } V \\ -u^{\text{inc}}(P), & \text{when } P \text{ outside } V. \end{cases} \tag{1.61}$$

One can show that the field at the observation point P generated by the equivalent sources, distributed over H_R, vanishes when the radius R of H_R tends to infinity. Note also that with $R \to \infty$, the surface S_R is transformed into the infinite plane S. Taking into account these observations, we finally obtain the following values for the function (1.57):

$$u_{s,1}^{sc} = \begin{cases} 0, & \text{in the region } z > 0 \\ -u^{\text{inc}}, & \text{in the region } z < 0. \end{cases} \tag{1.62}$$

The physical meaning of the field $u_{s,1}^{sc}$ is clear. It cancels the incident wave in the region $z < 0$, creating the complete shadow there. That is why we call this field the *shadow radiation* and denote it by u^{sh}:

$$u^{\text{sh}} = \frac{1}{4\pi}\int_S\left(u^{\text{inc}}\frac{\partial}{\partial n}\frac{e^{ikr}}{r} - \frac{\partial u^{\text{inc}}}{\partial n}\frac{e^{ikr}}{r}\right)ds. \tag{1.63}$$

With respect to this equation (!), the surface S can be interpreted as *perfectly absorbing* (i.e., *black*), as it does not reflect the incident wave (Ufimtsev, 1968, 1990, 1996, 2003).

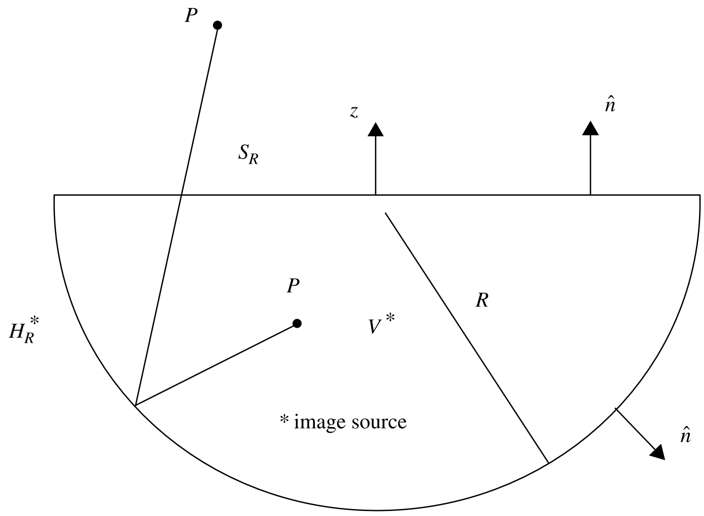

Figure 1.9 Illustration of the equivalency theorem applied to the reflected field. Here, as in Figure 1.8, S_R is a circular plate on the plane $z = 0$ with radius R, and H_R^* s a hemisphere with the same radius R.

To clarify the physical meaning of function $u_{s,2}^{sc}$, we introduce new denotations into Equation (1.58),

$$u_s^{refl} = -u^{inc}, \qquad \frac{\partial u_s^{refl}}{\partial n} = \frac{\partial u^{inc}}{\partial n}, \tag{1.64}$$

and apply the equivalency theorem to the surface $\Sigma = S_R + H_R^*$ shown in Figure 1.9. It is supposed that the image source, that is, the source of the reflected field, is inside the volume V^*.

According to the equivalency theorem,

$$u(P) = \frac{1}{4\pi} \oint_{S_R + H_R^*} \left(u^{refl} \frac{\partial}{\partial n} \frac{e^{ikr}}{r} - \frac{\partial u^{refl}}{\partial n} \frac{e^{ikr}}{r} \right) ds$$
$$= \begin{cases} 0, & \text{when } P \text{ inside } V^* \\ u^{refl}(P), & \text{when } P \text{ outside } V^*. \end{cases} \tag{1.65}$$

When $R \to \infty$, the integral over H_R^* tends to zero, S_R is transformed into the infinite plane S, and therefore

$$u_{s,2}^{sc} = u_s^{refl} = \frac{1}{4\pi} \int_S \left(u_s^{refl} \frac{\partial}{\partial n} \frac{e^{ikr}}{r} - \frac{\partial u_s^{refl}}{\partial n} \frac{e^{ikr}}{r} \right) ds$$
$$= \begin{cases} u_s^{refl}, & \text{in the region } z > 0 \\ 0, & \text{in the region } z < 0. \end{cases} \tag{1.66}$$

Thus, the function (1.58) represents the reflected field in the region $z > 0$.

The field (1.54) scattered by the hard infinite plane S also can be represented as the sum of the reflected field and the shadow radiation:

$$u_{\mathrm{h}}^{\mathrm{sc}} = u_{\mathrm{h}}^{\mathrm{refl}} + u^{\mathrm{sh}}, \tag{1.67}$$

where

$$u_{\mathrm{h}}^{\mathrm{refl}} = \frac{1}{4\pi}\int_S \left(u_{\mathrm{h}}^{\mathrm{refl}}\frac{\partial}{\partial n}\frac{\mathrm{e}^{ikr}}{r} - \frac{\partial u_{\mathrm{h}}^{\mathrm{refl}}}{\partial n}\frac{\mathrm{e}^{ikr}}{r}\right)\mathrm{d}s \tag{1.68}$$

and on the plane S

$$u_{\mathrm{h}}^{\mathrm{refl}} = u^{\mathrm{inc}}, \qquad \frac{\partial u_{\mathrm{h}}^{\mathrm{refl}}}{\partial n} = -\frac{\partial u^{\mathrm{inc}}}{\partial n}. \tag{1.69}$$

The shadow radiation does not depend on the boundary conditions and is the same both for the soft and hard planes. It is defined by Equation (1.63).

The above definitions of the reflected and shadow radiations are applicable to the PO field scattered by arbitrary soft and hard objects. In this case, however, the integration surface in (1.63), (1.66), and in (1.68) must be specified as the illuminated side (S_{il}) of the object (Fig. 1.4). Thus, in general,

$$u_{\mathrm{s,h}}^{\mathrm{PO}} = u_{\mathrm{s,h}}^{\mathrm{refl}} + u^{\mathrm{sh}} \tag{1.70}$$

where

$$u_{\mathrm{s,h}}^{\mathrm{refl}} = \frac{1}{4\pi}\int_{S_{\mathrm{il}}} \left(u_{\mathrm{s,h}}^{\mathrm{refl}}\frac{\partial}{\partial n}\frac{\mathrm{e}^{ikr}}{r} - \frac{\partial u_{\mathrm{s,h}}^{\mathrm{refl}}}{\partial n}\frac{\mathrm{e}^{ikr}}{r}\right)\mathrm{d}s, \tag{1.71}$$

$$u^{\mathrm{sh}} = \frac{1}{4\pi}\int_{S_{\mathrm{il}}} \left(u^{\mathrm{inc}}\frac{\partial}{\partial n}\frac{\mathrm{e}^{ikr}}{r} - \frac{\partial u^{\mathrm{inc}}}{\partial n}\frac{\mathrm{e}^{ikr}}{r}\right)\mathrm{d}s, \tag{1.72}$$

and $u_{\mathrm{s,h}}^{\mathrm{refl}}$, $\partial u_{\mathrm{s,h}}^{\mathrm{refl}}/\partial n$ are defined by Equations (1.64) and (1.69). Equation (1.72) can be interpreted as the generalization of the Kirchhoff definition for the *black bodies* suggested earlier by Ufimtsev (1968, 2003). It is clear that the PO formulation in the form of Equation (1.70) is valid for electromagnetic waves as well.

One should also notice another interesting relationship between the PO field and the shadow radiation. According to the definitions (1.31), (1.32) and (1.72) for these quantities, the shadow radiation can be represented in the form

$$u^{\text{sh}} = \frac{1}{2}(u_{\text{s}}^{\text{PO}} + u_{\text{h}}^{\text{PO}}). \tag{1.73}$$

If we now take into account Equation (1.37) and substitute it into (1.73), we immediately come to the fundamental conclusion that the shadow radiation exactly equals zero in the direction to the source of the incident wave. In other words, the *black bodies* (as they are defined above) do not generate the backscattering.

In contrast to the infinite plane problem, where the reflected field and shadow radiations exist in the separated half-spaces, the fields (1.71) and (1.72) caused by diffraction at the finite size objects exist in the whole surrounding space. However, their spatial distributions are different. The reflected field dominates in the ray region, and the shadow radiation concentrates at the shadow region and in its vicinity (Ufimtsev, 1968, 1990, 1996, 2003). Well-known manifestations of the shadow radiation are the phenomena Fresnel diffraction and forward scattering (Glaser, 1985; Ufimtsev, 1996; Willis, 1991).

Indeed, the diffraction bands bordering the geometrical optics shadow of opaque objects observed in Fresnel diffraction are nothing but the result of interference of an incident wave with shadow radiation. It is interesting that the history of diffraction as a science started in the seventeenth century with investigation of just this phenomenon (Grimaldi, Newton, Young, Fresnel). The forward scattering is the enhancement of the scattered field in the directions approaching the shadow boundary behind the object. It was extensively investigated both experimentally (Willis, 1991) and theoretically [Bowman et al. (1987)]. The numerical data for the field scattered by acoustically soft and hard objects, as well as perfectly conducting objects, presented in the work of Bowman et al. (1987), clearly illustrate the existence of this phenomenon. Our present analysis of the PO approximation reveals the nature of this phenomenon, which is inherent for scattering at any large opaque objects.

One should note that, due to transverse diffusion, the shadow radiation can penetrate far from the shadow region (Ufimtsev, 1990); see also Section 14.2 in the present book. Also, it gives origin to the edge waves, creeping waves, and surface diffracted rays (Ufimtsev, 1996, 2003).

1.3.5 Shadow Contour Theorem and the Total Scattering Cross-Section

Among the properties of shadow radiation, the most significant are the Shadow Contour Theorem and the Total Power of Shadow Radiation. They have already been established for electromagnetic waves (Ufimtsev, 1968, 1990, 1996, 2003) and will now be verified for acoustic waves.

Let us compare the shadow radiation generated by two scattering objects with different shapes, but with the same shadow contour (Fig. 1.10). Their illuminated sides are S_1 and S_2.

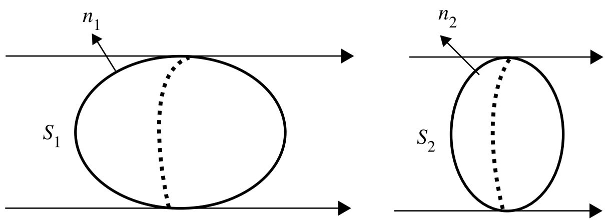

Figure 1.10 Two different objects with the same shadow contour (dotted line).

According to Equation (1.72),

$$u_1^{\text{sh}} = \frac{1}{4\pi}\int_{S_1}\left(u^{\text{inc}}\frac{\partial}{\partial n_1}\frac{\mathrm{e}^{ikr}}{r} - \frac{\partial u^{\text{inc}}}{\partial n_1}\frac{\mathrm{e}^{ikr}}{r}\right)\mathrm{d}s$$

and

$$u_2^{\text{sh}} = \frac{1}{4\pi}\int_{S_2}\left(u^{\text{inc}}\frac{\partial}{\partial n_2}\frac{\mathrm{e}^{ikr}}{r} - \frac{\partial u^{\text{inc}}}{\partial n_2}\frac{\mathrm{e}^{ikr}}{r}\right)\mathrm{d}s. \tag{1.74}$$

The difference of these quantities can be written as

$$u_1^{\text{sh}} - u_2^{\text{sh}} = \frac{1}{4\pi}\int_{S_1+S_2}\left(u^{\text{inc}}\frac{\partial}{\partial n}\frac{\mathrm{e}^{ikr}}{r} - \frac{\partial u^{\text{inc}}}{\partial n}\frac{\mathrm{e}^{ikr}}{r}\right)\mathrm{d}s, \tag{1.75}$$

where $\hat{n} = \hat{n}_1$, $\hat{n} = -\hat{n}_2$ is the external normal to the surface $S_1 + S_2$ (Fig. 1.11). As a result of the Helmholtz equivalence principle (Bakker and Copson, 1939), the quantity (1.75) equals zero for observation points outside the volume enclosed by surface $S_1 + S_2$. Therefore

$$u_1^{\text{sh}} = u_2^{\text{sh}}. \tag{1.76}$$

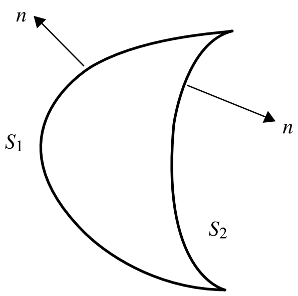

Figure 1.11 Surface $S_1 + S_2$ in an homogeneous medium. All sources and the observation points are outside the volume enclosed by this surface.

This equation represents the Shadow Contour Theorem:

The Shadow radiation does not depend on the whole shape of a scattering object and it is completely determined only by the size and geometry of the shadow contour.

Now let us evaluate the total power of the reflected field and shadow radiation. The total power of the reflected field can be written as

$$P_{\mathrm{s,h}}^{\mathrm{refl}} = \frac{1}{2}\mathrm{Re}\int_{S_{\mathrm{il}}} (p_{\mathrm{s,h}}^{\mathrm{refl}})^* (\vec{v}_{\mathrm{s,h}}^{\mathrm{refl}} \cdot \hat{n})\mathrm{d}s \tag{1.77}$$

where, according to Equations (1.2), (1.41), (1.64), and (1.69),

$$\begin{aligned} p_{\mathrm{s}}^{\mathrm{refl}} &= -p^{\mathrm{inc}} = -i\omega\rho u_0 \mathrm{e}^{ikz}, \qquad \vec{v}_{\mathrm{s}}^{\mathrm{refl}} \cdot \hat{n} = \vec{v}^{\mathrm{inc}} \cdot \hat{n} = iku_o \mathrm{e}^{ikz}(\hat{z}\cdot\hat{n}), \\ p_{\mathrm{h}}^{\mathrm{refl}} &= p^{\mathrm{inc}} = i\omega\rho u_0 \mathrm{e}^{ikz}, \qquad \vec{v}_{\mathrm{h}}^{\mathrm{refl}} \cdot \hat{n} = -\vec{v}^{\mathrm{inc}} \cdot \hat{n} = -iku_o \mathrm{e}^{ikz}(\hat{z}\cdot\hat{n}). \end{aligned} \tag{1.78}$$

Substitution of these quantities into Equation (1.77) results in

$$P_{\mathrm{s}}^{\mathrm{refl}} = P_{\mathrm{h}}^{\mathrm{refl}} = \frac{1}{2}k^2 ZA|u_0|^2 = P^{\mathrm{inc}} \cdot A \tag{1.79}$$

and

$$\sigma^{\mathrm{refl,tot}} = A \tag{1.80}$$

where A is the area of the shadow region cross-section (Fig. 1.4). These equations show that the total power of the reflected field exactly equals the power of the intercepted incident rays. The scattering object only distributes them in the surrounding space. By changing the shape of the illuminated surface S_{il}, one can significantly decrease the backscattering by deflection of the reflected rays from the direction back to the source of the incident wave. This is the first basic idea used in stealth technology.

The total power of the shadow radiation is determined by

$$P^{\mathrm{sh,tot}} = \frac{1}{2}\mathrm{Re}\int_{s_{\mathrm{il}}} (p^{\mathrm{inc}})^* [\vec{v}^{\mathrm{inc}} \cdot (-\hat{n})]\mathrm{d}s, \tag{1.81}$$

The minus sign in front of $\vec{n}$ is chosen because on the surface of black bodies no radiation/reflection in the positive normal direction exists. According to Equations (1.41) and (1.78)

$$p^{\mathrm{inc}} = i\omega\rho u_0 \mathrm{e}^{ikz}, \quad \text{and } (\vec{v}^{\mathrm{inc}} \cdot \hat{n}) = iku_0 \mathrm{e}^{ikz}(\hat{z}\cdot\hat{n}). \tag{1.82}$$

Substitution of Equation (1.82) into Equation (1.81) leads to

$$P^{\mathrm{sh,tot}} = \frac{1}{2}k^2 ZA|u_0|^2 = P^{\mathrm{inc}}A \tag{1.83}$$

and

$$\sigma^{\mathrm{sh,tot}} = A. \tag{1.84}$$

Thus, the shadow radiation power equals the reflected power, and their sum exactly equals the total scattered power (1.42). This result illustrates the physics behind the fundamental diffraction law (1.45). It shows that objects with soft and hard boundary conditions reveal a dual nature. They can be interpreted as if they are simultaneously perfectly reflecting (with reflection coefficients $\mathcal{R}_s = -1$ and $\mathcal{R}_h = 1$) and perfectly absorbing (with $\mathcal{R} = 0$); that is, black. This law can now be written in the form

$$\sigma^{\mathrm{tot}} = \sigma^{\mathrm{refl,tot}} + \sigma^{\mathrm{sh,tot}} = 2A, \tag{1.85}$$

where A is the area of the shadow region cross-section.

From the equation $\sigma^{\mathrm{refl,tot}} = A$, it is also clearly seen that the total power of the reflected waves does not depend on the object shaping if the area of shadow cross-section remains constant. However, this power can be decreased by absorbing coatings – this is the second basic idea of stealth technology. In contrast, the shadow radiation cannot be decreased by any absorbing coatings and it can be used for bistatic detection of large opaque objects with small backscattering cross-section (Ufimtsev, 1996).

1.3.6 Summary of Properties of Physical Optics Approximation

The PO approximation describes properly both all reflected rays away from the geometrical optics boundaries and the diffracted field near these boundaries, as well as near foci and caustics. Reflected rays are revealed by the asymptotic evaluation of the PO integrals. These integrals correctly predict the magnitude and position of main and near side lobes in the directivity pattern of the scattered field. However, these surface integrals are computer time consuming. Their transformation into line integrals reduces computer time, and is the subject of continuing research (Asvestas, 1985a,b, 1986, 1995; Gordon, 1994, 2003; Gordon and Bilow, 2002; Maggi, 1888; Meincke et al., 2003; Rubinowicz, 1917).

The *backscattered* field in the PO approximation possesses a special property. According to Equation (1.37), the fields scattered by the *soft* and *hard* objects (of the same shape and size) differ only in *sign*.

Separation of the PO field into the reflected field and the shadow radiation elucidates the scattering physics. In particular, it explains the fundamental law of diffraction theory, according to which the total scattering cross-section of large perfectly reflecting objects is double the area of their shadow cross-section. Well-known manifestations of shadow radiation are Fresnel diffraction and forward scattering.

The PO drawbacks are the following. It is not self-consistent. When the observation point approaches the scattering surface, the PO integrals do not reproduce the initial Geometrical Acoustics (GA) values for the surface field. Also, the PO field does not satisfy rigorously the boundary conditions and the reciprocity principle. The reason for these shortcomings is the Geometrical Optics (GO) approximation for the surface field, which does not include its diffraction components. The PO shortcomings are overcome in the PTD (Ufimtsev, 1962, 1991, 2003), which improves PO by taking into account the diffracted surface field.

1.4 NONUNIFORM COMPONENT OF INDUCED SURFACE FIELD

Surface fields u or $\partial u/\partial n$ induced by the incident wave on the scattering object can be considered as the sources of the scattered field. As noted in the Introduction, the central and original idea of PTD is the separation of these sources into *uniform* and *nonuniform* components:

$$j_{\mathrm{s,h}} = j^{(0)}_{\mathrm{s,h}} + j^{(1)}_{\mathrm{s,h}}. \tag{1.86}$$

The uniform component $j^{(0)}_{\mathrm{s,h}}$ is defined by Equation (1.31) and represents the surface field induced on the infinite plane tangent to the object (Figs. 1.4 and 1.12). In the case of the incident plane wave, this field is *uniformly* distributed over the tangent plane. Its amplitude is constant and its phase is a linear function of the plane coordinates. That is why this component is called *uniform*. According to the definition (1.31), it can also be called the *geometrical optics* or *ray* component.

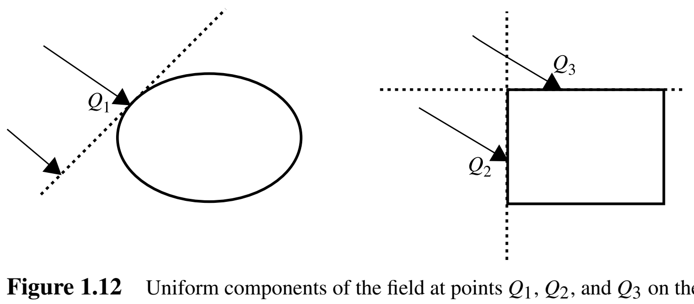

Figure 1.12 Uniform components of the field at points Q_1, Q_2, and Q_3 on the scattering objects are identical to those on the infinite tangential planes shown by the dotted lines.

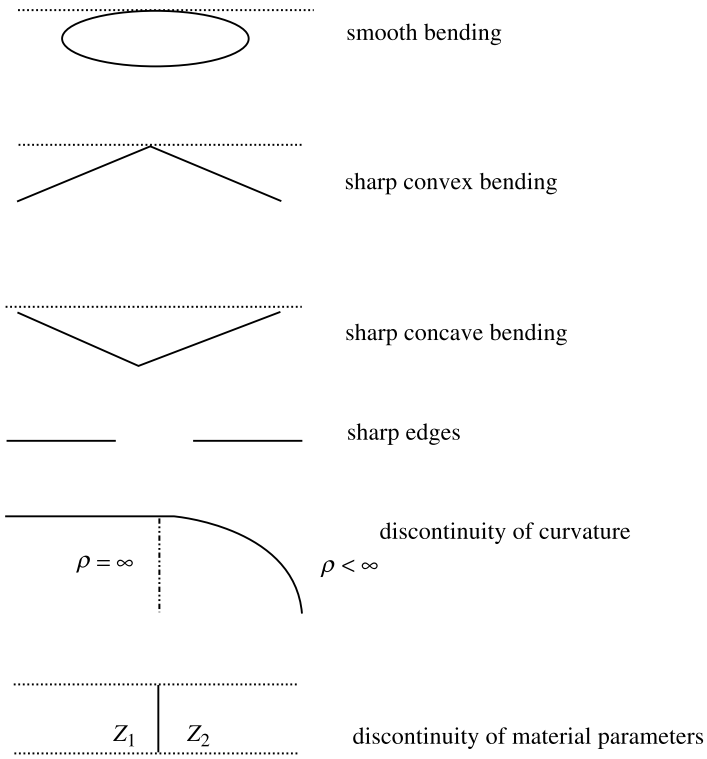

Figure 1.13 Different shapes and structures where the incident wave generates the *nonuniform* scattering sources.

In contrast, the *nonuniform* component $j^{(1)}_{s,h}$ is the *diffraction part* of the surface field. It is caused by diffraction due to any deviation of the scattering surface from that of the tangential infinite plane. These deviations can be a smooth or sharp bending, sharp edges, discontinuity of curvature, discontinuity of material properties, apertures, small bumps and dips, and so on. (Fig. 1.13).

If the scattering object is convex and smooth and its dimensions and radii of curvature are large compared to the wavelength, then the induced nonuniform component concentrates near the boundary between the illuminated and shadowed surfaces (Fig. 1.4). This component is described asymptotically by the Fock functions (Fock, 1965). From the physical point of view it represents creeping waves that radiate surface diffracted rays. If the object possesses sharp edges, the nonuniform component concentrates in their vicinity (Fig. 1.14) and it is described asymptotically by the Sommerfeld functions (Sommerfeld, 1935) presented in Chapter 2. This form of the nonuniform components radiates the edge waves that are often called *fringe* waves. Similarly the nonuniform sources near the vertices radiate vertex waves.

Taking into account the diffraction/nonuniform components of the surface fields, PTD overcomes the PO shortcomings and provides more accurate asymptotic results for high-frequency scattered fields. The PTD separation of the surface fields into uniform and nonuniform components has proved to be very productive and is often used

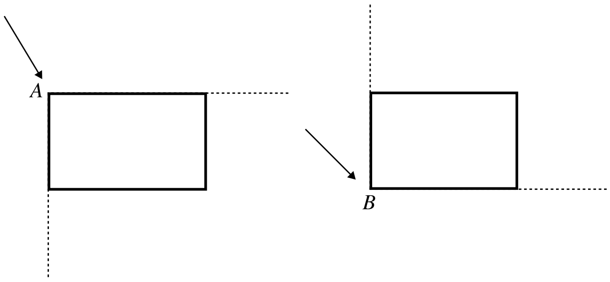

Figure 1.14 Nonuniform components of the surface field induced by the incident wave near edges *A* and *B* are asymptotically identical to those at the tangential wedges with infinite faces shown by dotted lines.

in diffraction theory. This concept is quite flexible. It can be extended for objects with other boundary conditions. It is also successfully used in hybrid techniques in combination with direct numerical methods. A proper choice of the uniform component depends on specific properties of the problem under investigation and can essentially facilitate its solution. See for example Ufimtsev (1998) and related references given in Ufimtsev (1996, 2003), as well as in the section "Additional references related to the PTD concept: Applications, modifications, and developments" shown at the end of this book.

1.5 ELECTROMAGNETIC WAVES

This section briefly presents the basic notions used in this book for the description of electromagnetic waves. This book studies the diffraction of electromagnetic waves at perfectly conducting bodies that are large compared to the wavelength. It is assumed that the waves and scattering objects are in free space (vacuum). The electric ($\vec{E}$) and magnetic vectors ($\vec{H}$) of the wave field are determined as

$$\vec{E} = \frac{i}{k} Z_0 \cdot [\nabla(\nabla \cdot \vec{A}^{\mathrm{e}}) + k^2 \vec{A}^{\mathrm{e}}] - \nabla \times \vec{A}^{\mathrm{m}} \tag{1.87}$$

and

$$\vec{H} = \frac{i}{kZ_0} [\nabla(\nabla \cdot \vec{A}^{\mathrm{m}}) + k^2 \vec{A}^{\mathrm{m}}] + \nabla \times \vec{A}^{\mathrm{e}}. \tag{1.88}$$

Here, $Z_0 = 1/Y_0 = \sqrt{\mu_0/\varepsilon_0} = 120\pi$ ohms is the impedance of free space, $k = \omega\sqrt{\varepsilon_0\mu_0} = 2\pi/\lambda$ is the wave number, and

$$\vec{A}^{\mathrm{e}} = \frac{1}{4\pi} \int \vec{j}^{\mathrm{e}} \frac{\mathrm{e}^{ikr}}{r} \mathrm{d}v \qquad \text{and} \qquad \vec{A}^{\mathrm{m}} = \frac{1}{4\pi} \int \vec{j}^{\mathrm{m}} \frac{\mathrm{e}^{ikr}}{r} \mathrm{d}v \tag{1.89}$$

are the electric and magnetic vector-potentials. They are the solutions of the equation

$$\Delta \vec{A}^{\mathrm{e,m}} + k^2 \vec{A}^{\mathrm{e,m}} = -\vec{j}^{\mathrm{e,m}}, \tag{1.90}$$

where $\vec{j}^{\mathrm{e}}$ ($\vec{j}^{\mathrm{m}}$) is the electric (magnetic) current density of the field sources.

Notice that these Equations (1.87) to (1.90) are more convenient for calculation than those usually accepted in books on engineering electromagnetics. Indeed, Equations (1.89) and (1.90) have exactly the same form both for electric and magnetic potentials. The second terms (out of brackets) in Equations (1.87) and (1.88) do not contain the factors $1/\mu_0$ and $1/\varepsilon_0$, which eventually disappear in the integral field expressions.

In the far zone from a scattering object, one can use the following approximation (similar to Equations (1.16) and (1.17):

$$\vec{A}^{\mathrm{e,m}} = \frac{1}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\int \vec{j}^{\mathrm{e,m}}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}v. \tag{1.91}$$

This leads to the field components

$$E_\vartheta = Z_0 H_\varphi = ik(Z_0 A^{\mathrm{e}}_\vartheta + A^{\mathrm{m}}_\varphi) \tag{1.92}$$

and

$$E_\varphi = -Z_0 H_\vartheta = ik(Z_0 A^{\mathrm{e}}_\varphi - A^{\mathrm{m}}_\vartheta). \tag{1.93}$$

The radial components E_{R}, H_{R} are of the order $1/R^2$ and are neglected here. The coordinate system is shown in Figure 1.2.

The scattering cross-section is determined by Equations (1.26) and (1.9) as

$$\sigma = 4\pi R^2 \left|\frac{\vec{E}^{\mathrm{sc}}}{\vec{E}^{\mathrm{inc}}}\right|^2. \tag{1.94}$$

Equation (1.45) for the total scattering cross-section is also valid for electromagnetic waves. In the case of an incident wave with a linear polarization, it can be represented in the form of Equation (1.53) as

$$\sigma^{\mathrm{tot}} = \frac{4\pi}{k}\mathrm{Im}[(E^{\mathrm{sc}}/E^{\mathrm{inc}})\cdot R\mathrm{e}^{-ikR}], \tag{1.95}$$

where E^{sc} is the field scattered in the forward direction.

The PO approximation for the far field scattered by perfectly conducting objects is defined by

$$\vec{A}^{\mathrm{e}} = \frac{1}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\mathrm{il}}} \vec{j}^{(0)}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}s, \qquad \vec{A}^{\mathrm{m}} = 0, \tag{1.96}$$

where S_{il} is the illuminated side of the scattering surface and

$$\vec{j}^{(0)} = 2[\hat{n}\times\vec{H}^{\mathrm{inc}}] \tag{1.97}$$

is the *uniform component* of the surface electric current induced by the incident wave on the illuminated side of a scattering object. The paper by Ufimtsev (1999) describes in detail the properties of the PO approximation (see also Ruck et al. (1970)).

Equations (4.1.12) and (4.1.13) of Ufimtsev (2003) show that in the PO approximation, the field backscattered by *convex perfectly conducting* objects does not depend on the polarization of the incident wave.

Consider another important consequence of the PO approximations (4.1.12) and (4.1.13) of Ufimtsev (2003). These equations were derived under the following conditions:

- The incident wave is a plane wave propagating in the direction

$$\hat{k}^i = \hat{y}\sin\gamma + \hat{z}\cos\gamma.$$

- The observation point is in the backscattering direction $\hat{m} = -\hat{k}^i$ (in the plane yoz ($\varphi = -\pi/2$)).
- Equation (4.1.12) is valid for the incident wave with E-polarization, $E_x^{\text{inc}} = E_{0x}\mathrm{e}^{ik(y\sin\gamma + z\cos\gamma)}$.
- Equation (4.1.13) is valid for the incident wave with H-polarization, $H_x^{\text{inc}} = H_{0x}\mathrm{e}^{ik(y\sin\gamma + z\cos\gamma)}$.

In view of these comments, the PO approximations (4.1.12) and (4.1.13) in Ufimtsev (2003) can be written as

$$E_x^{(0)} = -\frac{ik}{2\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}} E_x^{\text{inc}}\mathrm{e}^{-ikr'\cos\Omega}(\hat{k}^i\cdot\hat{n})\mathrm{d}s, \tag{1.98}$$

and

$$H_x^{(0)} = \frac{ik}{2\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}} H_x^{\text{inc}}\mathrm{e}^{-ikr'\cos\Omega}(\hat{k}^i\cdot\hat{n})\mathrm{d}s. \tag{1.99}$$

Comparison of these equations with Equation (1.37) reveals the following fundamental relationships, which exist between the PO approximations for *backscattered* acoustic and electromagnetic waves:

$$E_x^{(0)} = u_{\text{s}}^{(0)} \quad \text{if} \quad E_x^{\text{inc}} = u^{\text{inc}} \tag{1.100}$$

and

$$H_x^{(0)} = u_{\text{h}}^{(0)} \quad \text{if} \quad H_x^{\text{inc}} = u^{\text{inc}}. \tag{1.101}$$

Utilizing the vector equivalency theorems (Ufimtsev, 2003) and the idea of Section 1.3.4, one can represent the PO field in a form similar to Equation (1.70):

$$\vec{E}^{\text{PO}} \equiv \vec{E}^{(0)} = \vec{E}^{\text{refl}} + \vec{E}^{\text{sh}}, \qquad \vec{H}^{\text{PO}} \equiv \vec{H}^{(0)} = \vec{H}^{\text{refl}} + \vec{H}^{\text{sh}}. \tag{1.102}$$

Here, $\vec{E}^{\text{refl}}$, $\vec{H}^{\text{refl}}$ and $\vec{E}^{\text{sh}}$, $\vec{H}^{\text{sh}}$ are the reflected field and the shadow radiation, respectively. Their far-field approximations are

$$E_{\vartheta}^{\text{refl}} = \frac{ik}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}}\{Z_0[n\times\vec{H}^{\text{inc}}]\cdot\hat{\vartheta} + [\hat{n}\times\vec{E}^{\text{inc}}]\cdot\hat{\varphi}\}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}s, \tag{1.103}$$

$$E_{\varphi}^{\text{refl}} = \frac{ik}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}}\{Z_0[n\times\vec{H}^{\text{inc}}]\cdot\hat{\varphi} - [\hat{n}\times\vec{E}^{\text{inc}}]\cdot\hat{\vartheta}\}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}s, \tag{1.104}$$

$$E_{\vartheta}^{\text{sh}} = \frac{ik}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}}\{Z_0[n\times\vec{H}^{\text{inc}}]\cdot\hat{\vartheta} - [\hat{n}\times\vec{E}^{\text{inc}}]\cdot\hat{\varphi}\}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}s, \tag{1.105}$$

$$E_{\varphi}^{\text{sh}} = \frac{ik}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}}\{Z_0[n\times\vec{H}^{\text{inc}}]\cdot\hat{\varphi} + [\hat{n}\times\vec{E}^{\text{inc}}]\cdot\hat{\vartheta}\}\mathrm{e}^{-ikr'\cos\Omega}\,\mathrm{d}s, \tag{1.106}$$

$$\vec{H}^{\text{refl}} = [\nabla R\times\vec{E}^{\text{refl}}]/Z_0,$$

$$\vec{H}^{\text{sh}} = [\nabla R\times\vec{E}^{\text{sh}}]/Z_0. \tag{1.107}$$

Here, $\hat{\vartheta}\,(\hat{\varphi})$ is the unit vector in the direction of the angle $\vartheta\,(\varphi)$ increase.

Suppose that the incident wave is given as

$$E_x^{\text{inc}} = Z_0H_y^{\text{inc}} = E_{0x}\mathrm{e}^{ikz}. \tag{1.108}$$

Then, one can derive the following relationships:

$$E_x^{\text{sh}} = \frac{ik}{2\pi}E_{0x}A\frac{\mathrm{e}^{ikR}}{R}, \qquad E_x^{\text{refl}} = 0 \qquad (\vartheta = 0) \tag{1.109}$$

for the forward direction, and

$$E_x^{\text{sh}} = 0 \qquad (\vartheta = \pi) \tag{1.110}$$

for the backscattering direction. Here, A is the area of the shadow region cross-section (Fig. 1.4). Thus,

$$E_x^{\text{PO}} \equiv E_x^{\text{sh}} = E_{0x}\frac{ik}{2\pi}A\frac{\mathrm{e}^{ikz}}{z} \qquad (\vartheta = 0) \tag{1.111}$$

for the forward direction, and

$$E_x^{\text{PO}} = -\frac{ik}{2\pi}E_{0x}\frac{\mathrm{e}^{ikR}}{R}\int_{S_{\text{il}}}(\hat{n}\cdot\hat{z})\mathrm{e}^{i2kz'}\mathrm{d}s \qquad (\vartheta = \pi) \tag{1.112}$$

for the backscattering direction.

The Shadow Contour Theorem established in Section 1.3.5 for acoustic waves is also valid for electromagnetic waves.

Shadow radiation can be interpreted as the field scattered by black bodies. Chapter 1 of Ufimtsev (2003) presents explicit expressions and the results of numerical calculation for the field scattered by arbitrary 2-D black cylinders and black bodies of revolution.

The definition of the nonuniform component of the surface sources introduced in Section 1.4 is also applicable for electric surface currents. Its modification is presented below in Sections 7.9.1 and 7.9.2.

PROBLEMS

1.1 The incident wave $u^{\text{inc}} = u_0 \exp[-ik(x\cos\varphi_0 + y\sin\varphi_0)]$ excites the scattering sources $j_{\text{s}} = 2\partial u^{\text{inc}}/\partial y$ on the illuminated side ($y = +0$) of a soft infinite plane. $y = 0$ (Fig. P1.1). Start with the integral (1.10) and calculate the scattered field u_{s}^{sc} generated by these sources above and below the plane.

(**a**) Express the integral over the variable ζ through the Hankel function (3.7). Use the integral representation (3.8) of this function and obtain the Fourier integral for a plane wave.

(**b**) Consider the total field $u_{\text{s}}^{\text{t}} = u_{\text{s}}^{\text{inc}} + u_{\text{s}}^{\text{sc}}$ in the region $y < 0$ and realize the blocking role of the scattering sources j_{s}.

1.2 Solve the scattering problem similar to Problem 1.1, but for a hard reflecting plane.

1.3 The incident wave $E_z^{\text{inc}} = E_{0z} \exp[-ik(x\cos\varphi_0 + y\sin\varphi_0)]$ excites the surface current $\vec{j} = 2[\hat{y} \times \vec{H}^{\text{inc}}]$ on the illuminated side ($y = +0$) of a perfectly conducting infinite plane (Fig. P1. 1). Start with Equations (1.87) and (1.89) and calculate the scattered field E_z^{sc} generated by these currents above and below the plane.

(**a**) Express the integral over the variable ζ through the Hankel function (3.7). Use the integral representation (3.8) of this function and obtain the Fourier integral for a plane wave.

(**b**) Consider the total field $E_z^{\text{t}} = E_z^{\text{inc}} + E_z^{\text{sc}}$ in the region $y < 0$ and realize the blocking role of the surface currents j_z.

1.4 Solve the problem analogous to Problem 1.3 but with the incident wave $H_z^{\text{inc}} = H_{0z} \exp[-ik(x\cos\varphi_0 + y\sin\varphi_0)]$. Start with Equation (1.88).

1.5 Suppose that the incident wave $u^{\text{inc}} = u_0 \exp[ik(x\cos\phi_0 + y\sin\phi_0)]$ hits a soft strip as shown in Figure 5.1. Use Equation (1.71) and calculate the *reflected part* of the PO field scattered by this strip.

(**a**) Express the integral over the variable ζ through the Hankel function (3.7), apply its asymptotic approximation (2.29), and express the far field ($r \gg ka^2$) in closed form.

(**b**) Estimate the field in the directions $\phi = \phi_0, \phi = \pi - \phi_0, \phi = \pi + \phi_0$, and $\phi = -\phi_0$.

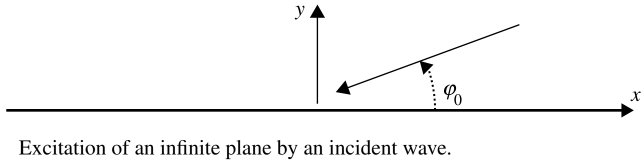

Figure P1.1 Excitation of an infinite plane by an incident wave.

(**c**) Apply the optical theorem (5.16) to the field in the direction $\phi = \pi - \phi_0$ and provide the geometrical interpretation of the total reflecting cross-section.

(**d**) Compute and plot the directivity pattern of the reflected field, setting $a = 2\lambda$, $\phi_0 = 45°$. What are the interesting properties of this field?

1.6 Solve the problem similar to Problem 1.5, but for a hard strip.

1.7 Suppose that the incident wave $u^{\text{inc}} = u_0 \exp[ik(x \cos\phi_0 + y \sin\phi_0)]$ hits a soft strip as shown in Figure 5.1. Use Equation (1.72) and calculate the *shadow radiation part* of the PO field scattered by this strip.

(**a**) Express the integral over the variable ζ through the Hankel function (3.7), apply its asymptotic approximation (2.29), and calculate the far field ($r \gg ka^2$) in closed form.

(**b**) Estimate the field in the directions $\phi = \phi_0$, $\phi = \pi - \phi_0$, and $\phi = \pi + \phi_0$.

(**c**) Apply the optical theorem (5.16) to the field in the direction $\phi = \phi_0$ and give the geometrical interpretation of the total power of the shadow radiation.

(**d**) Compute and plot the directivity pattern of the shadow radiation, setting $a = 2\lambda$, $\phi_0 = 45°$. What are the interesting properties of this field?

1.8 Is the difference between the reflected parts of the PO field scattered by soft and hard objects (of the same shape and size) illuminated by the same incident wave.

1.9 Is any difference between the shadow parts of the PO field scattered by soft and hard objects (of the same shape and size) illuminated by the same incident wave.

1.10 The incident wave $E_z^{\text{inc}} = E_{0z} \exp[ik(x \cos\phi_0 + y \sin\phi_0)]$ hits a perfectly conducting strip as shown in Figure 5.1. Calculate the *reflected part* of the PO scattered field.

(**a**) Start with Equations (1.87), (1.88) and (1.89). Apply $\vec{j}^{\text{e,refl}} = \hat{n} \times \vec{H}^{\text{inc}}$, $\vec{j}^{\text{m,refl}} = \hat{n} \times \vec{E}^{\text{inc}}$. Prepare the integral expression for the reflected field.

(**b**) Express the integral over the variable ζ through the Hankel function (3.7), apply its asymptotic approximation (2.29), and express the far field ($r \gg ka^2$) in closed form.

(**c**) Estimate the field in the directions $\phi = \phi_0, \phi = \pi - \phi_0, \phi = \pi + \phi_0$, and $\phi = -\phi_0$.

(**d**) Apply the optical theorem (5.16) to the field in the direction $\phi = \pi - \phi_0$ and provide the geometrical interpretation of the total reflecting cross-section.

(**e**) Compute and plot the directivity pattern of the reflected field, setting $a = 2\lambda$, $\phi_0 = 45°$. What are the interesting properties of this field?

1.11 The incident wave $E_z^{\text{inc}} = E_{0z} \exp[ik(x \cos\phi_0 + y \sin\phi_0)]$ hits a perfectly conducting strip as shown in Figure 5.1. Calculate the *shadow radiation part* of the PO scattered field.

(**a**) Start with Equations (1.87), (1.88) and (1.89). Apply

$$\vec{j}^{\text{e,sh}} = \hat{n} \times \vec{H}^{\text{inc}}, \vec{j}^{\text{m,sh}} = -\hat{n} \times \vec{E}^{\text{inc}}.$$

Prepare the integral expression for the shadow radiation.

(**b**) Express the integral over the variable ζ through the Hankel function (3.7), apply its asymptotic approximation (2.29), and calculate the far field ($r \gg ka^2$) in closed form.

(**c**) Estimate the field in the directions $\phi = \phi_0$, $\phi = \pi - \phi_0$, and $\phi = \pi + \phi_0$.

(**d**) Apply the optical theorem (5.16) to the field in the direction $\phi = \phi_0$ and provide the geometrical interpretation of the total power of the shadow radiation.

(**e**) Compute and plot the directivity pattern of the reflected field, setting $a = 2\lambda$, $\phi_0 = 45°$. What are the interesting properties of this field?