

Index

- Absolutely continuous, 281
Adapted parameter, 64, 70–2, 78, 88, 113
Adaptive
 kernel estimator, **180**, 198
 projection, **63**
 projection estimator, **81**, 92, 222
Additive functional, 213
Adjacent hypotheses, **107**
Approximation, 10
AR(1) process, **16**, **21**, 47, 79, 248, 253, 266, 289
 discretized Ornstein–Uhlenbeck, 201
AR(p) process, 291
ARB process, **254**
 standard, 255
ARC process
 estimation and prediction, **272**
ARH process, **243**
 with exogenous variables, 259
ARH(1) process, 72, 243, 246, 265
 Markovian, 249
 prediction, **269**, **279**
 standard, 248
ARH(p) process, 243, 259
ARH $^\infty$ (1) process, 263
ARHD process, 259
ARIMA process, 33
ARMA process, 45, **48**, 79
ARS process, 265
Autocorrelation
 function, 169, 171, 190, 211
 operator, 244, 248
 estimation and prediction, **277**
Autocovariance, 57, 64, 212, 290
 estimation of
 function, **275**
 operator, **263**
Bahadur efficiency, **110**
Banach space, 284
Bandwidth, 133
Bayesian
 estimator, 45, 52
 predictor, **31**
 risk, 31
Bernstein inequality, 141, 297
Berry–Esseen
 bound, 278
 inequality of multidimensional type, 98
Best linear
 error of prediction, 58
 predictor, 57, 261, 263
Best predictor, 287
 probabilistic, 269
Best unbiased
 estimator, 18–20, 33
 predictor, 18–20, 28, 33, 34
Billingsley inequality, 85, 140, 295
Birkhoff–Khinchine ergodic theorem, 215
Blackwell, *see also* Rao–Blackwell theorem
 algorithm, 41, 59
 problem, **8**
Bochner lemma, 135

- CAR process, **203, 257**
- Cauchy–Schwarz inequality, 284, 285
- Central limit theorem, 294
- Characteristic function, 50, 105, 114, 115
- Chi-square test, *see* Test of chi-square
- Complete statistics, 16
- Conditional, 12, 293
 - density, 9, 12, 31, 33
 - estimator, 31
 - distribution, 9, 11, 117
 - prediction, **122**
 - distribution function
 - prediction, **124, 129**
 - expectation, 10, 11, 15, 33, 44, 287
 - in function spaces, 256, 288
 - location parameter, 30
 - median, 30
 - mode, 30
 - occupation density, 225
 - probability, 7, 288
 - variance, 49
- Conditional independent, 12, 293
- Contiguity property, 109
- Continuous time random process, 289
- Convolution, 63, 134, 164, 209
- Correlation, *see also* Autocorrelation, 283
- Countable additivity, 281
- Counting
 - measure, 70, 223, 285
 - extended, 36
 - process, 8, 290
- Coupling lemma, 141, 297
- Covariance, *see also* Autocovariance
 - empirical function, 275, 276
 - empirical operator, 264
 - function, 275
 - matrix, 283
 - operator, 230, 264, 287
 - eigenelement, 277
 - estimation, **71, 264**
- Cramer’s conditions, 219, 297
- Cramer–Rao
 - bound, 24, 27, 39
 - inequality, 22
 - for unbiased predictors, 20, 22
 - multidimensional for
 - predictors, 34
- Cramer–von Mises distribution, 105
- Cramer–Wold device, 86, 219
- Cross-autocovariance operator
 - estimation, **268**
- Cross-covariance
 - matrix, 283
 - operator, 230, 256, 264, 287
- Davydov inequality, 296
 - Hilbertian version, 118
- Decomposition
 - canonical, 283
 - of the QPE, 10
 - spectral, 71, 266
 - Wold, 290
 - in Hilbert spaces, **235**
 - of a linear process, 291
- Density
 - derivatives estimation, **78**
 - empirical, 95, **209**
 - estimation
 - by kernel, **138, 163, 189**
 - by projection, **64, 78, 95**
 - estimator
 - by histogram, 97, 133
 - by kernel, 134, 164, 190
 - by projection, 97
 - by wavelet, 95
 - empirical, **212**
 - occupation, **209, 213, 216, 224**
 - unbiased estimator, 213, 224
- Diffusion process, 52, **170, 173, 224, 292**
 - multidimensional, **173, 175, 177, 179**
 - prediction, **52**
- Distribution, 282
 - estimation
 - in the discrete case, **70**
 - function, **71**
 - of a stochastic process, 289
- Dominance, 232, 259
- Dual, 284
- Efficiency
 - of Bahadur, **110**
 - of functional tests, **107**

Efficient

- estimator, 33, 52, 170, 220
 - unbiased, 23, 24, 28
- for predicting, 22, 27
- predictor, 21, 53
 - multidimensional, 36, 38
 - unbiased, 22–8

Eigen-

- element, 258, 264
- empirical, 267
- estimation of, **267**
- of covariance operator, 277
- of empirical covariance, 278
- value, 115, 116, 257
- vector, 71, 257, 268

Empirical

- autocovariance operator, 272
 - covariance function, 275, 276
 - covariance operator, 264
 - density, 95, **209**
 - density estimator, **212**
 - approximation, **220**
 - consistency, **214**
 - invariance, **213**
 - rate, 216
 - recursivity, **213**
 - eigenelement, 267
 - consistency, 268
 - error, 158
 - estimator, 49, 115
 - Fourier coefficient, 81
 - mean, 32, 214, **262**
 - mean operator, 272
 - measure, 134, 192, 209
 - in continuous time, 164, 213
 - moment, 27
 - operator, 261, 278
 - prediction error, 33
 - predictor, **28**
- Equivalent measures, 281
- risks, **54**
- Estimation problem, 10
- Expectation (or mean), 283
- Extended exponential model, 25

Factorization theorem, 12, 14

Filtering, 7

Filtration, 291

Forecasting, *see also* Prediction, 7

- a continuous time process, **9**
- a discrete time process, **8**
- climate variations, 279
- horizon, 9, 41
- ozone, 279

Foresight, 7

Functional

- autoregressive process
 - implementation, **279**
- law of the iterated logarithm, 220
- linear process, **229, 261**
- parameter, **63, 80, 86, 88, 92, 88, 92, 95,**
 - 149, 198
- random variable, 287
- test, **97, 113**
 - efficiency, **107**
 - for stochastic processes, **115**

Gaussian process, **13, 57, 112 169,**

- 171, 190, 203, 210, 257,
- 289

Glivenko–Cantelli theorem, 123

GSM, *see* Mixing processGuilbart space(s), **121, 122, 129**

Hilbert space, 284

Hilbert–Schmidt

- norm, 286
- operator, 71, 264, 267, 271,
- 286

Histogram, 97, 133, 134

Hoeffding inequality, 297

Implementation

- of adaptive functional
 - estimators, **92**
- of functional autoregressive
 - processes, **279**

Independent increments, 290

Innovation

- of H -valued process, 235
- of ARS process, 265
- of ARB process, 256
- of ARC process, 272
- of ARH process, 245, 252
- of CAR process, 258
- of MAH process, 242

- Innovation (*continued*)
 of sequence of real linear processes, 239
 process, 247, 291
- Ito integral, 14, 257, 292
- Kernel, 111, 117, 124, 277, 278
 estimator
 conditions \mathcal{K} , 135, 149, 164, 181, 190
 density, 134, 135, 164, 190, 220
 Epanechnikov, 135
 naive, 133, 135
 normal, 135, 148, 154, 202
 regression, 148, 181, 198
- finite-dimensional, 119
- inference by, **129**
 L^2 -consistency, 136
 density estimation, **138, 163, 190**
 from sampled data, **189**
 in continuous time, **163**
 in discrete time, **133**
 multivariate, 135
 prediction, **157, 186, 201**
 regression estimation, **148, 181, 198**
- of order k , 98, 107–111, 115
- predictor, 158, 186, 201
- reproducing, 97, 121
- Law of large numbers (strong), 294
- Lebesgue Nikodym theorem, 282
- Legendre polynomials, 88, 92, 120
- Lehmann–Scheffé theorem, *see also*
 Scheffé lemma, 16, 19
 for convex loss functions, 29
- Likelihood, *see also* Maximum likelihood estimator, 14, 26, 28, 37
- Linearly closed (sub)space, 230, 235, 249, 259
 projection over, **230**
- interpolation, 128, 276
- isometry, 292
- operator, 87, 264, 277, 286
 bounded, 255, 265, 277
 continuous, 118, 230, 232, 288
 prediction, 230
 predictor, **32, 56**
 process, 78, **227, 289, 291**
 functional, **229, 262**
 in Hilbert space, **235, 262**
 in the wide sense, 238
 invertible, 262
 prediction, **261**
 standard, 259
 Wold decomposition, 291
- regression, 33
 statistical predictor
- asymptotically optimal, 57, 58
- Lipschitz
 condition, 125
 process, 112
- Local time, *see also* Occupation density, **207, 225**
- Location parameter, 30
 conditional, 30
- Long memory process, 67
- Loss function, **29, 31**
- MA(g) process, 289
- MAH(1) process
 estimation, **279**
 in the wide sense, 241
- MAH(g) process, **239**
 connection with MA(q), 240
 in the wide sense, 239
- Markov process, 20, 54, 258, 293
 examples of, 14
 in the wide sense, 249, 293
 characterization, 249
 connection with ARH, 250
 prediction for, **117, 157, 186**
- Martingale, 32, 265, 290
 difference, 265, 269, 290
 semi-, **19**
- Maximum likelihood estimator,
 45
 ARIMA process, 33
 diffusion process, 52
 OU process, 50
 Poisson process, 37
 signal with Gaussian noise, 24
- Measurable space, 281
- Measure, 281
 absolutely continuous, 281
 product, 282
- Median (conditional), 31
- Mercer lemma, 277

Minimax

- asymptotically, 41, 43
- constant for density, 224
- estimator, 71
- rate, 46, 65, 77
 - for covariance operator, 72
 - for density, 167, 176, 224
 - for discrete distribution, 71
 - for distribution function, 71

Mixing

- coefficient, 55, 79, 115, 118, 295, 296
- condition, 93, 138, 153, 182, 199
- process
 - geometrically strongly, 47, 80, 82, 142, 150, 177, 186, 196, 201, 218, 271, 295
 - strongly, 295

Mode (conditional), 30

Moderate variation, 72

Multidimensional

- Berry–Esseen inequality, 98
- Cramer–Rao inequality, 34
- diffusion, **173**, 175, 177
- prediction, **33**, 39

Nondeterministic process, 56–9, 291

Nonparametric

- conditional density estimator, 31
- estimator
 - for density, 95, 97, 133, 163, 164, 212
 - for regression, 134, 158, 186
- framework, 30
- predictor, **117**, 158, 186, 201
- rate, **72**
- test of fit, 97

Occupation

- density, 209, 213, 216, 224
 - conditional, 225
 - criteria for existence, 210
- measure, 209

Operator, *see also* Linear operator

- adjoint, 286
- compact, 252, 286
- nilpotent, 247
- nuclear, 286
- of Hilbert–Schmidt, 286
- on Hilbert space, 286
- symmetric, 252, 286

Optimal, *see* Predictor, Rate of convergenceOracle, **66**

- bias of, 74
- rate of, 72, 78

Ornstein–Uhlenbeck process, **14**, **21**, **26**,

- 49**, 57, 172, **201**, **212**, 218, **246**, **256**, 292, 293

noncentered, **27**truncated, **241**

Orthogonal, 108, 281

- increments, 59
- projection method, 63
- projector, 10, 220, 270, 284, 287, 288
 - of LCS, 231, 234, 235
- strongly or weakly, 231, 234

Orthonormal (basis), 284

Parametric

- framework, 41
- prediction, **43**
- rate, **68**, 77, 88, **217**, 221

Pinelis–Sakhanenko inequality, 106, 263

Plan, 7

Point process, 9, 290

Poisson process, **14**, **17**, **19**, 20, **26**, **28**, **53**, 290sequence of, **36**Prediction, *see also* Statistical prediction, 8

- asymptotic, **41**
- by functional autoregressive processes, 279
- by kernel method, **157**, **186**, 201
- by projection method, **117**
- error, 118, 160
 - empirical, 33
 - linear, 58
 - probabilistic, 11
 - quadratic, 10, 17–9, 24, 28, 49, 54, 59, 118
 - quadratic integrated, 54
 - statistical, 11, 32
 - structural, 120
- for large time lags, **58**, 59
- for small time lags, **56**
- interval, 9, 51, 117, 125, **128**
- linear, 230
- mixed, 10, 11

- Prediction (*continued*)
 model, **9**, 38
 multidimensional, **33**, 39
 of a functional linear process, **261**
 of a Markov process, **117**, **157**, 186
 of ARC, **272**
 of ARH(1) process, **269**, **279**
 of autocorrelation operator, 277
 of conditional distribution, **122**
 function, **124**, 129
 of curves, **9**
 of diffusion process, **52**
 of electrocardiograms, 279
 of eurodollar rates, 279
 of functionals, **9**
 of traffic, 279
 parametric, **43**
 pure, 10
 statistical theory, **7**
- Predictor, *see also* Statistical predictor, 110
 B-valued, 34, 35
 Bayesian, **31**
 best, 287
 linear, 57, 261, 263
 probabilistic, 43, 269
 unbiased, 18–21, 27, 32, 34
 consistency, 123
 efficient, **21**, 36, 38, 53
 unbiased, 23–7
 empirical, **28**
 linear, **32**
 suboptimal, 56
 naive, 28, 32, 58, 59
 nonparametric, **117**, 157, 186, 201
 optimal, **15**, 39
 unbiased, 16, 17, 39
 plug-in, 41, 43, 44
 unbiased, 15, 16, 20, 21, 28, 32, 37
- Preference relation, 10, 29, 33
- Probabilistic
 best predictor, 43, 269
 prediction error, 11
- Probability, 281
 space, 281
- Process, *see* Stochastic process
- Projection
 adaptive, **63**, **81**, 92
 estimator, 63, 93
 estimator, 76, 97, 98, 113, 214, 220, 224
 inference by, **61**
 nonparametric test, **97**
 over LCS, 230
 prediction, **117**
- Pseudo-autoregressive representation, 269
- P -sufficient, **11**, 16, 29, 34, 36, 39
- Quadratic
 mean, 294
 prediction error, 10, 18, 19, 24, 28, 49,
 53, 59, 118
 integrated, 54
 variation, 224
- Radon Nikodym theorem, 281
- Random
 d -dimensional vector, 283
 correlation coefficient, 283
 correlation matrix, 283
 measure, 123
 process, *see* Stochastic process
 variable(s), 7, 282
 B-valued, 254, 258, 288
 C-valued, 273
 H-valued, 230, 231, 246, 286, 288
 H^∞ -valued, 263
 ℓ^2 -valued, 238, 254
 S-valued, 264
 expectation, mean or variance, 283
 functional, 287
 independent, 282
 integrable, 283
 positive, real or simple, 282
- Rao–Blackwell theorem, 15
 for convex loss functions, 28
- Rate of convergence
 of oracle, 78
 bandwidth-free, **164**, 170, 172, 175
 exponential, 107
 intermediate, **170**
 minimax, 46, 65, 77
 covariance operator, 72
 density, 167, 176, 224
 distribution function, 71
 nonparametric, **72**
 optimal, 79, **164**
 parametric, **68**, 77, 88, **167**, **217**, 221

- quasi-optimal, 143, 153
- superoptimal, **85, 164, 167**, 195, 209, 222
- Regression
 - affine, 11
 - estimation, **88**, 95, 134, **148, 181, 198, 225**
 - estimator
 - empirical, 225
 - kernel, 30, 158, 186, 206
 - regressogram, 134
 - linear, 33
 - parameter, 149, 181
 - polynomial, 11
 - test for, **113**
- Regressogram, 134
- Regular
 - process, 235, 291
 - ARH, 250
 - LPH, 236
 - MAH, 239
 - sequence of compact sets, 153, 184, 200
 - version, 288
- Reproducing kernel, 97, 121
 - Hilbert space, 121
- Riesz theorem, 284
- Sampled data, **223, 276**
 - inference for, **189**
- Sampling scheme
 - adequate, **193**
 - with fixed design, 195
 - with high rate, **190**, 198
- Sazonov inequality, 98, 114
- Scenario, 7
- Scheffé lemma, 216
- Separable, 281
- σ -algebra (or σ -field), 281
 - conditionally independent, 29
 - observed or non-observed, 7, 10
 - of invariant sets, 215
- σ -finite measure, 281
 - extended counting measure, 36
- Signal with Gaussian noise, **20**, 54
- Simple exponential smoothing, 32
- Simulations, 101, 116, **201**, 279
- Singular part of a process, 291
- Spectral
 - decomposition, 71, 266
 - density, 56, **64**, 78, **87**, 92
- Statistical
 - model, 9, 12
 - prediction, 7, 38
 - by kernel, **157, 160, 186**, 201
 - by plug-in, 43
 - by projection, **117**
 - empirical method, **271**
 - error, 11, 32
 - theory, **5**
 - predictor, 10, 30, 44, 48, 120, 261, 271, 278
 - asymptotically optimal, 57
- Stochastically independent, 282
 - convergence, 294
 - integral, 291
 - process, 289
 - (weakly) stationary, 290
 - distribution of, 289
 - ergodic, 214
 - in discrete or continuous time, 289
 - m -dependent, 297
 - nondeterministic, 56–9, 291
 - sample path, 289
 - singular part, 291
 - strictly stationary, 291
 - with independent increments, 20, 53
 - with long memory, 67
 - with stationary independent increments, 255, 289
 - WSH, 235
- Strong law of large numbers, 294
- Sufficient, 12–4
 - for predicting, *see* P-sufficient
- Test
 - asymptotic power, 109, 112, 115
 - Neyman–Pearson, 110
 - Bahadur efficiency, **110**
 - based on linear estimators, **101**
 - based on the uniform norm, **111**
 - consistency, **105**, 115
 - critical region, 100, 113
 - efficiency, **107**
 - for absence of mixture, **100**
 - for regression, **112**
 - for stochastic processes, **115**
 - functional, **95**, 107, 112

Test (*continued*)

- of chi-square, 97, 101, 114–6
- of Neyman–Pearson, 107, 109

- of noninfluence, 113

- statistic, 97, 114

Unbiased

- density estimator, 213, 234

- estimator, 15, 18–20, 23, 24, 27, 47, 65, 66, 68

- predictor, 15–21, 23–28, 32–4, 38, 39

- Validation, 33

- Variance, 283

Wavelet

- method, 279

- thresholding, 95

- Weak convergence, 294

- White noise, 58, 78, 238, 241, 289

- in ℓ^2 , 254

- in C spaces, 277

- in S spaces, 265

- in Hilbert spaces, 236, 237, 239, 241, 244, 246, 252, 262

- strong, 16, 33, 49, 87, 289

- in Banach spaces, 255

- in Hilbert spaces, 237, 248, 265, 277, 279

- Wiener process, 190, 289, 290

- bilateral, 242, 255, 257, 292

- standard, 24, 170, 289

- standard d-dimensional, 173

- standard bilateral, 14, 28, 246

- Wold decomposition, 290

- in Hilbert spaces, 235

- of a linear process, 291

- Wong process, 57, 206