# 1

# Introduction

This book is about robust control design. To truly understand robust control design, we first need to understand basic concepts of systems and control. We will introduce systems and control theory in this chapter. We will also give an overview of the book.

## 1.1 SYSTEMS AND CONTROL

A Google search in August 2006 found more than 5 billion entries for the word 'system'. So what is a system? There are many definitions, depending on the areas of application or interest. For example, according to The Free Dictionary by Farlax (http://www.thefreedictionary.com/system), a system is:

1. A group of interacting, interrelated, or interdependent elements forming a complex whole.
2. A functionally related group of elements, especially:
   (a) the human body regarded as a functional physiological unit.
   (b) an organism as a whole, especially with regard to its vital processes or functions.
   (c) a group of physiologically or anatomically complementary organs or parts: the nervous system; the skeletal system.

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(d) a group of interacting mechanical or electrical components.
(e) a network of structures and channels, as for communication, travel, or distribution.
(f) a network of related computer software, hardware, and data transmission devices.

3. An organized set of interrelated ideas or principles.
4. A social, economic, or political organizational form.
5. A naturally occurring group of objects or phenomena: the solar system.
6. A set of objects or phenomena grouped together for classification or analysis.
7. A condition of harmonious, orderly interaction.
8. An organized and coordinated method; a procedure.
9. The prevailing social order; the establishment.

All the above definitions are appropriate for some applications. However, in this book, we define a system as an assemblage of objects, real or abstract, that has some inputs and some outputs (Figure 1.1).

There are many examples of systems: an automobile whose input is the position of the gas pedal and whose output is the speed, a bank account whose input is the fund deposited and whose output is the interest generated, a traffic light whose input is the command indicated by green, yellow, or red lights and whose output is the traffic flow, and a dryer whose input is different dry circles and whose output is dry cloth.

To better understand systems, we shall classify them into different types. We will not classify systems according to their physical appearance, but rather according to their mathematical properties. Mathematically, we can view a system as a mapping $S: U \to Y$ from its input $u$ to its output $y = S(u)$.

The first classification is whether a system is linear or nonlinear. A system is linear if its input–output relation is linear; that is, for all inputs $u_1$ and $u_2$

$$y_1 = S(u_1) \wedge y_2 = S(u_2) \Rightarrow \alpha_1 y_1 + \alpha_2 y_2 = S(\alpha_1 u_1 + \alpha_2 u_2) \tag{1.1}$$

where $\alpha_1$ and $\alpha_2$ are any constants, $\wedge$ means 'and', and $\Rightarrow$ means 'implies'. Equation (1.1) says that if $y_1$ is the output when the input is $u_1$ and $y_2$ is the output when the input is $u_2$, then $\alpha_1 y_1 + \alpha_2 y_2$ is the output when the input is $\alpha_1 u_1 + \alpha_2 u_2$. If there exist some inputs $u_1$ and $u_2$ such that Equation (1.1) is not satisfied, then the system is nonlinear.

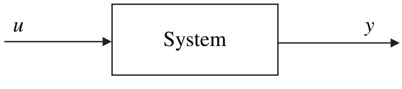


**Figure 1.1** A system with input $u$ and output $y$.

Let us consider the system of a bank account. If the interest rate is fixed at 3%, then the system is linear because the interest generated by the account is proportional to the balance of the account: \$100 will generate \$3, \$1000 000 will generate \$30 000, etc. However, in order to attract large deposits, a bank may use progressive interest rates. For example, the first \$10 000 of the balance earns an interest rate of 2% and the rest earns an interest rate of 4%. The account of this type is nonlinear because the interest generated by the account is not proportional to the balance of the account: \$100 will generate \$2 and \$1000 000 will generate $\$10\,000 \times 0.02 + 990\,000 \times 0.04 = \$39\,800$.

The second classification of systems is whether a system is time-invariant or time-varying. A system is time-invariant if its input–output relation does not change over time; that is, for any input $u$ applied at different times,

$$y(t) = S(u(t)) \Rightarrow y(t+T) = S(u(t+T)) \tag{1.2}$$

where $T$ is any constant time delay. If there exist some input $u$ and some constant $T$ such that Equation (1.2) is not satisfied, then the system is time-varying.

Consider again the system of a bank account. The system is time-invariant if the interest rate does not change over time. It is time-varying if the interest rate changes over time, which is most common in our daily experience.

The third classification of systems is whether a system has single input and single output (SISO) or multiple inputs and multiple outputs (MIMO). This classification requires no further explanation.

The last classification of systems is whether a system is a continuous-time or a discrete-time system. A system is a continuous-time system if its input and output are functions of a continuous time variable. All physical systems are continuous-time systems. However, nowadays, many physical systems are controlled by computers rather than by analogue devices. For computer control, input and output signals must be sampled. After a continuous-time signal $x(t)$ is sampled, it becomes a discrete-time signal $x(t_k)$, where $t_k$ is the $k$th sampling time. In this book, we will study only continuous-time systems.

Our goal is to control a system to achieve some objectives. Generally speaking, the control objectives can be classified to ensure either stability or optimality, or both of a system. Stability means that the system will not 'blow up'; that is, the output of the system will not become unbounded as long as its input is bounded. This is a basic requirement of most systems that we encounter. Optimality means that the system performance will be optimal in some sense. For example, we may want an automobile to consume the least fuel; or we may want a bank account to generate most interest. In this book, we will discuss stability in Chapter 3 and optimality in Chapter 4.

To achieve stability or optimality, some control needs to be used. Generally speaking, two types of control can be used: (1) feedback or closed-loop control; and (2) open-loop control.

In feedback control, the controller knows the output of the system and uses this information in its control. A feedback control system is shown in Figure 1.2. Most control systems we see in our daily life are feedback control systems. For example, most control systems in an automobile, such as engine control, throttle control, cruise control, and power train control are feedback controls. So are temperature controls in modern houses or controls in ovens and refrigerators.

In open-loop control, the controller does not know the output of the system, as shown in Figure 1.3. Open-loop control is used if it is hard or meaningless to measure the output. There are not many, but some examples of open-loop control in existence. Most traffic controls are open-loop control because the controllers do not know the traffic flow that is being controlled. In most cases, washers and dryers are open-loop controlled, because it is hard to measure the cleanness or dryness of cloth.

Needless to say, feedback control has many advantages over open-loop control. Many unstable systems can be stabilized by feedback controls but cannot be stabilized by open-loop control. Feedback can often handle disturbance much better than open-loop control. Optimization can also be achieved using feedback. Since open-loop control is relatively easy to design and less frequently used in practice, almost all controls addressed in the control theory are feedback control. Most methods developed in control theory are for feedback control. This is also true in this book. We will investigate feedback control systems in this book.

To control a system, we first need to obtain a mathematical model of the system. In the development of the control theory, two main modelling frameworks have been proposed. One uses transfer functions and the other

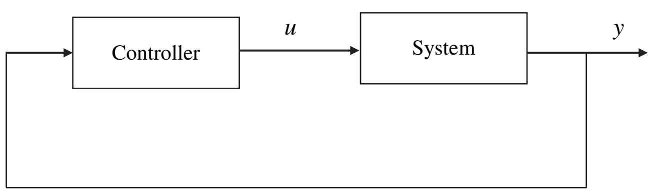


**Figure 1.2** A feedback control system.

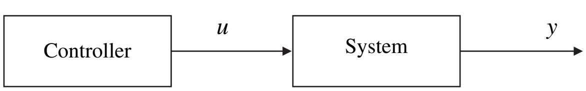


**Figure 1.3** An open-loop control system.

uses state space representations. The methods developed using transfer functions are sometimes called 'classical control'. The methods developed using state space representations are sometimes called 'modern control'. In this book, we will mainly use state space representations to model systems.

Our focus is on robust control design. Robust control is related to modelling and model uncertainties. No matter how hard we try, no model is completely accurate. Every model has errors or uncertainties. If a control will work under uncertainties, we say that the control is robust. Robust control design tries to design a control that has good tolerance to modelling errors. There are several approaches available for robust control and robust control design. In this book, we will present two popular approaches: the parametric approach and the $H_\infty/H_2$ approach. More importantly, we will present a new approach to robust control design. This new approach is 'indirect' in the following sense: it translates a robust control problem into an optimal control problem. Since we know how to solve a large class of optimal control problems, this optimal control approach allows us to solve some robust control problems that cannot be easily solved otherwise. Furthermore, this approach is easy to understand and easy to apply to practical problems.

To build the foundation for the optimal control approach, we will first present the fundamentals of control theory, stability theory, and optimal control.

## 1.2 MODERN CONTROL THEORY

We will start this book with a comprehensive review of modern control theory in Chapter 2. We will use general state space models to describe systems:

$$\dot{x} = f(x, u, t)$$
$$y = g(x, u, t)$$

where $f: R^n \times R^m \times R \to R^n$ and $g: R^n \times R^m \times R \to R^p$ are nonlinear functions. $\dot{x} = f(x, u, t)$ are state equations and $y = g(x, u, t)$ are output equations. Derivation of these equations is illustrated in Appendix A, where we model various electrical, mechanical and other practical systems.

Chapter 2 will focus on a linear time-invariant system of the form

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

We will study its responses and their properties. We will also study its transfer function. We note that for a given system, its state space representation or realization is not unique. Different representations are related by some similarity transformations. Some representations are more useful in control, including the Jordan canonical form, controllable canonical form, and observable canonical form.

Controllable and observable canonical forms are related to two important properties of systems: controllability and observability. Intuitively, a system is called controllable if all its states can be controlled in the sense that they can be driven to anywhere using some input, and a system is called observable if all its states can be observed in the sense that their values can be determined from the output. For linear time-invariant systems, these two properties can be easily checked by checking the ranks of some controllability or observability matrices.

The importance of controllability is due to the fact that if a system is controllable, then we can move or place its poles or eigenvalues in arbitrary places in the complex plane by using state feedback. We show how this can be done in three steps. First, we show how to design a state feedback for a system in controllable canonical form. We then show how to do this for general single-input systems. Finally, we show how to design a state feedback for a multi-input system.

Using state feedback requires that all state variables are available for control. This in turn requires that there are sensors to measure all state variables. This requirement is sometimes impractical and most times too expensive to satisfy. Furthermore, this requirement is also unnecessary because even if the state variables are not directly measurable, they can be estimated from the output of the system, if the system is observable. Such estimation is achieved by an observer. An observer is a linear time-invariant system whose inputs are the input and output of the system to be observed, and whose output is the estimate of the state variables. The performance of the observer is determined by its poles, which can be placed arbitrarily if the system is observable. The nice thing about feedback control is that the use of the observer does not change the poles determined by the state feedback. This separation principle allows us to design state feedback and an observer separately.

## 1.3 STABILITY

In Chapter 3, we will review the basic theory of stability. Intuitively, stability means that, without inputs, a system's response will converge to some equilibrium. Consider a general nonlinear system

$$\dot{x} = A(x)$$

where $x \in R^n$ are the state variables and $A : R^n \to R^n$ is a (nonlinear) function. Assume $A(0) = 0$, the equilibrium point $x_0 = 0$ is asymptotically stable if there exists a neighbourhood of $x_0 = 0$ such that if the system starts in the neighbourhood then its trajectory converges to the equilibrium point $x_0 = 0$ as $t \to \infty$.

Determining stability of a system is not easy if the system is nonlinear. One approach often used is the Lyapunov approach, which can be explained as follows: given a system, let us define some suitable 'energy' function of the system. This function must have the property that it is zero at the origin and positive elsewhere. Assume further that the system dynamics are such that the energy of the system is monotonically decreasing with time and hence eventually reduces to zero. Then the trajectories of the system have no other place to go but the origin. Therefore, the system is asymptotically stable. This generalized energy function is called a Lyapunov function. The Lyapunov approach will be used in deriving the results on our optimal control approach to robust control design.

On the other hand, for a linear time-invariant system

$$\dot{x} = Ax$$

its stability is determined by its characteristic polynomial

$$\varphi(s) = |sI - A| = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

and its corresponding roots, which are eigenvalues or poles. A linear time-invariant system is asymptotically stable if and only if all the roots of its characteristic polynomial are in the open left half of the $s$-plane.

If the numerical values of matrix $A$ are known, then we can always find the numerical values of its eigenvalues and hence determine the stability of the system. However, if symbolic values are used or, for any other reasons, we do not want to calculate the eigenvalues explicitly, then two other criterions can be used to determine the stability of a system.

The Routh–Hurwitz criterion is a method to determine the locations of roots of a polynomial $\varphi(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$ with constant real coefficients with respect to the left half of the $s$-plane without actually solving for the roots. It involves first constructing a Routh table and then checking the number of sign changes of the elements of the first column of the table, which is equal to the number of roots outside the open left half of the complex plane.

The second criterion is the Nyquist criterion. Unlike the Routh–Hurwitz criterion, the Nyquist criterion is a frequency domain method based on the frequency response of a linear time-invariant system. To use the Nyquist criterion to check the stability of a system with the characteristic equation given by $1 + G(s)H(s) = 0$, we first construct a Nyquist plot of $G(s)H(s)$.

For the system to be stable, the Nyquist plot of $G(s)H(s)$ must encircle the $(-1, j0)$ point as many times as the number of poles of $G(s)H(s)$ that are in the right half of the $s$-plane.

Chapter 3 also discusses two other properties of a linear time-invariant system: stabilizability and detectability. A system is stabilizable if all unstable eigenvalues are controllable. Obviously, stabilizability is weaker than controllability. It is the weakest condition that allows us to stabilize a system using feedback. Dually, a system is detectable if all unstable eigenvalues are observable.

## 1.4 OPTIMAL CONTROL

After we stabilize a system, the next thing we want to do is to optimize the system performance. Optimal control will be discussed in Chapter 3. This topic is not only important in its own right, but also serves as the basis of our optimal control approach to robust control design.

We formulate an optimal control problem for a general nonlinear system

$$\dot{x} = f(x, u)$$

so as to minimize the following cost functional

$$J(x, t) = \int_t^{t_f} L(x, u)\mathrm{d}\tau$$

where $t$ is the current time, $t_f$ is the terminating time, $x = x(t)$ is the current state, and $L(x, u)$ characterizes the cost objective.

We will derive the solution to the optimal control problem from the principle of optimality, which states that if a control is optimal from some initial state, then it must satisfy the following property: after any initial period, the control for the remaining period must also be optimal with regard to the state resulting from the control of the initial period. Applying the principle of optimality to the optimal control problem, we can derive the Hamilton–Jacobi–Bellman equation that must be satisfied by any solution to the optimal control problem.

It is not always easy to solve the Hamilton–Jacobi–Bellman equation, especially for nonlinear systems. However, if the system is linear and the cost function is quadratic with infinite horizon; that is

$$\dot{x} = Ax + Bu$$

$$J(x, t) = \int_t^{\infty} (x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru)\mathrm{d}\tau$$

then the Hamilton–Jacobi–Bellman equation is reduced to the following algebraic Riccati equation

$$SA + A^{\mathrm{T}}S + Q - SBR^{-1}B^{\mathrm{T}}S = 0$$

Solving the above equation for $S$, we can obtain the solution to the optimal control problem as

$$u^* = -R^{-1}B^{\mathrm{T}}Sx$$

The above optimal control problem is also called a linear quadratic regulator (LQR) problem.

The problem dual to the optimal control problem is to design an optimal observer, more commonly known as the Kalman or Kalman–Bucy filter. Deriving results on the Kalman filter often requires knowledge and background on stochastic processes. However, we will provide a new method to derive the Kalman filter in Chapter 4 without using results on stochastic processes.

## 1.5 OPTIMAL CONTROL APPROACH

The main focus of this book is of course on the optimal control approach to robust control design. We will discuss this approach starting in Chapter 5, where we present the optimal control approach for linear systems. The system to be controlled is described by

$$\dot{x} = A(p)x + Bu$$

where $p$ represents uncertainty. The goal is to design a state feedback to stabilize the system for all possible $p$ within given bounds. The solution to this robust problem depends on whether the uncertainty satisfies a matching condition, which requires that the uncertainty is within the range of $B$.

If the uncertainty satisfies the matching condition, then the solution to the robust control problem always exists and can be obtained easily by solving an LQR problem. The LQR problem is obtained by including the bounds on the uncertainty in the cost functional. The proof that the solution to the LQR problem is a solution to the robust control problem is based on the properties of the optimal control, as described by the Hamilton–Jacobi–Bellman equation. Furthermore, if the matching condition is satisfied, we can also solve a robust pole placement problem by placing the poles of the controlled system to the left of $-\gamma$, where $\gamma$ is some arbitrary positive real number, as long as the uncertainty is within the bounds.

If the uncertainty does not satisfy the matching condition, then the problem is much more complex. We first need to decompose the uncertainty into the matched part and the unmatched part. We will use an augmented control to deal with the unmatched uncertainty. Robust control may or may not be possible, depending on whether a sufficient condition is satisfied. This conclusion is in sync with the results obtained by other researchers in the field.

Chapter 5 also discusses how to handle uncertainty in the input matrix; that is, the uncertain system has the form

$$\dot{x} = A(p)x + BD(p)u$$

Method for this case is similar but the derivation is more complex.

The optimal control approach to nonlinear systems will be presented in Chapter 6. The idea is similar to the idea for linear systems: we will translate a robust control problem into an optimal control problem. However, because the system is nonlinear, it is more difficult to solve the optimal control problem. Hence, some numerical solutions or other methods may need to be used, although this is outside the scope of this book.

As in the case for linear systems, the procedure for systems satisfying the matching condition is quite different from the procedure for systems not satisfying the matching condition. For systems satisfying the matching condition, the solution to the optimal control problem is guaranteed to be a solution to the robust control problem. Therefore, as long as we can find an analytic or numerical solution to the optimal control problem, we have a solution to the robust control problem. For systems not satisfying the matching condition, the solution to the optimal control problem is a solution to the robust control problem only if a certain sufficient condition is satisfied. If the unmatched part of the uncertainty is too large, the sufficient condition is unlikely to be satisfied. Again, this is not surprising in view of results obtained by other researchers.

## 1.6 KHARITONOV APPROACH

A book on robust control design would not be complete without presenting the parametric approach, sometimes called the Kharitonov approach. The Kharitonov approach is an excellent method for robust analysis of control systems. To some degree, it can also be used for robust control design. We will discuss the Kharitonov approach in Chapter 7.

The Kharitonov approach considers a system with the following characteristic polynomial

$$\varphi(s, p) = p_0 + p_1 s + \cdots + p_{n-1} s^{n-1} + p_n s^n$$

where $p_i \in [p_i^-, p_i^+]$, $i = 0, 1, \ldots, n$ are coefficients whose values are uncertain, but we know their lower and upper bounds. The Kharitonov theorem states that the stability of the following four polynomials is necessary and sufficient for the stability of all polynomials with the uncertainty within the bounds:

$$
\begin{aligned}
K_1(s) &= p_0^- + p_1^- s + p_2^+ s^2 + p_3^+ s^3 + p_4^- s^4 + p_5^- s^5 + \cdots \\
K_2(s) &= p_0^- + p_1^+ s + p_2^+ s^2 + p_3^- s^3 + p_4^- s^4 + p_5^+ s^5 + \cdots \\
K_3(s) &= p_0^+ + p_1^- s + p_2^- s^2 + p_3^+ s^3 + p_4^+ s^4 + p_5^- s^5 + \cdots \\
K_4(s) &= p_0^+ + p_1^+ s + p_2^- s^2 + p_3^- s^3 + p_4^+ s^4 + p_5^+ s^5 + \cdots
\end{aligned}
$$

To prove the Kharitonov theorem, we need a few preliminary results. These preliminary results will also be proven in Chapter 7.

To compare the optimal control approach with the Kharitonov approach, we note that the optimal control approach is inherently a design tool, in the sense that it will design a controller that can robustly stabilize the system; while the Kharitonov approach is inherently an analysis tool, in the sense that, given a (closed-loop) system, it will analyse and verify if the system is robustly stable.

## 1.7 $H_\infty$ AND $H_2$ CONTROL

It is not easy to summarize the $H_\infty/H_2$ control in one chapter, but that is what we will do in Chapter 8. We will start with the introduction of function spaces. In particular, $H_\infty$ denotes the Banach space of all complex valued functions $F: C \to C$ that are analytic and bounded in the open right half of the complex plane and are bounded on the imaginary axis $jR$ with its $H_\infty$ norm defined as

$$\|F\|_\infty = \sup_{\omega \in R} |F(j\omega)|$$

$H_2$ denotes the Hilbert space of all complex valued functions $F: C \to C$ that are analytic and bounded in the open right half of the complex plane and the following integral is bounded

$$\int_{-\infty}^{\infty} \overline{F(j\omega)} F(j\omega) d\omega < \infty$$

The $H_2$ norm can then be defined as

$$\|F\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{F(j\omega)} F(j\omega) \mathrm{d}\omega}$$

We will show how to calculate the $H_\infty$ and $H_2$ norms.

To discuss robustness under uncertainty, we will separate the uncertainty from the nominal system and put the uncertainty in the feedback loop. We will prove a small-gain theorem which states intuitively that the perturbed closed-loop system is stable if the $H_\infty$ norm of the loop is less than one. From the small-gain theorem, we can determine the bounds on the uncertainty that guarantee the stability of the perturbed system.

We will then show that $H_2/H_\infty$ control synthesis boils down to designing a controller for the nominal system such that its $H_\infty/H_2$ norm is minimized. Note that the $H_2/H_\infty$ approach is very different from the optimal control approach. In the optimal control approach, we start with the bounds on uncertainties. We then design a controller based on these bounds. As the result, if the controller exists, then it is guaranteed to robustly stabilize the perturbed system. On the other hand, in the $H_2/H_\infty$ approach, the bounds on uncertainties are not given in advance. The synthesis will try to achieve the largest tolerance range for the uncertainty. However, there is no guarantee that the range is large enough to cover all possible uncertainties. In other words, the $H_2/H_\infty$approach cannot guarantee the robustness of the resulting controller. The approach will do its best to make the resulting controller robust. Whether this best is good enough depends on the nature of the uncertainty.

## 1.8 APPLICATIONS

We will present three practical applications of the optimal control approach to robust control design. These applications will be presented in Chapters 9, 10, and 11.

The first application is robust active damping for stability enhancement of vibration systems. Many practical systems such as buildings, flexible structures, and vehicles, exhibit vibration. How to reduce (damp) vibration is an important control problem. We will be interested in active damping that uses external force to actively control the system to reduce the vibration. The system will be modelled as

$$M_0\ddot{x} + A_0 x = B_0 u + C_0 f_0(x, \dot{x})$$

where $M_0$ is the mass matrix, $A_0$ is the stiffness matrix, and $f_0(x, \dot{x})$ is the uncertainty. We will introduce a special inner product and the associated energy norm. The solution to the robust damping problem will be obtained by translating it into an optimal control problem. The control law will be obtained by solving an LQR problem.

The second application is robust control of robot manipulators. The dynamics of a robot manipulator is modelled as

$$M(q)\ddot{q} + V(q, \dot{q}) + U(\dot{q}) + W(q) = \tau$$

where $q$ is the generalized coordinate vector, $\tau$ is the generalized force vector, $M(q)$ is the inertia matrix, $V(q, \dot{q})$ is the Coriolis/centripetal vector, $W(q)$ is the gravity vector, and $U(\dot{q})$ is the friction vector. Based on this model, we will formulate the robust control problem when the load and other parameters are uncertain. The resulting robust control problem satisfies the matching condition. However, there is also uncertainty in the input matrix. We will use the method in Chapter 5 to solve the robust control problem. We will apply the control law obtained to a two-joint SCARA-type robot and simulate the controlled system.

The third and last application is the hovering control of a vertical/short takeoff and landing (V/STOL) aircraft. The aircraft state is simply the positions, $\tilde{x}, \tilde{y}$ of the aircraft centre of mass, the roll angle $\theta$ of the aircraft, and the corresponding velocities $\dot{\tilde{x}}, \dot{\tilde{y}}, \dot{\theta}$. The control inputs $U_t, U_m$ are, respectively, the thrust (directed out the bottom of the aircraft) and the rolling moment about the aircraft centre of mass. The dynamics of the aircraft can be written as

$$\begin{aligned} m\ddot{\tilde{x}} &= -U_t \sin\theta + \varepsilon_0 U_m \cos\theta \\ m\ddot{\tilde{y}} &= U_t \cos\theta + \varepsilon_0 U_m \sin\theta - mg \\ J\ddot{\theta} &= U_m \end{aligned}$$

where $\varepsilon_0 > 0$ is a coefficient describing the coupling between the rolling moment and the lateral force on the aircraft. We will design a robust control to take care of the coupling between the rolling moment and the lateral force on the aircraft. We will solve a nonlinear optimal control problem analytically to obtain a nonlinear robust control law.

## 1.9 USE OF THIS BOOK

By selecting different chapters, this book can be used in the following three courses.

Chapters 1–5 and Appendix A can be used for an undergraduate/graduate course on modern control theory. These parts cover the following topics:

1. Modelling and responses of systems (Appendix A and Chapter 2).

2. Properties of linear time-invariant systems (Chapters 2 and 3), including controllability, observability, stability, stabilizability, and detectability.
3. Control synthesis for linear time-invariant systems (Chapter 2): pole placement and observer design.
4. Introduction to optimal control and the Kalman filter (Chapter 4).
5. Introduction to robust control design (Chapter 5).

Chapters 5–8 can be used for a graduate level course on robust control design. Such a course will cover the following topics:

1. Optimal control approach to robust control design for linear systems (Chapter 5).
2. Optimal control approach to robust control design for nonlinear systems (Chapter 6).
3. Robust control of parametric systems using the Kharitonov theorem (Chapter 7)
4. $H_\infty$ and $H_2$ robust control design (Chapter 8).

Finally, Chapters 5–6 and Chapters 9–11 can be used for an application-orientated course on robust control design using the optimal control approach, which covers the following topics:

1. Optimal control approach to robust control design for linear systems (Chapter 5).
2. Optimal control approach to robust control design for nonlinear systems (Chapter 6).
3. Robust active damping for vibration systems (Chapter 9).
4. Robust control of robot manipulators (Chapter 10).
5. Hovering control of (V/STOL) aircraft (Chapter 11).