Chapter 5

EQUATIONS WITH TWO VARIABLES

Chapter Check-In

- ❑ Solving systems of equations
- ❑ Addition/Subtraction method
- ❑ Substitution method
- ❑ Graphing method

If you have two equations with the same two unknowns in each, you can solve for both unknowns.

Solving Systems of Equations (Simultaneous Equations)

There are three common methods for solving: addition/subtraction, substitution, and graphing.

Addition/subtraction method

To use the addition/subtraction method,

1. **Multiply one or both equations by some number to make the number in front of one of the letters (unknowns) the same in each equation.**
2. **Add or subtract the two equations to eliminate one letter.**
3. **Solve for the other unknown.**
4. **Insert the value of the first unknown in one of the original equations to solve for the second unknown.**

Example 1: Solve for *x* and *y*.

$$3x + 3y = 24$$
$$2x + y = 13$$

First multiply the bottom equation by 3. Now the *y* is preceded by a 3 in each equation.

$$3x + 3y = 24 \qquad\qquad 3x + 3y = 24$$
$$3(2x) + 3(y) = 3(13) \qquad\qquad 6x + 3y = 39$$

Now the equations can be subtracted, eliminating the *y* terms.

$$\begin{array}{r} 3x + 3y = 24 \\ -6x + -3y = -39 \\ \hline -3x = -15 \end{array}$$

$$\frac{-3x}{-3} = \frac{-15}{-3}$$

$$x = 5$$

Now insert $x = 5$ in one of the original equations to solve for *y*.

$$2x + y = 13$$
$$2(5) + y = 13$$

$$\begin{array}{r} 10 + y = 13 \\ -10 -10 \\ \hline y = 3 \end{array}$$

Answer: $x = 5$, $y = 3$

Of course, if the number in front of a letter is already the same in each equation, you do not have to change either equation. Simply add or subtract.

Example 2: Solve for *x* and *y.*

$$x + y = 7$$
$$x - y = 3$$

$$\begin{array}{r} x + y = 7 \\ \underline{x - y = 3} \\ 2x \quad = 10 \end{array}$$

$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

Now inserting 5 for *x* in the first equation gives

$$\begin{array}{r} 5 + y = \ \ 7 \\ \underline{-5 \qquad -5} \\ y = \ \ 2 \end{array}$$

Answer: $x = 5$, $y = 2$

You should note that this method will not work when the two equations are, in fact, the same.

Example 3: Solve for *a* and *b.*

$$3a + 4b = 2$$
$$6a + 8b = 4$$

The second equation is actually the first equation multiplied by 2. In this instance, the *system is unsolvable.*

Example 4: Solve for *p* and *q.*

$$3p + 4q = 9$$
$$2p + 2q = 6$$

Multiply the second equation by 2.

$$(2)2p + (2)2q = (2)6$$
$$4p + 4q = 12$$

Now subtract the equations.

$$\begin{array}{r} 3p + 4q = \ \ 9 \\ \underline{(-)\,4p + 4q = \ 12} \\ -p \qquad = -3 \\ p \qquad = \ \ 3 \end{array}$$

Now that you know $p = 3$, you may plug in 3 for p in either of the two original equations to find q.

$$3p + 4q = 9$$
$$3(3) + 4q = 9$$
$$9 + 4q = 9$$
$$4q = 0$$
$$q = 0$$

Answer: $p = 3$, $q = 0$

Substitution method

Sometimes a system is more easily solved by the *substitution method.* This method involves substituting one equation into another.

Example 5: Solve for x and y.

$$x = y + 8$$
$$x + 3y = 48$$

From the first equation, substitute $(y + 8)$ for x in the second equation.

$$(y + 8) + 3y = 48$$

Now solve for y. Simplify by combining y's.

$$\begin{aligned} 4y + 8 &= 48 \\ -8 \quad &\;\; -8 \\ \hline 4y \quad &= 40 \end{aligned}$$
$$\frac{4y}{4} = \frac{40}{4}$$
$$y = 10$$

Now insert $y = 10$ in one of the original equations.

$$x = y + 8$$
$$x = 10 + 8$$
$$x = 18$$

Answer: $y = 10$, $x = 18$

Graphing method

Another method of solving equations is by *graphing* each equation on a coordinate graph. The coordinates of the intersection will be the solution to the system. If you are unfamiliar with coordinate graphing, carefully review the chapter on coordinate geometry (see Chapter 9) before attempting this method.

Example 6: Solve the system by graphing

$$x = 4 + y$$

$$x - 3y = 4$$

First, find three values for x and y that satisfy each equation. (Although only two points are necessary to determine a straight line, finding a third point is a good way of checking.)

$x = 4 + y$

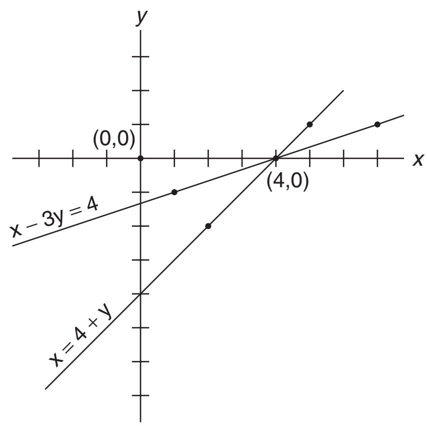
x	y
4	0
2	-2
5	1

$x - 3y = 4$

x	y
1	-1
4	0
7	1

Now graph the two lines on the coordinate plane, as shown in Figure 5-1.

Figure 5-1 A graph of lines $x = 4 + y$ and $x - 3y = 4$ indicating solution.



The point where the two lines cross (4, 0) is the solution of the system.

If the lines are parallel, they do not intersect, and therefore, there is *no solution to that system.*

Chapter Checkout

Q&A

1. Solve for x and y:

$8x + 2y = 7$

$3x - 4y = 5$

2. Solve for a and b:

$a = b + 1$

$a + 2b = 7$

Answers: 1. $x = 1,\ y = -\frac{1}{2}$ **2.** $a = 3,\ b = 2$