

Preface to Volume I

Financial risk management is a new quantitative discipline. Its development began during the 1970s, spurred on by the first Basel Accord, between the G10 countries, which covered the regulation of banking risk. Over the past 30 years banks have begun to understand the risks they take, and substantial progress has been made, particularly in the area of market risks. Here the availability of market data and the incentive to reduce regulatory capital charges through proper assessment of risks has provided a catalyst to the development of market risk management software. Nowadays this software is used not only by banks, but also by asset managers, hedge funds, insurance firms and corporate treasurers.

Understanding market risk is the first step towards managing market risk. Yet, despite the progress that has been made over the last 30 years, there is still a long way to go before even the major banks and other large financial institutions will really know their risks. At the time of writing there is a substantial barrier to progress in the profession, which is the refusal by many to acknowledge just how mathematical a subject risk management really is.

Asset management is an older discipline than financial risk management, yet it remains at a less advanced stage of quantitative development. Unfortunately the terms ‘equity analyst’, ‘bond analyst’ and more generally ‘financial analyst’ are something of a misnomer, since little analysis in the mathematical sense is required for these roles. I discovered this to my cost when I took a position as a ‘bond analyst’ after completing a postdoctoral fellowship in algebraic number theory.

One reason for the lack of rigorous quantitative analysis amongst asset managers is that, traditionally, managers were restricted to investing in cash equities or bonds, which are relatively simple to analyse compared with swaps, options and other derivatives. Also regulators have set few barriers to entry. Almost anyone can set up an asset management company or hedge fund, irrespective of their quantitative background, and risk-based capital requirements are not imposed. Instead the risks are borne by the investors, not the asset manager or hedge fund.

The duty of the fund manager is to be able to describe the risks to their investors accurately. Fund managers have been sued for not doing this properly. But a legal threat has less impact on good practice than the global regulatory rules that are imposed on banks, and this is why risk management in banking has developed faster than it has in asset management. Still, there is a very long way to go in both professions before a firm could claim that it has achieved ‘best practice’ in market risk assessment, despite the claims that are currently made.

At the time of writing there is a huge demand for properly qualified financial risk managers and asset managers, and this book represents the first step towards such qualification. With this book readers will master the basics of the mathematical subjects that lay the foundations

for financial risk management and asset management. Readers will fall into two categories. The first category contains those who have been working in the financial profession, during which time they will have gained some knowledge of markets and instruments. But they will not progress to risk management, except at a very superficial level, unless they understand the topics in this book. The second category contains those readers with a grounding in mathematics, such as a university degree in a quantitative discipline. Readers will be introduced to financial concepts through mathematical applications, so they will be able to identify which parts of mathematics are relevant to solving problems in finance, as well as learning the basics of financial analysis (in the mathematical sense) and how to apply their skills to particular problems in financial risk management and asset management.

AIMS AND SCOPE

This book is designed as a text for introductory university and professional courses in quantitative finance. The level should be accessible to anyone with a moderate understanding of mathematics at the high school level, and no prior knowledge of finance is necessary. For ease of exposition the emphasis is on understanding ideas rather than on mathematical rigour, although the latter has not been sacrificed as it is in some other introductory level texts. Illustrative examples are provided immediately after the introduction of each new concept in order to make the exposition accessible to a wide audience.

Some other books with similar titles are available. These tend to fall into one of two main categories:

- Those aimed at ‘quants’ whose job it is to price and hedge derivative products. These books, which include the collection by Paul Wilmott (2006, 2007), focus on continuous time finance, and on stochastic calculus and partial differential equations in particular. They are usually written at a higher mathematical level than the present text but have fewer numerical and empirical examples.
- Those which focus on discrete time mathematics, including statistics, linear algebra and linear regression. Among these books are Watsham and Parramore (1996) and Teall and Hasan (2002), which are written at a lower mathematical level and are less comprehensive than the present text.

Continuous time finance and discrete time finance are subjects that have evolved separately, even though they approach similar problems. As a result two different types of notation are used for the same object and the same model is expressed in two different ways. One of the features that makes this book so different from many others is that I focus on *both* continuous and discrete time finance, and explain how the two areas meet.

Although the four volumes of *Market Risk Analysis* are very much interlinked, each book is self-contained. This book could easily be adopted as a stand-alone course text in quantitative finance or quantitative risk management, leaving more advanced students to follow up cross references to later volumes only if they wish. The other volumes in *Market Risk Analysis* are:

Volume II: *Practical Financial Econometrics*

Volume III: *Pricing, Hedging and Trading Financial Instruments*

Volume IV: *Value at Risk Models*.

OUTLINE OF VOLUME I

This volume contains sufficient material for a two-semester course that focuses on basic mathematics for finance or financial risk management. Because finance is the study of the behaviour of agents operating in financial markets, it has a lot in common with economics. This is a so-called ‘soft science’ because it attempts to model the behaviour of human beings. Human behaviour is relatively unpredictable compared with repetitive physical phenomena. Hence the mathematical foundations of economic and econometric models, such as *utility theory* and *regression analysis*, form part of the essential mathematical toolkit for the financial analyst or market risk manager. Also, since the prices of liquid financial instruments are determined by demand and supply, they do not obey precise rules of behaviour with established analytic solutions. As a result we must often have recourse to *numerical methods* to resolve financial problems. Of course, to understand these subjects fully we must first introduce readers to the elementary concepts in the four core mathematics subjects of calculus, linear algebra, probability and statistics. Besides, these subjects have far-reaching applications to finance in their own right, as we shall see.

The introduction to Chapter 1, *Basic Calculus for Finance*, defines some fundamental financial terminology. Then the chapter describes the mathematics of graphs and equations, functions of one and of several variables, differentiation, optimization and integration. We use these concepts to define the return on a portfolio, in both discrete and continuous time, discrete and continuous compounding of the return on an investment, geometric Brownian motion and the ‘Greeks’ of an option. The last section focuses on Taylor expansion, since this is used so widely in continuous time finance and all three subsequent volumes of *Market Risk Analysis* will make extensive use of this technique. The examples given here are the delta–gamma–vega approximation to the change in an option price and the duration–convexity approximation to the change in a bond price, when their underlying risk factors change.

Chapter 2, *Essential Linear Algebra for Finance*, focuses on the applications of matrix algebra to modelling linear portfolios. Starting from the basic algebra of vectors, matrices, determinants and quadratic forms, we then focus on the properties of covariance and correlation matrices, and their eigenvectors and eigenvalues in particular, since these lay the foundations for principal component analysis (PCA). PCA is very widely used, mainly in discrete time finance, and particularly to orthogonalize and reduce the dimensions of the risk factor space for interest rate sensitive instruments and options portfolios. A case study in this chapter applies PCA to European equity indices, and several more case studies are given in subsequent volumes of *Market Risk Analysis*. A very good free downloadable Excel add-in has been used for these case studies and examples. Further details are given in the chapter.

Chapter 3, *Probability and Statistics*, covers the probabilistic and statistical models that we use to analyse the evolution of financial asset prices or interest rates. Starting from the basic concepts of a random variable, a probability distribution, quantiles and population and sample moments, we then provide a catalogue of probability distributions. We describe the theoretical properties of each distribution and give examples of practical applications to finance. Stable distributions and kernel estimates are also covered, because they have broad applications to financial risk management. The sections on statistical inference and maximum likelihood lay the foundations for Chapter 4. Finally, we focus on the continuous time and discrete time statistical models for the evolution of financial asset prices and returns, which are further developed in Volume III.

Much of the material in Volume II rests on the *Introduction to Linear Regression* given in Chapter 4. Here we start from the basic, simple linear model, showing how to estimate and draw inferences on the parameters, and explaining the standard diagnostic tests for a regression model. We explain how to detect autocorrelation and heteroscedasticity in the error process, and the causes and consequences of this. Then we use matrix notation to present the general multivariate linear regression model and show how to estimate such a model using both the Excel data analysis tools and the matrix operations in Excel. The chapter concludes with a long survey of applications of regression to finance and risk management, which includes many references to later volumes of *Market Risk Analysis* where the applied regression models are implemented and discussed in finer detail.

Chapter 5 covers *Numerical Methods in Finance*. Iterative methods form the basis for numerical optimization, which has a huge range of applications to finance from finding optimal portfolios to estimating parameters of GARCH models. Extrapolation and interpolation techniques such as cubic splines are illustrated by fitting currency option smiles and yield curves. Binomial lattices are applied to price European and American options consistently with the Black–Scholes–Merton model, and Monte Carlo simulation is applied to simulate correlated geometric Brownian motions, amongst other illustrative examples. As usual, all of these are contained in an Excel workbook for the chapter on the CD-ROM, more specific details of which are given below.

The presentation in Chapter 6, *Introduction to Portfolio Theory*, follows the chronological development of the subject, beginning with decision theory and utility functions, which were pioneered by Von Neumann and Morgenstern (1947). We describe some standard utility functions that display different risk aversion characteristics and show how an investor's utility determines his optimal portfolio. Then we solve the portfolio allocation decision for a risk averse investor, following and then generalizing the classical problem of portfolio selection that was introduced by Markowitz (1959). This lays the foundation for our review of the theory of asset pricing, and our critique of the many risk adjusted performance metrics that are commonly used by asset managers.

ABOUT THE CD-ROM

My golden rule of teaching has always been to provide copious examples, and whenever possible to illustrate every formula by replicating it in an Excel spreadsheet. Virtually all the concepts in this book are illustrated using numerical and empirical examples, and the Excel workbooks for each chapter may be found on the accompanying CD-ROM.

Within these spreadsheets readers may change any parameters of the problem (the parameters are indicated in *red*) and see the new solution (the output is indicated in *blue*). Rather than using VBA code, which will be obscure to many readers, I have encoded the formulae directly into the spreadsheet. Thus the reader need only click on a cell to read the formula. Whenever a data analysis tool such as regression or a numerical tool such as Solver is used, clear instructions are given in the text, and/or using comments and screenshots in the spreadsheet. Hence the spreadsheets are designed to offer tutors the possibility to set, as exercises for their courses, an unlimited number of variations on the examples in the text.

Several case studies, based on complete and up-to-date financial data, and all graphs and tables in the text are also contained in the Excel workbooks on the CD-ROM. The case study data can be used by tutors or researchers since they were obtained from free internet

sources, and references for updating the data are provided. Also the graphs and tables can be modified if required, and copied and pasted as enhanced metafiles into lecture notes based on this book.

ACKNOWLEDGEMENTS

During many years of teaching mathematics at the introductory level I believe I have learned how to communicate the important concepts clearly and without stressing students with unnecessary details. I have benefited from teaching undergraduate students at the University of Sussex from the mid-1980s to the mid-1990s and, for the past 10 years, from teaching master's courses in market risk, volatility analysis and quantitative methods at the ICMA Centre at the University of Reading. The last of these, the core course in quantitative finance, is quite challenging since we often have around 200 students on different master's degrees with very diverse backgrounds. The student feedback has been invaluable, and has helped me develop a skill that I have tried to exercise in writing this book. That is, to communicate worthwhile and interesting information to two very different types of students simultaneously. This way, the book has been aimed at those with a quantitative background but little knowledge of finance *and* those with some understanding of finance but few mathematical skills.

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