# PART 1

# PHASE LOCK WITHOUT NOISE

# CHAPTER 1

# INTRODUCTION

## 1.1 WHAT IS A PHASE-LOCKED LOOP (PLL)?

A phase-locked loop is a circuit that synchronizes the signal from an oscillator with a second input signal, called the reference, so that they operate at the same frequency. The synchronized oscillator is commonly a voltage-controlled oscillator (VCO), so we will usually use these terms interchangeably. The loop synchronizes the VCO to the reference by comparing their phases and controlling the VCO in a manner that tends to maintain a constant phase relationship between the two. In some types of phase-locked loops (PLLs) this phase relationship is held constant. In other types it is allowed to vary somewhat. But the frequency is always synchronized—otherwise the loop is said to be "out of lock."

## 1.2 WHY USE A PHASE-LOCKED LOOP?

Phase-locked loops are often used because they provide filtering to the phase or frequency of a signal that is similar to what is provided to voltage or current waveforms by ordinary electronic filters. The designer has some control over the manner in which the phase (or frequency) of the VCO follows a changing reference phase (or a changing reference frequency—one cannot occur without the other). The loop can be made to follow quickly or to follow sluggishly. This capability is particularly valuable in removing the effects of noise on the reference or on the synchronized oscillator or, with astute design, on both.

*Phase-Lock Basics, Second Edition.* By William F. Egan

If the reference signal is supposed to be a constant-amplitude phase-modulated signal [e.g., a frequency-modulated (FM) radio signal], the spectrum of the VCO will be a cleaned-up version of the reference spectrum. The PLL, while reproducing the reference signal, rejects all amplitude modulation noise and all other noise that is separated sufficiently in frequency from the signal. It acts like a filter that tracks the signal frequency. In fact, it can provide filtering that ordinary filters cannot because it can follow a signal whose frequency varies slowly by an amount that is greater than the filter bandwidth. For example, a PLL could be designed to filter out noise that is more than 1 kHz from a signal as the signal drifts by say 10 kHz.

Another use of the PLL is in phase or frequency modulating and demodulating signals. This is possible because there exist, within the PLL, a voltage that is proportional to the frequency of the reference and another that follows its phase. Not only can these be extracted for demodulation, but the loop can be forced to produce phase or frequency changes that are proportional to voltages that are injected into the loop, and thus to provide modulation.

Phase-locked loops are also used in frequency synthesis where the VCO oscillates at a selectable multiple of the reference. In this case the oscillator is synchronized with the reference without having the same frequency. However, the combination of the controlled oscillator and the subsequent frequency divider, which outputs the reference frequency, can be looked upon as a synchronized oscillator whose frequency is equal to the reference frequency.

Other PLLs, which are seemingly very different, involve things as diverse as rotating machinery and computer programs.

## 1.3 SCOPE OF THIS BOOK

This text describes the fundamentals of PLLs, principles that apply to all types of PLLs. It emphasizes the details that apply to the processing of analog electronic signals, as discussed in the first three paragraphs of the previous section, leaving details that are more peculiar to frequency synthesis to other works (see Section 21.2). However, at the end of Part 1, we will illustrate how the theory developed here can be used to analyze many applications, including synthesizers and other circuits usable in wireless and telecommunications.

The text concentrates on second-order loops because higher-order loops have too many parameters to make general discussions practical and because higher-order loops can often be approximated by second-order loops. The effect of such approximations will be discussed, too. A special class of third-order loops is analyzed and described in Appendix 10.A.

While the main body of text stands by itself, additional material is available from appendices that can be downloaded from the Wiley Internet site, as explained in Appendix 21.B. This material includes additional text, problems for most chapters and the answers to those problems, MATLAB® programs (scripts), and Microsoft® Excel spreadsheets. Appendices show how to use MATLAB or Excel as aids in understanding and designing PLLs.

The MATLAB scripts can run under either the Student or Professional editions of MATLAB. (Some scripts require the Control or Signal Processing Toolbox, which will be identified near the beginning of the script.) Refer to notes in the scripts and to the instructional and help material that comes with MATLAB or is available online to learn how to use the scripts and how to enhance and expand them. Scripts tend to increase in complexity as we advance through the text. They are intended as learning and research aids. While enhancing the understanding of the text, they also depend on an understanding of that material. They are not foolproof. Some of the scripts contain user-interface features that can be incorporated into others; there is a tradeoff between user-friendliness and simplicity.

Excel is a very adaptable program that can be of great help to the engineer. The graphs of time-domain and frequency-domain loop responses in this book were generated using Excel.

Appendices that are available online are briefly described in the text. Their section numbers may be preceded by the letter "i" to quickly indicate that they are online.

## 1.4 BASIC LOOP

A block diagram of the basic PLL is shown in Fig. 1.1. While the synchronized oscillator is usually a VCO, one whose frequency changes in response to a control voltage, $u_2$, it is also possible that $u_2$ could be a current and the synchronized oscillator would thus be a current-controlled oscillator (ICO). However, we will consider it a VCO—the transition between the two is easily understood.

The phase detector compares the reference to the VCO's output and produces a signal $u_1$, which changes in proportion to the difference in their phases. This is processed by the loop filter to provide the oscillator control signal $u_2$. The loop filter can be as simple as a conductor ($u_2 = u_1$) or a flat amplifier ($u_2 = K_{\text{LF}} u_1$), but it is usually designed to provide some advantageous response characteristic.

If the output frequency, $\omega_{\text{out}} = d\varphi_{\text{out}}/dt$, should be greater than the reference frequency, $\omega_{\text{in}} = d\varphi_{\text{in}}/dt$, then $u_1 \sim (\varphi_{\text{in}} - \varphi_{\text{out}} - \Theta)$ would necessarily decrease with time, causing $u_2$ to decrease, which, in turn, would cause $\omega_{\text{out}}$ to decrease, bringing $\omega_{\text{out}}$ down toward $\omega_{\text{in}}$. Thus the PLL provides negative feedback to keep the output

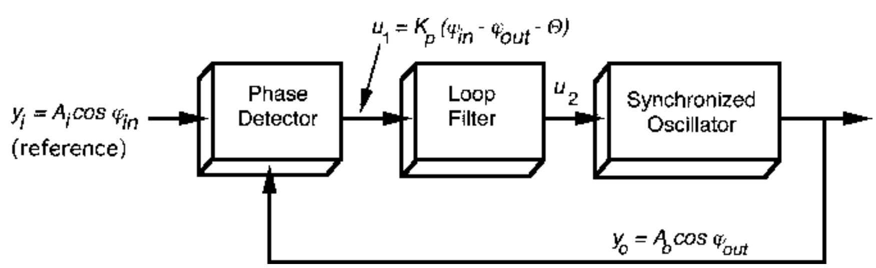


**Fig. 1.1** Basic phase-locked loop; Θ is an offset that depends on the type of phase detector.

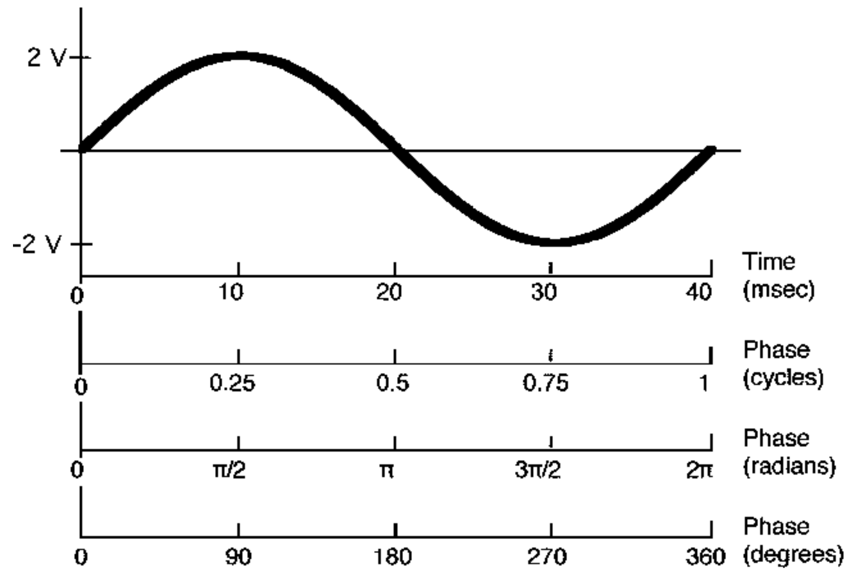


**Fig. 1.2** A 2-V 25-Hz sine wave showing various measures of the abscissa.

frequency $\omega_{\text{out}}$ equal to the reference frequency $\omega_{\text{in}}$. The output amplitude $A_o$ is constant and independent of the input amplitude $A_i$.

## 1.5 PHASE DEFINITIONS

Figure 1.2 shows one cycle of a sinusoid and various ways of measuring the abscissa value. The first measure is time. The other measures are of phase, which indicates the time relative to the waveform's period. The figure illustrates how phase can be measured in any of three common units, cycles (c), radians (rad), or degrees (deg). Because of this we should be careful to carry the units in our calculations. Otherwise errors, typically of $2\pi$, begin to appear in important computed parameters. For example, if $\varphi$ equals $10t^2$ rad/sec$^2$ (a linear frequency sweep), then the frequency is

$$\omega = \frac{d\varphi}{dt} = 20t \text{ rad/sec}^2. \tag{1.1}$$

The frequency can also be expressed as[1]

$$f = (20t \text{ rad/sec}^2)(1\text{c}/2\pi \text{ rad}) = 3.18t \text{ c/sec}^2 = 3.18t \text{ Hz/sec}. \tag{1.2}$$

[1] Note that these expressions produce correct units for phase and frequency when time units are included with $t$. For example, at $t = 3$ sec, $\varphi = 10(3 \text{ sec})^2 \text{ rad/sec}^2 = 90$ rad, $\omega = 20(3 \text{ sec})\text{rad/sec}^2 = 60$ rad/sec, and $f = 3.18(3 \text{ sec})\text{Hz/sec} = 9.54$ Hz.

In this example the change of symbol between $\omega$ and $f$ is a clue to the units, but it is more efficient to carry units than to memorize a different set of formulas for each set of units. The task of maintaining units is made more difficult, however, because of the widespread practice of not considering the radian a unit (i.e., considering angles measured in radians to be without units) and the common practice of not incorporating units in equations involving Laplace transforms (see Section 1.9) or derivatives and integrals of some functions. An alternative to carrying units might be to employ always a set of formulas that use one type of measure, say radians, and each time to convert to those units. This can be awkward in a field that deals so much with angle units and in which various angle units are more natural to use in various situations. In this text the usual procedure will be to carry units, with one exception that we will discuss at the end of this chapter.

**Example 1.1 Preservation of Units** The time derivative of 1 V sin $kt$ is commonly said to be

$$\frac{d}{dt} 1\text{V} \sin kt = k \text{ V} \cos kt \tag{1.3a}$$

but the units are wrong. For example, if $k = 5$ Hz, Eq. (1.3a) gives a time derivative at $t = 0.01$ sec of [5 c-V/sec cos 0.05 c]. The cosine could conceivably be obtained from a table written in cycle units or any table could be used with conversion of 0.05 c to the correct units (e.g., 18°), but cycle-volts/second (c-V/sec) are not proper units for the slope of a voltage waveform. We are able to use Eq. (1.3a) correctly only because we know that it requires that $k$ be in radians/second and because we know that radian units may not be carried. We would therefore begin by converting 5 Hz to $10\pi$ rad/sec. Then we would drop the radian unit to obtain the correct answer without explicitly converting units.

The formula that correctly carries units gives the derivative of sin $kt$ as ($k$/radian) cos $kt$ so that Eq. (1.3a) becomes

$$\frac{d}{dt} 1 \text{ V} \sin kt = \frac{k}{\text{rad}} \text{ V} \cos kt \Rightarrow \frac{5 \text{ c-V}}{\text{rad-sec}} \cos 0.05 \text{ c} = 10\pi \text{ V/sec } \cos 0.05 \text{ c}. \tag{1.3b}$$

Equation (1.3b) permits the use of any units.

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Figure 1.3 shows how the phase difference between two sine waves is defined. At time $t_x$ the phase difference is

$$\Delta\varphi(t_x) = \varphi_2(t_x) - \varphi_1(t_x) = \sin^{-1} v_2/A_2 - \sin^{-1} v_1/A_1. \tag{1.4}$$

This is true even if the frequencies of the two sinusoids are not the same. However, some phase detectors are sensitive to the zero crossings of the waveforms. For these the quantity measured is really

$$\Delta\varphi = (\Delta t/T) \text{ cycles}. \tag{1.5}$$

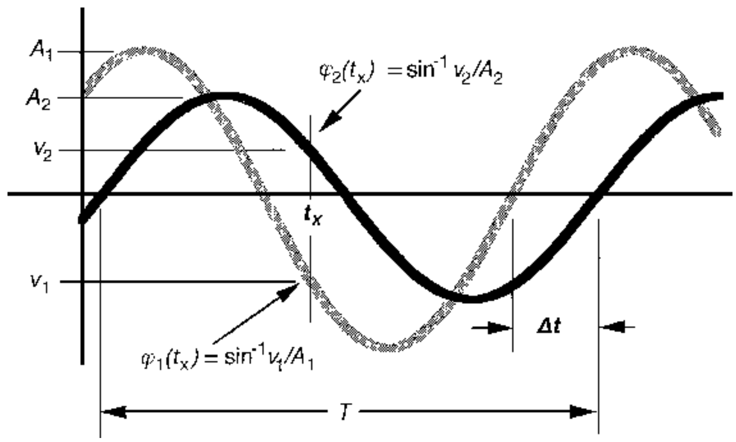


**Fig. 1.3** Defining relative phase.

If the frequencies differ, the phase of one of the waveforms is used as a reference and $T$ is taken from that waveform. In that case what we are actually measuring is the phase of the reference waveform at the zero crossing of the second waveform.

Equation (1.5) not only illustrates the carrying of units but also the efficiency of using whatever units are most convenient.

## 1.6 PHASE DETECTOR

Figure 1.4 shows the type of response that we would like to get from a phase detector (PD). It produces a voltage proportional to the difference in phases of the reference $\varphi_{\text{in}}$ and the VCO output, which is also the loop output, $\varphi_{\text{out}}$. The constant of proportionality, $K_p$, is the gain of the phase detector, relating its output voltage change to its input phase change. For this case

$$K_p = A/(2\pi \text{ rad}) = A/(1 \text{ c}). \tag{1.6}$$

For example, if $A = 3$ V, then $K_p = 3$ V/c $= 0.48$ V/rad.

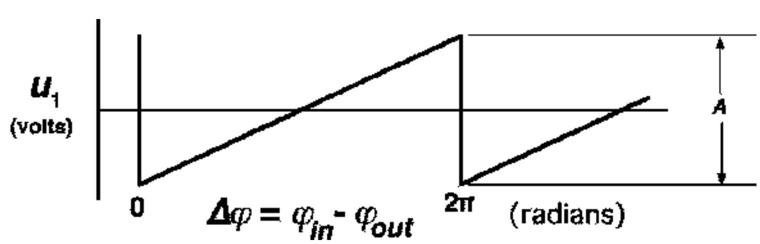


**Fig. 1.4** A "Linear" phase detector characteristic. The characteristic is linear but only over one cycle of phase range.

The PD response will always involve a nonlinearity. Of course, all representations of the responses of physical things involve a nonlinearity for large enough values, but, in the case of the PD, this nonlinearity can easily occur in a region of operation, or at least of consideration. The PD in Fig. 1.4 has a linear range of 1 cycle. Often the linear range is smaller than a cycle, but some types have a lineear range of 2 cycles.

A common type of PD, the balanced mixer, may produce a sinusoidal response like that shown in Fig. 1.5. The gain is the slope of the characteristic,

$$u_1 = B \cos \Delta\varphi, \tag{1.7}$$

$$K_p = \frac{du_1}{d\Delta\varphi} = \frac{d(B\cos\Delta\varphi)}{b \to B\ d\Delta\varphi} = \frac{-B\sin\Delta\varphi}{\text{rad}}, \tag{1.8}$$

where

$$\Delta\varphi = \varphi_{\text{in}} - \varphi_{\text{out}}. \tag{1.9}$$

The most desirable operating point is at $\Delta\varphi = -\pi/2$, where the slope is maximum and equals

$$K'_p \equiv K_p(\Delta\varphi = -\pi/2) = B/\text{rad} \tag{1.10}$$

so Eq. (1.7) can be rewritten

$$u_1 = K'_p \text{ rad } \cos\ \Delta\varphi. \tag{1.11}$$

Thus $K'_p$, the maximum value of $K_p$, for a sinusoid equals its amplitude per radian. Often $K_p$ is used to mean this maximum value. Obviously, this characteristic has no truly linear range, but $K'_p \geq K_p \geq 0.7\ K'_p$ for a range of ±45° about $\Delta\varphi = -\pi/2 = -90°$.

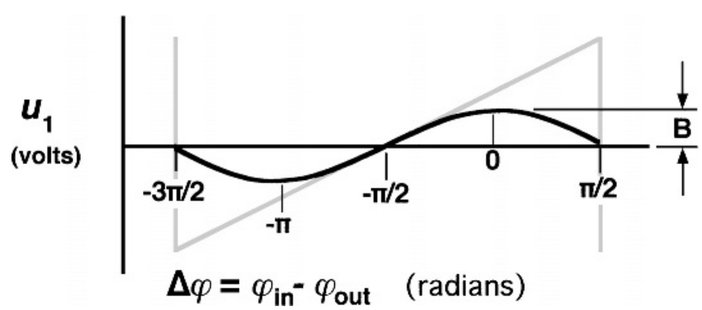


**Fig. 1.5** Sinusoidal phase-detector characteristic, superimposed on the linear response. Both of these have the same gain $K'_p$ at $\pi/2$ radians.

## 1.7 COMBINED GAIN

In its simplest form, the loop filter in Fig. 1.1 consists merely of an amplifier, in which case

$$\frac{du_2}{du_1} \equiv K_{\mathrm{LF}}. \tag{1.12}$$

Just as the PD can have a nonlinear response characteristic, so can the VCO. But it too can be characterized at some operating point by the gain there:

$$\frac{d\omega}{du_2} \equiv K_v, \tag{1.13}$$

where $K_v$ has units of radians per second per volt, or

$$\frac{df}{du_2} \equiv K_v, \tag{1.14}$$

where the units are hertz per volt.

If we break the loop's feedback path in order to determine the gain around the loop (the "open-loop gain"), we see that

$$\frac{d\omega_{\mathrm{out}}}{d\Delta\varphi} = K_p K_{\mathrm{LF}} K_v \triangleq K. \tag{1.15}$$

Checking the units of $K_p$, $K_{\mathrm{LF}}$, and $K_v$, we can see that the units of $K$ should be reciprocal seconds, $\mathrm{sec}^{-1}$. This is not quite the entire open-loop transfer function, which is the ratio of $d\omega_{\mathrm{out}}$ at the output of the opened loop to $d\omega_{\mathrm{out}}$ at its input, and which has no units.

**Example 1.2 Combined Gain** A phase detector, in the region of operation, has a characteristic with a slope of 0.1 V per radian so

$$K_p = 0.1 \text{ V/rad}.$$

The loop filter is an amplifier with a voltage gain of 5 so

$$K_{\mathrm{LF}} = 5.$$

The slope of the VCO tuning curve at the operating frequency is 2 MHz/V so

$$K_v = 2 \times 10^6 \text{ Hz/V}.$$

The combined gain is, from (1.15),

$$K = 10^6 \frac{\text{Hz}}{\text{rad}} = 10^6 \frac{\text{c}}{\text{sec-rad}} = 10^6 \frac{\text{c}}{\text{sec-rad}} \frac{2\pi \text{ rad}}{\text{c}} = 6.28 \times 10^6 \text{ sec}^{-1}.$$

If the input phase $\varphi_{\text{in}}$ suddenly changes by 0.01 radian, the immediate result will be a change of voltage at the phase detector output of

$$0.01 \text{ rad}(K_p) = 10^{-3} \text{ V}.$$

This will cause a change $K_{\text{LF}}$ larger at the loop filter output, that is $5 \times 10^{-3}$ V. This will produce a frequency change at the VCO output of

$$5 \times 10^{-3} \text{V}(K_V) = 10^4 \text{ Hz}.$$

Or we can also simply multiply the phase change by $K$ to obtain the frequency change

$$0.01 \text{ rad}(6.28 \times 10^6 \text{ sec}^{-1}) = 6.28 \times 10^4 \text{ rad/sec} = 1 \times 10^4 \text{ Hz}.$$

In a locked loop, this freqenncy change causes the phase at the VCO output to begin to advance more rapidly than the input phase and thus to begin the process of restoring the phase difference that existed before the step in input phase.

## 1.8 OPERATING RANGE

Assuming a PLL that is initially locked, what limitation is there on the range of $\omega_{\text{in}}$ over which lock can be maintained? As the input frequency $\omega_{\text{in}}$ is slowly lowered, the operating point will move down the ramp in Fig. 1.4 to lower the VCO's frequency $\omega_{\text{out}}$, and keep it equal to $\omega_{\text{in}}$. At some point $\omega_{\text{in}}$ may become so low that lock will be lost because it is not within the capability of the PD to generate a voltage that is low enough to cause $\omega_{\text{out}}$ to equal $\omega_{\text{in}}$. Likewise there is a limit on how high $\omega_{\text{in}}$ can go before the operating point runs off the upper extremity of the PD characteristic. The difference between these two extremes is called the "hold-in"or "synchronization" range.

From Fig. 1.4 we can see that the output of a PD with the sawtooth characteristic can vary over a total range of $A$. By Eq. (1.6), this equals $2\pi K_p$ rad. We can also write this as $\pm \pi K_p$ rad about the midpoint We obtain the corresponding change in $\omega_{\text{out}}$ by multiplying the change in PD output by $K_{\text{LF}} K_v$, giving a total hold-in or synchronization range of

$$\pm\Omega_{H,\text{sawtooth}} = \pm \pi K_p K_{\text{LF}} K_v \text{ rad} = \pm \pi K \text{ rad}. \tag{1.16}$$

Note that, since the units of $K$ are reciprocal seconds, the units of $\Omega_H$ are radians/second, as they should be. If there is significant curvature in the VCO tuning characteristic over $\pm\Omega_H$, an average value of $K_v$ should be used. In practice, however, the actual values of $u_2$ that correspond to the extremes of the PD range must be found before the region of operation of the VCO can be determined so it is as easy to find $\Omega_H$ from the tuning curve as it is to determine the average value of $K_v$. This is illustrated in Fig. 1.6.

From Fig. 1.5 we can see that the output of a PD with a sinusoidal characteristic can vary over a total range of $2B$. By Eq. (1.11), this equals $2K'_p$ rad. The hold-in range for this PD is therefore

$$\pm\Omega_{H,\ \text{sinusoidal}} = \pm K' \,\text{rad}, \tag{1.17}$$

where $K' = K'_p K_{\text{LF}} K_v$, the maximum value of $K$. Thus, for the same maximum gain, the hold-in range of the sawtooth PD is $\pi$ larger than that of the sinusoidal PD. Assuming other components are linear, we can write the forward transfer function for a loop with this phase detector as

$$\Delta f = K' \cos\, \Delta\varphi. \tag{1.18}$$

Here the $\Delta f$ and $\Delta\varphi$ are deviations from midrange. In practice, the range of other components, such as the VCO or an operational amplifier, can be more limiting than that of the phase detector. However, these limitations are not fundamental—they can normally be increased, whereas the range of any phase detector has an inherent limit.

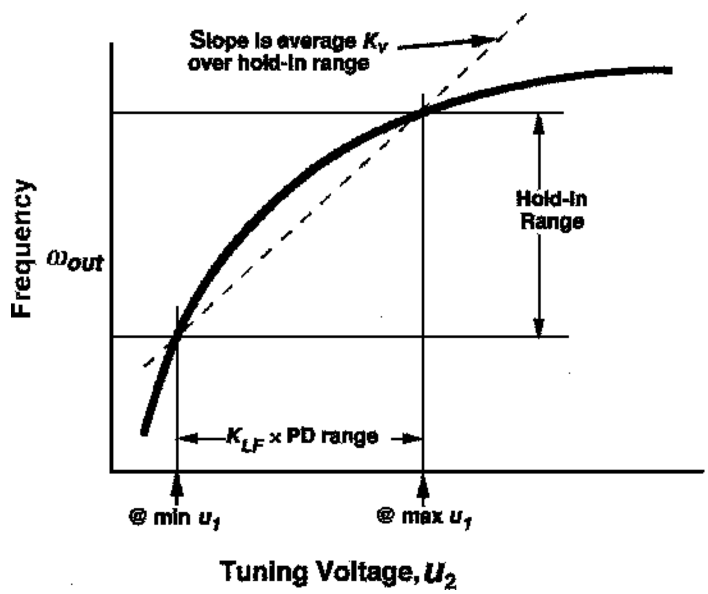


**Fig. 1.6** Typical VCO tuning curve with the average slope over the hold-in range shown.

For this reason, the phase detector is assumed to be the range-limiting element in most PLL analyses.

While we have assumed a simple loop filter, Eqs. (1.16)–(1.18) apply also for more complex loops whose low-frequency filter gains are $K_{\mathrm{LF}}$.

**Example 1.3 Hold-in Range** The parameters in Example 1.2 describe a loop when the phase is in the center of the phase detector's range (output midway between maximum and minimum). Assume the other components have linear responses.

If the phase detector is sinusoidal, (1.17) indicates a hold-in range of

$$\pm K' \,\mathrm{rad} = \pm(6.28 \times 10^6 \ \mathrm{sec}^{-1})\,\mathrm{rad} = \pm 6.28 \times 10^6 \,\mathrm{rad/sec} = \pm 1\,\mathrm{MHz}.$$

Thus the VCO can be tuned ±1 MHz without loss of lock.

If the phase detector has a sawtooth characteristic, (1.16) gives the hold-in range as

$$\begin{aligned} \pm \pi K \,\mathrm{rad} &= \pm 3.14(6.28 \times 10^6 \ \mathrm{sec}^{-1})\,\mathrm{rad} \\ &= \pm 1.97 \times 10^7 \,\mathrm{rad/sec} = \pm 3.14\,\mathrm{MHz}. \end{aligned}$$

## 1.9 UNITS AND THE LAPLACE VARIABLE *s*

The Laplace variable $s$ represents $d/dt$, so it should have units of reciprocal seconds. When $1/s$ represents $\int dt$, again, $s$ should have units of reciprocal seconds. But then, for consistency of units, $s = j\omega + \sigma$ should be written $s = j\omega\,\mathrm{rad} + \sigma/\mathrm{neper}$. The open-loop transfer function should be written $-G = -K/(j\omega/\mathrm{rad})$. How can we prevent the confusion and wrong answers that can be caused by not carrying units and yet not be overburdened by nontraditional rules involving radian units? Perhaps the best solution is to *drop or add radians, and only radians, as required*. For example, if I want the hold-in range in hertz and I use Eq. (1.16) but drop the "rad," the hold-in range is $2\pi K$. The unit of $K$ is reciprocal seconds, but this is not a proper unit for frequency. To obtain proper units, we add "rad," obtaining $2\pi K$ rad with units of radians/second for hold-in range. We cannot mistake this for the hold-in range in hertz because we do not treat cycles as arbitrarily as we do radians. To obtain hold-in range in hertz, we must convert Eq. (1.16) as follows:

$$2\Omega_{H,\ \mathrm{sawtooth}} = 2\pi K \,\mathrm{rad} \frac{\mathrm{c}}{2\pi \,\mathrm{rad}} = K\mathrm{c}. \tag{1.19}$$

This has units of cycles/second = hertz (c/sec = Hz), as desired. In fact, even though the equalities above are valid, Ω, which we might assume from the symbol to be in radians/second, has here been equated to hold-in range in hertz. This gives a hint of the danger in relying on the symbol to differentiate between radians/second and hertz.