

## Preface

This is a textbook on rigid body mechanics written for graduate and advanced undergraduate students of science and engineering. The primary reason for writing the book was to give an account of the subject which was firmly grounded in both the classical and geometrical foundations of the subject. The book is intended to be accessible to a student who is well prepared in linear algebra and advanced calculus, who has had an introductory course in mechanics and who has a certain degree of mathematical maturity. Any mathematics needed beyond this is included in the text.

Chapter 1 deals with the rotations, the basic operation in rigid body theory. Rotations are presented in several parameterizations including axis angle, Euler angle, quaternion, and Cayley–Klein parameters. The rotations form a Lie group which underlies all of rigid body mechanics.

Chapter 2 studies rigid body motions, angular velocity, and the physical concepts of angular momentum and kinetic energy. The fundamental idea of angular velocity is straight from the Lie algebra theory. These concepts are illustrated with several examples from physics and engineering.

Chapter 3 studies rigid body dynamics in vector, Lagrangian, and Hamiltonian formulations. This chapter introduces many geometric concepts as dynamics occurs on differential manifolds and for rigid body mechanics the manifold is often a Lie group. The idea of the adjoint action is seen to be basic to the rigid body equations of motion. This chapter contains many examples from physics and engineering.

Chapter 4 considers the dynamics of constrained systems. Here Lagrange multipliers are introduced and several ways of determining or eliminating them are considered. This topic is rich in geometrical interactions and there are several examples, some standard and some not.

Chapter 5 considers the integrable problems of free rotation, Lagrange's top, and the gyrostat. The Kowalevsky top and Lax equations are also considered. Geometrical topics include the Poincaré construction, the geometric phase, and

Liouville tori. Verification and validation are of the utmost importance in the world of scientific and engineering computing and analytical solutions are treasured. In addition to the validation service they also play an important role in developing our intuitive understanding of the subject, not to mention their intrinsic and historical worth.

The importance of numerical methods in today's applications of mechanics cannot be overstated. Chapter 6 discusses classical numerical methods and more recent methods tailored specifically for Lie groups. This chapter includes a case study of a complicated rigid body motion, the wobblestone, which is naturally studied with numerical methods.

The final chapter, Chapter 7, applies the previous material to phenomena ranging in scale from the astronomical to the molecular. The largest scale concerns precession and nutation of the Earth. The Earth is nonspherical – an oblate spheroid to first approximation – and the axis of the Earth's rotation is observed to move on the celestial sphere. Most of this motion is attributed to the torque exerted on the nonspherical Earth by the Moon and the Sun and rigid body dynamics explains the effect. Next we study satellite gravity gradient stabilization. If Earth's satellites are not stabilized by some mechanism, they will tumble – as do the asteroids – and will not be useful platforms. One stabilization mechanism uses the gradient in the Earth's gravitational field and the basics of this mechanism are explained by rigid body theory. On the same scale, but down to the Earth, we consider the motion of a multibody, a mechanical system consisting of interconnected rigid bodies. Rigid body dynamics is used to study the motion of a robot arm. At the smallest scale we examine the techniques of molecular dynamics. Physicists and chemists study properties of matter by simulating the motions of a large number of interacting molecules. At the most basic level this is a problem in quantum mechanics. However, there are properties which can be calculated by idealizing the material to be a collection of interacting rigid bodies.

Three appendices are provided on spherical trigonometry, elliptic functions and Lie groups and Lie algebras. Lie groups and Lie algebras unify the subject. Appendix C provides background on all the Lie group concepts used in the text.

Some choices made in the course of writing the book should be mentioned. The application of mechanics often boils down to making a calculation and getting the useful number. I have tried to keep calculations foremost in mind. There are no theorem–proof structures in the book. Rather observations are made and substantiated by methods which are decidedly computational. Rarely are equations put in dimensionless form because one can rely on dimensional checks to avoid algebra mistakes and misconceived physics. Of course, when it comes time to compute one is well advised to pay attention to

the scalings. Various notations are used in the text ranging from “old tensor” to intrinsic operators independent of coordinates and the concise notations of [1,23] which facilitate the computations. There is a glossary in Appendix D of the notations used. Examples are set aside in italic type and end with the symbol  $\diamond$ .

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