PART I

Measurements

1 Apparent Diameter: Observed Diameter of Celestial Bodies

Celestial bodies are natural objects in the sky, such as the Sun, moons, planets, and stars. The apparent diameter of a celestial body is not the actual diameter, but how large the diameter appears to be as viewed from Earth. Two objects of different actual diameters may have the same apparent diameter.

In this project, you will compare the apparent diameters of the Moon and the Sun and discover how their distances from Earth affect their apparent diameters. You will demonstrate and calculate the greatest and least distance ratios of the Sun and the Moon from Earth. You will determine a method of using the ratio of apparent diameter to apparent distance to determine the Moon's angular diameter, which is its apparent diameter measured in radians or degrees. You will also research different instruments of measuring angular size, such as the cross-staff and sextant.

Getting Started

Purpose: To determine how distance from Earth affects the apparent diameter of celestial bodies.

Materials

drawing compass
ruler
sheet of yellow construction paper
scissors
¼ inch (0.63 cm) one-hole paper punch
white index card
masking tape
yardstick (meterstick)
3 yards (3 m) of thin string

Procedure

1. Prepare a data table like Table 1.1.
2. Use the compass to draw a 1-inch (2.5-cm)-diameter circle on the yellow paper. Cut out the circle. Call this diameter D_1.

3. Use the paper punch to make a hole in the center near the edge of one short side of the index card. The diameter of the hole is ¼ or 0.25 inch (0.63 cm). Call it D_2.

4. Tape the yellow circle to a wall at eye level. A path of at least 3 yards (3 m) must be clear in front of the wall. Use tape to secure one end of the string to the center of the circle.

5. Thread the free end of the string through the hole in the index card.

6. Standing close to the wall, hold the bottom edge of the index card with both hands so that the hole in the card is at the top.

7. With your hands at arm's length in front of your face, stand so that the hole in the card is centered on the yellow circle.

8. Close one eye and look through the hole in the card with your open eye. As you continue to look through the hole, slowly back away from the yellow circle, letting the card move along the string. Stop when the outer edge of the yellow circle and the edge of the hole line up precisely.

9. In this position, continue to hold the card at arm's length with one hand and with the other hand pull the string taut between your open eye and the yellow circle.

10. Ask your helper to measure and record two distances: d_1, the length of the string from your face to the center of the yellow circle, and d_2, the length of the string from your face to the hole in the index card (see Figure 1.1). Record the distances in columns 4 and 5 of your data table for test 1.

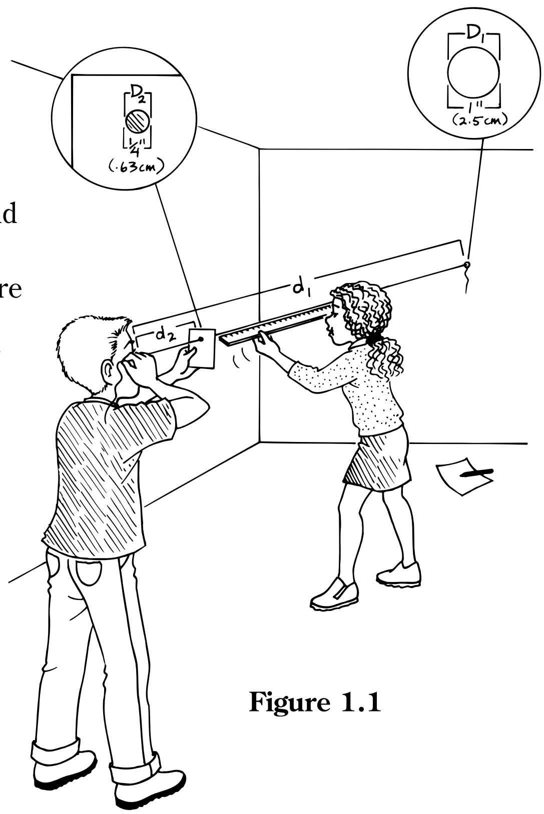


Figure 1.1

11. The ratio of the diameters compared to the ratio of the distances is:

$$D_1/D_2 = d_1/d_2$$

- D_1/D_2 = 1 inch/0.25 in = 4/1 (2.5 cm/0.63 = 4/1)

- Use the distance measurements from step 10 to calculate the distance ratio, d_1/d_2, by dividing the numerator by the denominator. Record the distance ratio in column 7 of your data table for test 1.

12. Repeat steps 6 to 11 four times, recording the results for tests 2 to 5.

13. Average the distance measurements for d_1 and d_2. Record the averages in columns 4 and 5 of your data table.

14. Average the calculations for d_2/d_1. Record the average in column 7 of your data table.

Table 1.1 Distance/Diameter Data

Test	Diameter, inches (cm)		Distance, inches (cm)		Diameter Ratio	Distance Ratio
	D_1	D_2	d_1	d_2	D_1/D_2	d_1/d_2
1	1 (2.5)	0.25 (0.63)			4/1	
2	1 (2.5)	0.25 (0.63)			4/1	
3	1 (2.5)	0.25 (0.63)			4/1	
4	1 (2.5)	0.25 (0.63)			4/1	
5	1 (2.5)	0.25 (0.63)			4/1	
Average	1 (2.5)	0.25 (0.63)			4/1	

Results

The ratio of the diameters and the ratio of the distances are both equal to or close to 4/1, or 4 to 1. Because these diameter and distance ratios are the same, the small hole appears to be the same size as the larger paper circle when viewed at a distance from the circle.

Why?

As you move away from the wall, the **apparent diameter** (how large an object's diameter appears to be from a specific distance) of the yellow circle decreases. Since the ratio of the diameter of the circle to that of the hole is 4 to 1, the yellow circle appears to be the same size as the hole in the card when the ratio of their distances from your eye is also 4 to 1. The same thing can be true of **celestial bodies,** which are natural objects in the sky such as the Sun and the Moon, when viewed from Earth. The ratio of the actual diameter of the Sun to that of the Moon is 400 to 1, and the average ratio of the distance of the Sun from Earth to that of the Moon is also approximately 400 to 1. Because the diameter of the Sun is about 400 times the diameter of the Moon, and because the Sun is also 400 times farther away from Earth than is the Moon, the Moon and the Sun have the same apparent diameter.

Try New Approaches

1a. Since the actual diameters of the Sun and the Moon don't change, their diameter ratio is always about 400 to 1. However, the distances of the Sun and the Moon from Earth do change somewhat. This is because Earth's orbit around the Sun and the Moon's orbit around Earth are not perfect circles. Instead they are slightly flattened circles called **ellipses** (see Figure 1.2). Therefore the distance ratio of the Sun and the Moon from Earth is sometimes more and sometimes less than 400 to 1. Determine how an increase in this distance ratio affects the apparent diameters of the Sun and the Moon. The distance ratio is greatest when Earth is at **aphelion** (point in a planet's orbit farthest from the Sun) and the Moon is at **perigee** (the orbital point of the Moon or man-made satellite at the least distance from Earth). To demonstrate this, stand so that d_1 and d_2 are at the average measurements in the experiment and bend your arm so that d_2 is shorter and d_1 is longer. Note how the yellow circle fits into the hole in the card.

b. The least distance ratio of the Sun and the Moon from Earth occurs when Earth is at **perihelion** (the point in a planet's orbit at the least distance from the Sun) and the Moon is at **apogee** (point in the Moon's or a man-made satellite's orbit farthest from Earth). To demonstrate this, repeat the previous experiment; turn slightly so that you extend your arm farther so that d_2 is longer and d_1 is shorter. Note how the yellow circle fits into the hole in the card.

2a. Calculate the greatest distance ratio of the Sun and the Moon from Earth. At aphelion, Earth is about 95 million miles (152 million km) from the Sun. When the Moon is at perigee, it is about 227,063 miles (363,300 km) from Earth.

b. Calculate the least distance ratio. At perihelion, Earth is 91.9 million miles (147 million km) from the Sun. When the Moon is at apogee, it is 253,437 miles (405,500 km) from Earth. **Science Fair Hint:** Make a drawing like the one in Figure 1.2 to represent the positions of Earth, the Moon, and the Sun at the greatest and least distance ratios. Add your calculated distance ratios to your diagram. Note that your diagram does not have to be drawn to scale and the ellipses can be exaggerated for illustrative purposes. Do make a note on your diagram explaining this.

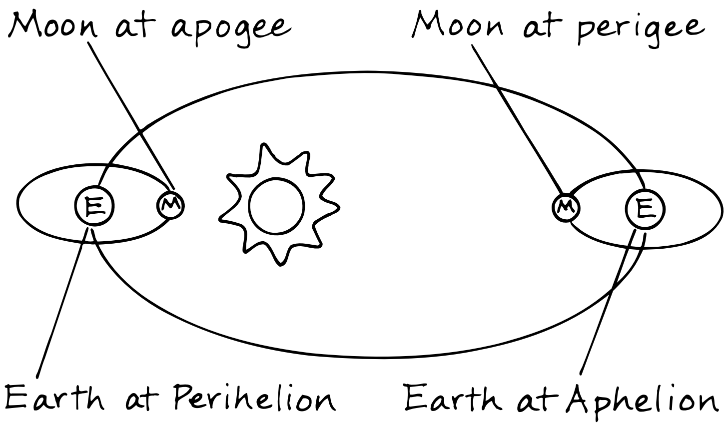


Figure 1.2

Design Your Own Experiment

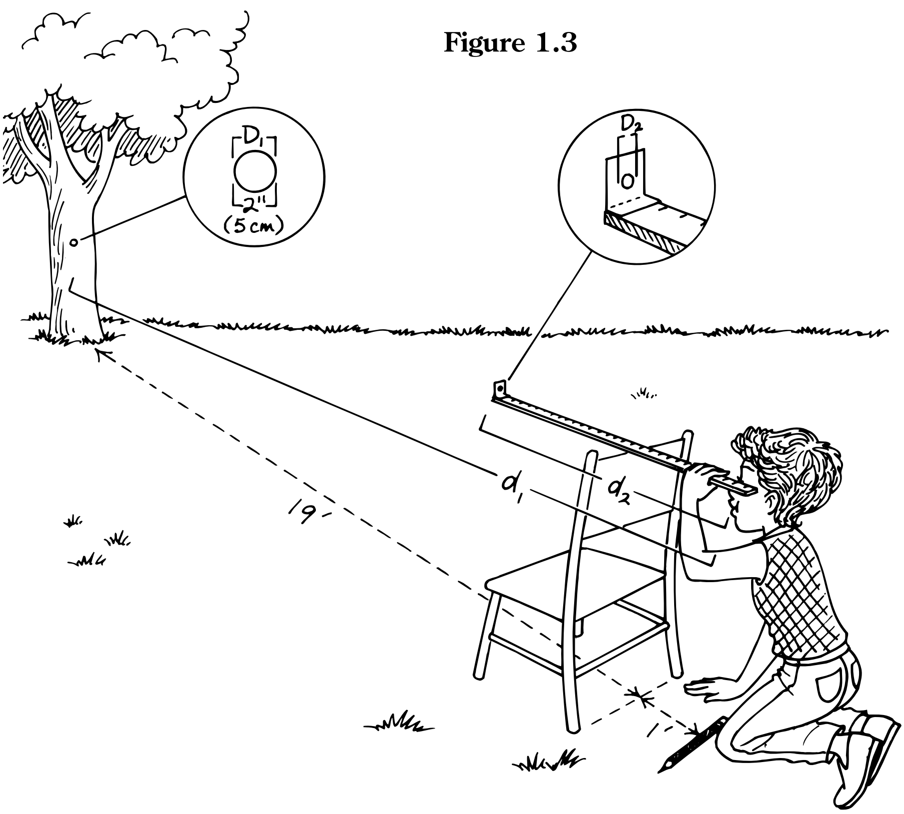
1a. Design an experiment to demonstrate how the ratio of the apparent diameter to the apparent distance of the Moon can be used to determine the Moon's **angular diameter** (apparent diameter, measured in radians or degrees). One way is to cut a 2-inch (5-cm) -diameter circle from an index card or other stiff paper to represent the Moon. Call this diameter D_1. Tape the paper moon to a tree or other outdoor structure. Set a chair on the ground so that it faces the tree with its back 19 feet (5.7 m) from the tree.

Prepare a measuring instrument. First, cut a 1-by-2-inch (2.5-cm-by-5-cm) piece from an index card. Using a paper punch, make a hole in the center of the card. The diameter of the hole, ¼ inch (0.63 cm), is the apparent diameter of the paper circle (the Moon) and is called D_2. Bend the card about ½ inch (1.25 cm) from one short end, and tape the card to the zero end of a yardstick (meterstick). The hole in the card should be just above the surface of the measuring stick (see Figure 1.3). Rest the measuring stick on the back of the chair, aiming the end with the card toward the paper moon. Lay a pencil on the ground parallel to and 1 foot (30 cm) from the back of the chair.

Kneel by the pencil with the edge of the measuring stick next to one side of your face. Close one eye and use your open eye to look at the paper moon through the hole in the card. Keeping your head still, slide the measuring stick back and forth along the side of your face until the circle on the tree exactly fills the hole in the card. Record the measurement on the stick at the point even with your open eye as d_2. This is the apparent distance to the paper moon from an actual distance of 20 feet (6 m), which is d_1. Use the following equation to calculate the angular size(S), in degrees, of the paper moon:

$$S = 57.3° \times D_2/d_2$$

Figure 1.3



Note: D_2/d_2 yields a number without a unit of measurement. When no unit of measurement is indicated in giving the measure of an angle, the angle is understood to be expressed in radians. To express the angle in degrees, the conversion 57.3° per 1 radian is used.

Repeat the measurement four times and calculate an average. ***Note:*** Make sure D_2 and d_2 are expressed in the same units. For example, if D_2 is in inches, d_2 must be also.

b. In your demonstration, how does the ratio of actual diameter, D_1, to actual distance of the Moon, d_1, compare to the ratio of apparent diameter to apparent distance of the Moon, $D_1/d_1 = D_2/d_2$? Use the measurements to calculate each ratio by dividing the numerator by the denominator.

2a. During a full moon, use the instrument in the previous investigation to determine the angular diameter of the Moon. Take five measurements and, using the method in Appendix 1, determine your random error of measurement. **CAUTION:** *Do not attempt this experiment with the Sun, because looking at the Sun even for just a few seconds can cause permanent damage to your eyes.*

b. The angular diameter of the Moon varies from 0.49° at apogee to 0.55° at perigee. Using 0.5°, the average angular diameter of the full moon, the average angular diameter measured in the previous experiment, and the method in Appendix 2, calculate the relative error (also called percentage error) of your investigative results. What might account for any relative measuring error?

Get the Facts

The cross-staff and sextant are other instruments used to measure how large the angular measurement between two points appears to be, called the *angular distance* and angular diameter. Find out more about these instruments. For information, see the next project and also Richard Moeschl, *Exploring the Sky* (Chicago: Chicago Review Press, 1993), pp. 115–123.