# Important Linear Motion Equations

(C Exam)

$$\textbf{Displacement } \mathbf{x}(t) = \int \mathbf{v}(t)\,dt$$

(C Exam)

$$\mathbf{v}_{INSTANTANEOUS} = \mathbf{v}(t) = d\mathbf{x}/dt = \int \mathbf{a}(t)\,dt$$

(C Exam)

$$\mathbf{a}_{INSTANTANEOUS} = \mathbf{a}(t) = d\mathbf{v}/dt = d^2\mathbf{x}/dt^2$$

$$\mathbf{a}_{AVE} = \Delta\mathbf{v}/\Delta t$$

$$\mathbf{v}_{AVE} = \Delta\mathbf{x}/\Delta t$$

$$\mathbf{v}_F = \mathbf{v}_o + \mathbf{a}t \quad \text{[For constant accelerations]}$$

$$\mathbf{d} = \mathbf{s}(t) = \mathbf{v}_o t + \frac{1}{2}\mathbf{a}t^2 \quad \text{[For constant accelerations]}$$

$$\mathbf{v}_F^2 = \mathbf{v}_o^2 + \mathbf{2ad} \quad \text{[For constant accelerations]}$$

**Note: All of these linear motion equations also hold for the analogous quantities in rotational motion. "Rotational Motion and Torque" later in this chapter will cover these equations, along with other concepts.**

## Example (B Exam)

The position of a particle moving along the **x**-axis is given as the following:

$$\mathbf{x}_2 = 8.8\text{m}$$
$$\mathbf{x}_1 = 5.5\text{m}$$

In a time interval of 3.0 seconds, what is the particle's average velocity?

## Solution

The average velocity is $\mathbf{x}_2 - \mathbf{x}_1 / t_2 - t_1$, or 3.3m/3.0 sec = **1.1 m/s**

## Example (C Exam)

The position of a particle moving along the **x**-axis is given as the following:

$$\mathbf{x} = 5.4 + 6.8\,t + 10.0\,t^2 \text{ (in meters)}$$

What is the velocity at time = 2.0 sec?

## Solution

Take the first derivative with respect to time:

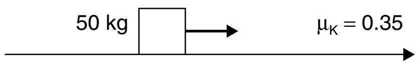
$$\mathbf{v}(t) = \frac{d\mathbf{x}}{dt} = 0 + 6.8 + 20.0t$$

This indicates that the velocity is changing with time and that when t = 2.0 sec, the velocity will be (6.8 m/s) + (20.0 m/$s^2$)(2.0 sec) = **46.8 m/s.**

each other. They both accelerate in opposite directions. The skater with the larger mass has the lesser acceleration and the smaller skater's acceleration is greater than that of the larger skater. The skaters attain unequal velocities in opposite directions. This involves the Law of Conservation of Momentum, which is covered in the next chapter.

## Example 1 (B Exam)

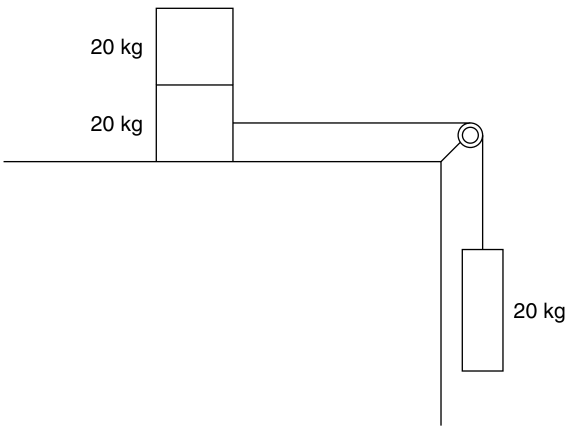
A box of physics textbooks is pulled along a rough floor at a constant speed. If the coefficient of kinetic friction between the box and the floor is 0.35, and the box has a mass of 50 kg, what is the frictional force on the box?




## Solution

Because the box is not accelerating, no net force exists. Therefore, the parallel and the frictional forces are equal and opposite. Because $\mu = \mathbf{F_F} / \mathbf{F_N}$ ,

$$\mathbf{F_F} = \mu\, \mathbf{F_N} = (0.35)(50\text{ kg})(9.8\text{ m/s}^2) = \mathbf{170\ N}$$




## Example 2

Two 20 kg masses rest on a horizontal table. The bottom mass is connected to a string of negligible mass, which passes over a frictionless pulley with negligible mass and supports a third 20 kg mass that hangs over the side of the table. The coefficient of kinetic friction between the table and the stacked masses is 0.15.

**(a)** What is the net force on the system?
**(b)** What is the acceleration of the system?
**(c)** What is the minimum coefficient of friction between the two vertically stacked masses that will prevent the top one from sliding off as they move?

## Solution

**(a)**

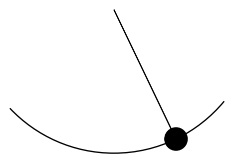
$$\begin{aligned} F_{NET} &= \mathbf{F_{WT}} - \mathbf{F_F} = (20\text{ kg})(\mathbf{g}) - \mu\mathbf{F_N} \\ &= (20\text{ kg})(9.8\text{ m/s}^2) - (0.15)(40\text{ kg})(9.8\text{ m/s}^2) = 196\text{ N} - 59\text{ N} \\ &= \mathbf{137\ N} \end{aligned}$$

**(b)**

$$\mathbf{a} = \frac{\mathbf{F_{NET}}}{\Sigma m} = \frac{137\text{N}}{60\text{kg}} = \mathbf{2.3\ m/s^2}$$

**9.** A 6 kg skateboard rolls away from its owner at a speed of 5 m/s. If the coefficient of rolling friction between the wheels and the pavement is 0.2, it will come to a stop in

**(A)** 0.5 second.
**(B)** 1.0 second.
**(C)** 1.5 seconds.
**(D)** 2.0 seconds.
**(E)** 2.5 seconds.



**10.** A pendulum with a period of 2 seconds at sea level has its length doubled. Its new period is now most nearly

**(A)** 1 second.
**(B)** 2 seconds.
**(C)** 3 seconds.
**(D)** 4 seconds.
**(E)** 5 seconds.

## Answers to Multiple-Choice Questions

**1.** **(B)** $\mathbf{v_F = v_O + at} = (2\text{ m/s}^2)(3\text{ sec}) = 6$ m/s final velocity.

For the average velocity: $\mathbf{v_{AVE}} = (0 + 6\text{ m/s})/2 = \mathbf{3\ m/s}$

**2.** **(C)** Again, $\mathbf{v_F = v_O + at} = (2\text{ m/s}^2)(2\text{ sec}) = 4$ m/s final velocity.

The average velocity is 2 m/s and (2 m/s)(2 sec) = **4 m**

**3.** **(C)** $\mathbf{a} = (\frac{d\mathbf{v}}{dt}) = (1.2)\text{ t m/s}^2$ and $(1.2\text{ m/s}^2)(2) = \mathbf{2.4\ m/s^2}$

**4.** **(E)** $\mathbf{v} = \frac{d\mathbf{x}}{dt}$, and integrate **v**(t) to find **x**:

$\int_0^3 \mathbf{v}(t) = \mathbf{x}(t)$ $\qquad \int_0^3 (3 + 0.6\ t^2)\ dt = 3t + (0.6)(1/3)t^3 \Big|_0^3 = 9 + (0.6)(9)$
$= \mathbf{14.4\ m}$

**5.** **(E)** The net force on the tire is $\mathbf{F_{NET}} = m\mathbf{g} \sin 30°$

$= (25\text{ kg})(10\text{ m/s}^2)(0.5) = \mathbf{125\ N}$

**6.** **(B)** $E_P = E_{K\ LINEAR} + E_{K\ ROT}$

$m\mathbf{gh} = \frac{1}{2} m\mathbf{v}^2 + \frac{1}{2} \mathbf{I\omega}^2$ $\qquad h = 10\text{ m} \sin 30° = 5\text{m}$

$\mathbf{v} = \sqrt{\mathbf{gh}} = \sqrt{(10\text{m/s}^2)(5\text{m})} = \sqrt{50} \approx \mathbf{7\ m/s}$

**7.** **(C)** At its maximum height, the ball's vertical speed is 0.

Using $v_F^{\mathbf{2}} = \mathbf{v_O^2 + 2ad}$, $\mathbf{v_{O\,VERT}} = 10 \sin 45° = 10\ (\frac{2^{1/2}}{2}) = 5(2^{1/2})$

$0 = [5(2^{1/2})]^2 + (2)(-10\text{ m/s}^2)(\mathbf{d})$

$\mathbf{d} = \frac{-50\text{m/s}^2}{-20\text{m/s}^2} = \mathbf{2.5\ m}$