Example 2 (B & C Exams)

Determine the linear velocity **v** of a communications satellite a distance equal to the Earth's radius above the Earth.

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
Earth's radius = 6.37×10^6 m
Earth's mass = 5.98×10^{24} kg

Solution

The gravitational force on the satellite equals the centripetal force on the satellite.

$\mathbf{F_G} = Gm_1m_2/\mathbf{r}^2$
$\mathbf{F_C} = m\mathbf{v^2}/\mathbf{r}$

Setting them equal,

$$Gm_Em_S/\mathbf{r}^2 = m_S\mathbf{v}^2/\mathbf{r}$$

Canceling m_S and **r** gives $\mathbf{v} = \sqrt{\frac{Gm_E}{\mathbf{2r_E}}}$ and substituting numerical values yields $\mathbf{5.59 \times 10^3}$ **m/s**.

In problems involving a car traveling over a circular-shaped hill and just staying on the hill at the very top, let the car's weight equal the centripetal force.

Example 3 (B & C Exams)

A cart of mass 50 kg rides over a semicircular bridge of radius 10 m. What is the maximum speed critical velocity crit, the cart can have without lifting off the bridge?

Solution

Set the cart's weight equal to the centripetal force on it. (The inertia of the cart away from the center of the bridge "equals" the centripetal force on it, which in turn equals the cart's weight.)

$$\mathbf{F_{WT} = F_C}$$
$$m\boldsymbol{g} = m\mathbf{v^2/r}$$

Canceling the mass results in the following:

$$\mathbf{v} = \sqrt{\mathbf{gr}}$$
$$= \sqrt{(9.8\,\text{m/s}^2)(10\,\text{m})}$$
$$\mathbf{= 9.9\ m/s}$$

Note that the mass is inconsequential.

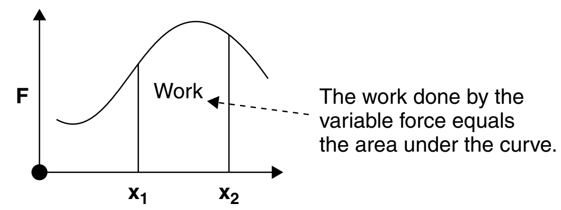
Variable Forces: Elastic Forces and Springs

In considering variable forces, the most common are springs and variations of elastic forces, both being functions of the spring constant, k, and the stretch displacement, **x**. Since **F** = –k**x** (Hooke's Law: Where **F** is the restoring force, k is the spring constant, or springiness of the spring, **x** is the stretch displacement, and the negative sign signifies that the spring pushes back in the opposite direction), the work done *on* or *by* a spring is given by $\mathbf{W = \overline{F}d}$**,** where **W** is work in Joules, $\mathbf{\overline{F}}$ is the average force exerted on or by the spring, and **d** is displacement.

Variable Forces (C Exam)

(Work is covered in a later chapter, along with momentum, energy, and power.) For work done by a variable force, it is necessary to take the definite integral of the force over the extent of displacement. For a variable force **F**, which acts from point $\mathbf{x_1}$ to point $\mathbf{x_2}$, the work done would be as follows:

$\mathbf{W} = \int_{x_1}^{x_2} \mathbf{F}(\mathbf{x})\,\mathbf{dx}_2$. The graph of force **F** versus displacement **x** is given by the following:



Example 1 (C Exam)

A certain spring of negligible mass has a spring constant k of 10 N/m. How much work is required to stretch it 20 cm from its rest position?

Solution

Use the following formula:

$$\begin{aligned}\mathbf{W} &= \int_0^{.2} -\mathrm{k}\mathbf{x}\mathrm{d}\mathbf{x} \\ &= -(\tfrac{1}{2})\mathrm{k}\ \mathbf{x}^2 \Big|_0^{.2} \\ &= (-\tfrac{1}{2})(10\mathrm{N/m})(0.2\mathrm{m})^2\end{aligned}$$

Since the work is done by an applied force, the work is positive:

$$\mathbf{= 0.2\ J}$$

Example 2 (B Exam)

A certain spring of negligible mass has a spring constant k of 10 N/m. How much work is required to stretch it 20 cm from its rest position?

Solution

Using Hooke's Law and the work definition, $\mathbf{W} = \overline{\mathbf{F}}\mathbf{d}$, **W** = The average spring force (**F**/2) times the distance stretched. Since the maximum spring force is **F** = –k**x**, the average force is **F**/2 = –k**x**/2. Work is done on the spring and is therefore positive:

$$(-\mathrm{k}\mathbf{x}/2)(\mathbf{x}) = |(-10\ \mathrm{N/m})(0.20\ \mathrm{m}/2)(0.2\ \mathrm{m})| = \mathbf{0.2\ J}$$