

# Mathematics: Content Knowledge Practice Test 1

**Directions:** Each of the questions or incomplete statements is followed by four suggested answers or completions. Select the one that is best in each case and then fill in the corresponding lettered space on the answer sheet.

1. Rose took a cab from the bus station to her home. She gave the cab driver \$20, which included the fare and a tip of \$2. The cab company charges \$3 for the first mile and 75 cents for each additional  $\frac{1}{2}$  mile. How many miles is Rose's home from the bus station?

A. 10 miles  
B. 11 miles  
C. 20 miles  
D. 21 miles

2. For which of the following expressions is  $a + b$  a factor?

I.  $a^2 - b^2$   
II.  $a^2 + b^2$   
III.  $a^3 - b^3$   
IV.  $a^3 + b^3$

A. I and II only  
B. I and IV only  
C. II and III only  
D. III and IV only

3. A student needs an average of at least 80 on four tests to earn a grade of B in algebra. The student has grades of 78, 91, and 75 on the first three tests. What is the *lowest* grade the student can make on the fourth test and still receive a B in the algebra class?

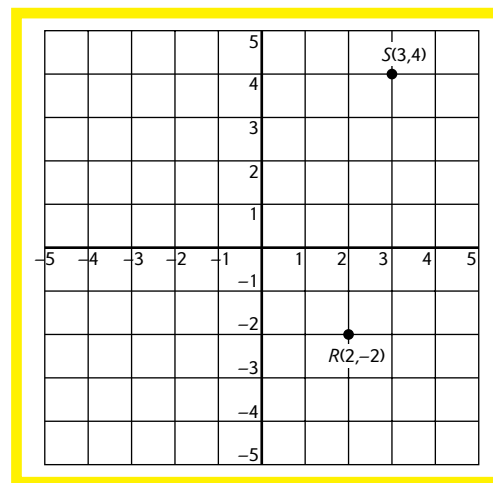
A. 99  
B. 82  
C. 80  
D. 76

4. Which of the following statements can be proven using the principle of mathematical induction?

A.  $\sin^2\theta + \cos^2\theta = 1$ , where  $\theta$  is a real number.  
B.  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ , where  $x$  is a real number.  
C.  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ , where  $n$  is a natural number.  
D.  $\int_a^b f(x) dx = F(b) - F(a)$ , where  $F'(b) = f(x)$  and  $a, b$  are real numbers.

5. Which of the following expressions is equivalent to the expression  $(m^2 + 4)^{-\frac{1}{2}}$ ?

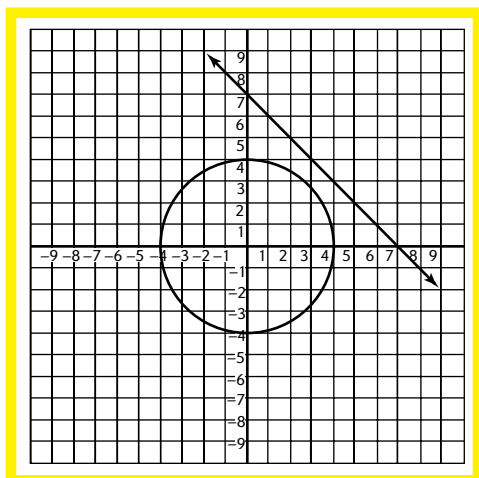
A.  $-\frac{m^2 + 4}{2}$   
B.  $-\sqrt{m^2 + 4}$   
C.  $\frac{1}{\sqrt{m^2 + 4}}$   
D.  $\frac{1}{m + 2}$



6. What is the distance between the two points R and S in the graph shown here?

A.  $\sqrt{37}$  units  
B.  $\sqrt{29}$  units  
C. 7 units  
D.  $\sqrt{49}$  units

GO ON TO THE NEXT PAGE



7. Which of the following statements is true about the solution set of the system of equations represented by the graphs shown?

- A. The system of equations has no solution because the two graphs do not intersect.
- B. The solution set is  $\{-4, 4, 7\}$ .
- C. An  $x$ -value that satisfies the system is  $\frac{7}{2} + i\frac{\sqrt{41}}{2}$ .
- D. An  $x$ -value that satisfies the system is  $\frac{7}{2} - i\frac{\sqrt{17}}{2}$ .

8. Which property of the complex numbers is illustrated here?

$$(5 - 3i) + (2 + 6i) + (1 - 2i) = (7 + 3i) + (1 - 2i) = (5 - 3i) + (3 + 4i) = 8 + i$$

- A. the distributive property
- B. the associative property of addition
- C. the commutative property of addition
- D. the existence of an additive inverse

9. Using a digital thermometer, every morning at 8 A.M. for 5 days, a scientist measures the temperature in degrees Celsius of a lake. The temperature readings are  $10.6^\circ$ ,  $9.2^\circ$ ,  $9.1^\circ$ ,  $10.8^\circ$ , and  $10.3^\circ$ . Before the temperature measurements are numerically summarized, the scientist discovers that the digital thermometer used is off by 0.7 degrees. Which of the following is a true statement that can be made about the scientist's data?

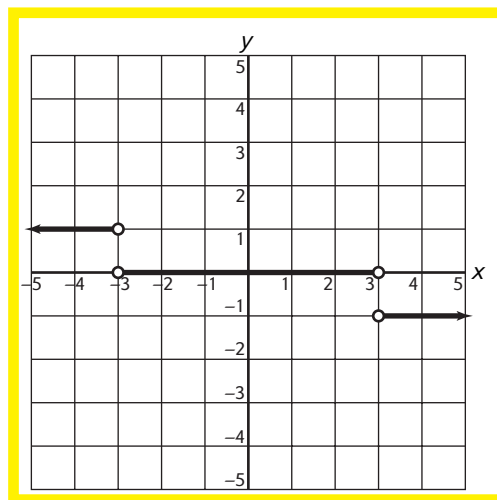
- A. The data are unreliable.
- B. The data are imprecise.
- C. The data are inconclusive.
- D. The data are inaccurate.

10. A rectangular-shaped swimming pool is 50 feet wide and 60 feet long with an uneven bottom surface. The table that follows shows the depth of the water at intervals of 10 feet.

<i>Interval</i>	<i>Depth</i>
0 feet	3.0
10 feet	5.1
20 feet	7.5
30 feet	9.6
40 feet	10.5
50 feet	11.5
60 feet	12.0

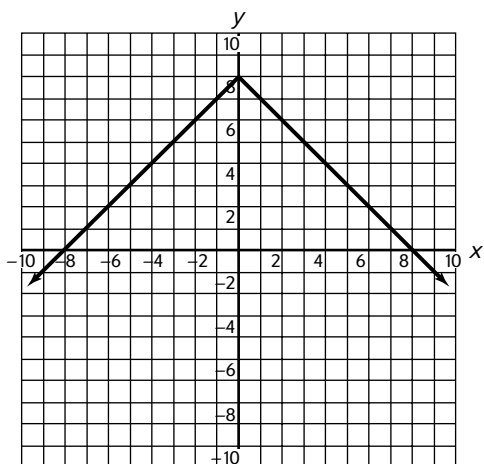
Using the data in the table, the best approximation of the volume of the water in the pool is:

- A.  $22,500 \text{ ft}^3$ .
- B.  $23,600 \text{ ft}^3$ .
- C.  $25,850 \text{ ft}^3$ .
- D.  $28,100 \text{ ft}^3$ .

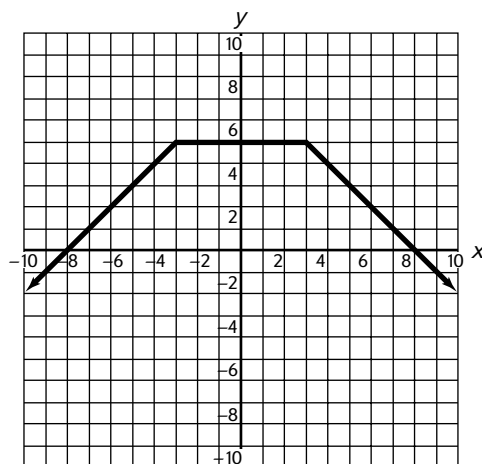


32. The figure is a graph of  $y = f(x)$ . Which of the following graphs is a possible representation of  $f'$ ?

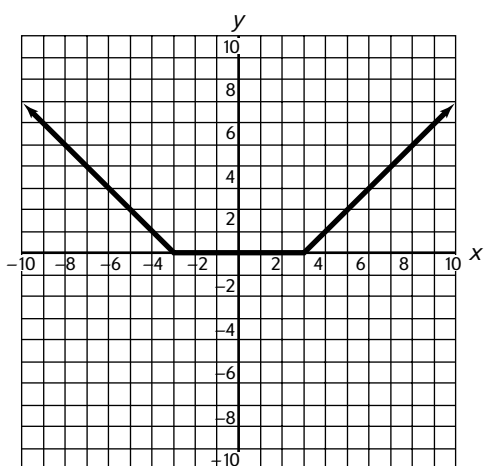
A.



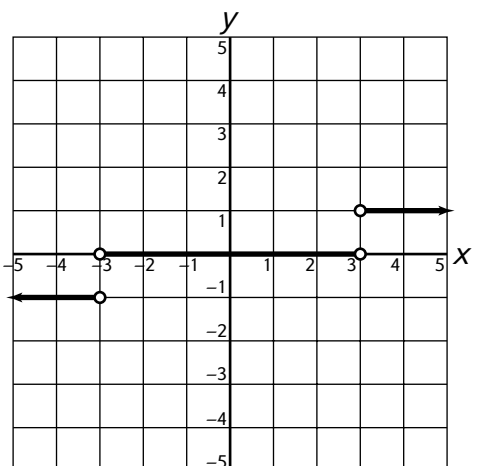
C.



B.



D.



23. A. The domain of  $y = \frac{x^2 + x - 2000}{100}$  is the set consisting of all its possible  $x$ -values; that is,  $D_f = \{x \mid 100 < x < 400\}$ . The range of  $y = \frac{x^2 + x - 2000}{100}$  is the set consisting of all its possible  $y$ -values; that is,  $R_f = \{y \mid \frac{100^2 + 100 - 2000}{100} < y < \frac{400^2 + 400 - 2000}{100}\} = R_f = \{y \mid 81 < y < 1584\}$ . Thus, Choice A is the correct response.

**Tip: After you compute the  $y$ -value corresponding to  $x = 100$ , use the Recall Entry feature of your graphing calculator to compute the  $y$ -value corresponding to  $x = 400$ .**

Choices B and C are incorrect because these choices use  $y$ -values for the domain instead of  $x$ -values. Choice D is incorrect because the range of the function is incorrect in this answer choice.

24. D. Only the statement given in Choice D will always be true about the engineer's function. By definition, each first component (temperature value) is paired with one and only one second component (current value). None of the other statements are guaranteed to be true about the engineer's function.
25. D. A graph of a function must pass the vertical line test: If any vertical line can be drawn so that it cuts the graph of a relation in more than one point, the relation is *not* a function. Examining the graph, you can see that the graph passes the vertical line test when  $x$  is any real number. Therefore,  $R$  is a function on the domain  $= \{x \mid x \text{ is a real number}\}$ , Choice D. **The domains given in the other answer choices are too restrictive.**
26. C. Subtracting a positive constant  $h$  from  $x$  will result in a horizontal shift of  $h$  units to the right. The graph of  $f(x) = (x - 5)^3$  is the same as the graph of  $f(x) = x^3$  shifted right by 5 units. Eliminate choices A and B because these shifts are vertical. Eliminate D because the shift is to the left, not to the right.

**Tip: If you are unsure whether the shift is to the right or left, graph the two functions on your graphing calculator to check.**

27. B. Analyze the problem. Lake A starts at a population of 1000 (in 2005) and increases an average of  $20t$  fish after  $t$  years. Lake B starts at a population of 2000 (in 2005) and decreases an average of  $15t$  fish after  $t$  years. Devise a plan. Write an expression that represents the bass population in each lake  $t$  years after 2005, and set the two expressions equal to each other:  $1000 + 20t = 2000 - 15t$ , Choice B. The other answer choices result if you analyze the problem incorrectly.
28. A. Everywhere you have  $x$  in  $g(x) = x + 2$ , you must substitute  $\frac{2x+6}{x+2}$ , and then simplify, if possible:

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{2x+6}{x+2}\right) = \frac{2x+6}{x+2} + 2 = \frac{2x+6}{x+2} + \frac{2(x+2)}{x+2} = \frac{2x+6}{x+2} + \frac{2x+4}{x+2} = \frac{4x+10}{x+2}, \text{ Choice A.}$$

Choice B is what you get if you find  $(f \circ g)(x) = f(g(x))$  instead of  $(g \circ f)(x) = g(f(x))$ . Choice C results if you make a simplifying error when finding  $(g \circ f)(x) = g(f(x))$ . Choice D results if you make the mistake of finding  $f(g(x))$  instead of  $g(f(x))$ , and you make a simplifying error as well.

29. D. Analyze the problem. The phrase "a long extended period of time" is a clue that this is a calculus problem in which you need to find the limit of a function as the variable approaches infinity. Devise a plan. To answer the question, you will need to find the limit of the function  $Q(d) = \frac{5(6d+14)}{d+7}$  as  $d$  approaches infinity:

$$\lim_{d \rightarrow \infty} \frac{5(6d+14)}{d+7} = \lim_{d \rightarrow \infty} \frac{30d+70}{d+7} = \lim_{d \rightarrow \infty} \frac{30 + \frac{70}{d}}{1 + \frac{7}{d}} = \frac{30+0}{1+0} = \frac{30}{1} = 30 \text{ units, Choice D. Choices A and C result if}$$

you evaluate the limit as  $d$  approaches infinity incorrectly. Choice B results if you find the limit as  $d$  approaches zero, instead of the limit as  $d$  approaches infinity.

30. A. The value of the first derivative at each point  $x_1, x_2, x_3,$  and  $x_4$  is the instantaneous rate of change of the acceleration curve at that point. The acceleration is changing most rapidly at point  $x_1$  because the magnitude of the change is greatest at this point, Choice A.