Precision, Accuracy, and Approximate Error

For this topic, you must be able to analyze precision, accuracy, and approximate error in measurement situations (*Mathematics: Content Knowledge (0061) Test at a Glance*, page 3).

In the physical world, measurement of continuous quantities is always approximate. The precision and accuracy of the measurement relate to the worthiness of the approximation.

Precision refers to the degree to which a measurement is repeatable and reliable; that is, consistently getting the same data each time the measurement is taken. The precision of a measurement depends on the magnitude of the smallest measuring unit used to obtain the measurement (for example, to the nearest meter, to the nearest centimeter, to the nearest millimeter, and so on). In theory, the smaller the measurement unit used, the more precise the measurement.

Accuracy refers to the degree to which a measurement is true or correct. A measurement can be precise without being accurate. This can occur, for example, when a measuring instrument needs adjustment, so that the measurements obtained, no matter how precisely measured, are inaccurate.

The amount of error involved in a physical measurement is the **approximate error** of the measurement. The **maximum possible error** of a measurement is half the magnitude of the smallest measurement unit used to obtain the measurement. For example, if the smallest measurement unit is 1 inch, the maximum possible error is 0.5 inch.

The most accurate way of expressing a measurement is as an interval. For instance, a measurement of 10 inches, to the nearest inch, should be reported as 10 inches ± 0.5 inches. In other words, the true measurement lies between 9.5 inches and 10.5 inches. Closer approximations can be obtained by refining the measurement to a higher degree of precision (for example, by measuring to the nearest half-inch).

Results of calculations with approximate measurement should not be reported with a degree of precision that would be misleading, that is, suggesting a degree of accuracy greater than the actual accuracy that could be obtained using the approximate measurements. Generally, such calculations should be rounded to have the same precision as the measurement with least precision in the calculation.

Informal Approximation Concepts

For this topic, you must be able to apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations (*Mathematics: Content Knowledge (0061) Test at a Glance*, page 3).

Methods of **successive approximation** can be used to approximate the area of plane regions. The area of the region is approximated using the sum of the areas of a sequence of rectangles. One technique is to partition the region in two different ways—so that one partitioning overestimates the area, yielding an **upper bound,** and the other partitioning underestimates the area, yielding a **lower bound.** Sequences of increasingly accurate approximations are obtained by refining the precision of the partitioning. The upper and lower bounds get increasingly close to each other, and their average approaches the true area of the plane region. The volume of a solid can be approximated in a similar manner using the sum of the volumes of a sequence of geometric solids.

Sigma notation is an abbreviated way to represent the sums of the areas of the rectangles or the sums of the volumes of the geometric solids. For instance, a Riemann sum for the area under the curve of a continuous function on the interval $[a, b]$, using n rectangles of equal length, n x_i, has the form $\sum_{i=1}^{n} f(\varepsilon_i)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and $\{\varepsilon_i \mid x_{i-1} \le \varepsilon_i < x_i,$ $i = 1, 2, \ldots, n\}$ are an associated network of points such that the point ε_i can be any point in the subinterval $[x_{i-1}, x_i]$; but, for convenience, can consist of the left endpoints of each subinterval, or the right endpoints of each subinterval, or the midpoints of each subinterval, or so on.