The techniques of algebra and the powerful tools of calculus are needed for dealing with continuous processes. For example, functions with graphs that have no gaps, jumps, or holes are continuous functions. Applications of continuous functions are found in many fields such as in the sciences and in business and industry.

# Modeling and Solving Problems

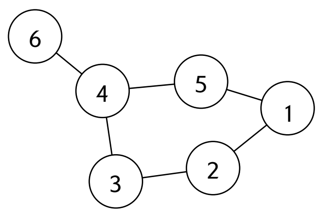
For this topic, you must be able to use difference equations, vertex-edge graphs, trees, and networks to model and solve problems (*Mathematics: Content Knowledge Test (0061) at a Glance*, page 6).

A **difference equation** is an equation that describes sequential, step-by-step change. A difference equation has the form $f(n + 1) - f(n) = g(n)$, where $n$ is a positive integer.

**Note: A difference equation is the discrete mathematics analog to the first derivative equation given by $f'(x) = g(x)$. (See the section titled "Derivatives" in the chapter titled "Calculus" for a discussion of the first derivative $f'(x)$.)**

A difference equation for the first difference, call it $\triangle a_n$, of a sequence $\{a_n\}$ is given by $a_{n+1} - a_n = \triangle a_n$. The first difference describes the rate of growth or decline of the sequence. When $\triangle a_n$ is positive, the terms of the sequence are increasing; when $\triangle a_n$ is negative, the terms of the sequence are decreasing. When $\triangle a_n$ is constant, the sequence is growing or declining at a constant rate and the relationship between terms is linear. The **second difference** of a sequence is given by $\triangle a_{n+1} - \triangle a_n$. When the second difference is constant, the relationship between terms of the sequence is quadratic.

A **vertex-edge graph** is a discrete structure consisting of a nonempty, finite set of **vertices** (also called **nodes**) and a set of **edges** (also called **lines**) **connecting** these vertices. Vertex-edge graphs are commonly used to model and solve problems involving optimal situations for networks, paths, schedules, and relationships among finitely many objects. The following figure is a vertex-edge graph with six vertices and six edges.




A **vertex** that meets an edge is an **endpoint** of that edge, and the edge is said to be **incident** with that vertex.

Two edges are **adjacent** if they share a common vertex. Two vertices are **adjacent** if they are joined by a common edge.

A **loop** is an edge that is incident with exactly one vertex; that is, both endpoints of the edge are the same.

A **link** is an edge that has two different endpoints.

The **degree** of a vertex is the number of edges that are incident to that vertex (with loops being counted twice).

The **Degree Sum Formula** states that the sum of the degrees of the vertices of a graph is an even number equal to twice the number of edges of the graph. Consequently, there are no graphs with an odd number of vertices of odd degree.

A **path** in a vertex-edge graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex. A **simple path** is a path in which no edge is repeated.

A **circuit** in a vertex-edge graph is a path that begins and ends at the same vertex. A **simple circuit** is a circuit in which no edge is repeated.