Order of Operations

First, operations enclosed in Parentheses (or other grouping symbol, if present)
Next, Exponentiation
Then, Multiplication and Division in the order in which they occur from left to right
Last, Addition and Subtraction in the order in which they occur from left to right

Tip: Note that multiplication does not have to be done before division, or addition before subtraction. You multiply and divide in the order they occur in the problem. Similarly, you add and subtract in the order they occur in the problem. That's why there are parentheses around MD and AS in PE(MD)(AS).

The rules for performing operations with complex numbers follow. Keep in mind that since the coefficients x and y in a complex number $x + yi$ are real numbers, the computations involving the real number coefficients must adhere to Rules 1 through 10 given previously.

Rule 1. Addition of two complex numbers: $(x + yi) + (u + vi) = (x + u) + (y + v)i$

Tip: Add the real parts. Add the imaginary parts.

Rule 2. Subtraction of two complex numbers: $(x + yi) - (u + vi) = (x - u) + (y - v)i$

Tip: Subtract the real parts. Subtract the imaginary parts.

When multiplying complex numbers, it is important to remember that $i^2 = -1$.

Rule 3. Multiplication of two complex numbers: $(x + yi)(u + vi) = (xu - yv) + (xv + yu)i$

Tip: Use F.O.I.L. (first terms, outer terms, inner terms, and last terms) to perform the multiplication.

The complex numbers $x + yi$ and $x - yi$ are **complex conjugates** of each other. The product of a complex number and its conjugate is a real number. Specifically, $(x + yi)(x - yi) = x^2 + y^2$. This concept is used in the division of complex numbers.

Rule 4. Division of two complex numbers: $\frac{x+yi}{u+vi} = \frac{(x+yi)(u-vi)}{(u+vi)(u-vi)} = \frac{(xu+yv)+(yu-xv)i}{u^2+v^2} = \frac{(xu+yv)}{u^2+v^2} + \frac{(yu-xv)}{u^2+v^2}i$

Tip: Multiply the numerator and denominator by the conjugate of the denominator.

Properties of Number Systems

For this topic, you must be able to compare and contrast properties (for example, closure, commutativity, associativity, distributivity) of number systems under various operations (*Mathematics: Content Knowledge (0061) Test at a Glance*, page 3).

The set of real numbers has the following **field properties** under addition and multiplication:

Closure Property: $a + b$ and ab are real numbers.
Commutative Property: $a + b = b + a$ and $ab = ba$.

1. Look for a greatest common monomial factor.
2. If a factor is a binomial, check for
   difference of two squares: $x^2 - y^2 = (x + y)(x - y)$
   sum of two cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
   difference of two cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.
3. If a factor is a trinomial, check for
   general factorable quadratic: $x^2 + (a + b)x + ab = (x + a)(x + b)$
   $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
   perfect trinomial square: $a^2x^2 + 2abxy + b^2y^2 = (ax + by)^2$
   $a^2x^2 - 2abxy + b^2y^2 = (ax - by)^2$
4. If a factor has four terms, look for a grouping arrangement that will work for factoring by grouping.
5. Write the original polynomial as the product of all the factors obtained. Check to make sure that all polynomial factors except monomial factors are prime.
6. Check by multiplying the factors to obtain the original polynomial.

A **rational expression** is an algebraic fraction in which both the numerator and denominator are polynomials. Values for which the denominator is zero are **excluded.** The principles used in the study of arithmetic fractions are generalized in work with rational expressions.

To **multiply** rational expressions, factor completely, cancel (meaning "divide out") common factors, and then multiply the remaining numerator factors to obtain the numerator of the product and the remaining denominator factors to obtain the denominator of the product. Canceling is permitted for factors only—do not cancel terms! To **divide** rational expressions, multiply the dividend by the reciprocal of the divisor.

To **add** or **subtract** rational expressions, find a common denominator, express each term as an equivalent rational expression having the common denominator, and then add or subtract numerators, writing the result over the common denominator, and simplify.

A **complex fraction** is a fraction that contains fractions in its numerator or denominator or both. A complex fraction can be simplified by treating it as a division problem or by multiplying its numerator and denominator by the least common denominator of the fractions they contain.

Every positive number has two square roots that are equal in absolute value and opposite in sign. The *positive* square root is called the **principal square root** of the number. If $a^n = x$, a is called an ***n*th root** of x, where n is a natural number. A *positive* real number x has exactly one real positive nth root whether n is even or odd; and *every* real number x has exactly one real nth root when n is odd. Negative numbers do not have real nth roots when n is even. If n is a natural number, the nth root of zero is zero. The quantity $\sqrt[n]{a}$ is called a **radical;** a is called the **radicand;** n is called the **index** and indicates which root is desired. If no index is written, it is understood to be 2, and the radical expression indicates the *principal* square root of the radicand.

We have the following rules for radicals when x, y ∈ real numbers, m, n ∈ positive integers, and the radical expression denotes a real number:

Rules for Radicals

$\sqrt[n]{x^n} = x$ if n is odd

$\sqrt[n]{x^n} = |x|$ if n is even

$\sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\left(\sqrt[n]{x}\right)\left(\sqrt[n]{y}\right) = \sqrt[n]{xy}$

$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}, (y \neq 0)$

$\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$

$\sqrt[pn]{x^{pm}} = \sqrt[n]{x^m}$

$a\left(\sqrt[n]{x}\right) + b\left(\sqrt[n]{x}\right) = (a + b)\left(\sqrt[n]{x}\right)$